Magnetic-Field-Dependent Transmission Phase of a Double-Dot System in a Quantum Ring

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The Aharonov-Bohm effect is measured in a four-terminal open ring geometry. Two quantum dots are embedded in the structure, one in each of the two interfering paths. The number of electrons in the two dots can be controlled independently. The transmission phase is measured as electrons are added to or taken away from the individual quantum dots. Although the measured phase shifts are in qualitative agreement with theoretical predictions, the phase evolution exhibits unexpected dependence on the magnetic field. Phase lapses are found only in certain ranges of the magnetic field.

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The phase difference of interfering paths in coherent quantum rings can be detected in the conductance by measuring Aharonov-Bohm (AB) oscillations. Keeping the transmission phase of one path constant (reference path), the phase change of the other can be measured. This technique opens the possibility to investigate the transmission phase which contains information complementary to the transmission probability. Multiterminal devices avoid the restriction found in two-terminal devices, where the phase is locked to 0 or π [1].

Ringlike interference geometries have been exploited for many experiments [2–12]. Electronic phase studies were pioneered in Refs. [2–6]. Partial phase coherence of electron transport through a Coulomb blockaded quantum dot was demonstrated in a two-terminal device [2]. The phase of the reflection coefficient of a quantum dot in the quantum Hall regime was measured in Ref. [3]. The expected transmission phase change of π over a quantum dot transmission resonance was demonstrated in a multiterminal configuration [4]. Between resonances phase lapses were found rather than the almost constant phase expected in the simplest model. Similar experiments were carried out on a Kondo-correlated system [5,6].

The issue of the phase lapses has been addressed by theory (for a review, see [13]). The transmission phase has to be distinguished from the Friedel phase which fulfills a general sum rule and is a monotonic function of energy, whereas the transmission phase can show phase lapses of π when the transmission goes through zero [14–16]. The measured phase may also depend on the details of the entire interferometer, on interactions [17], and on the dot-lead coupling [18]. The phase determined experimentally is the transmission phase through the dot, only if the system couples negligibly to its environment [19]. A tunneling-Hamiltonian approach predicted the phase to be influenced by the number and width of leads connecting to the ring [20,21]. In a scattering matrix approach, the limit of many occupied modes in the channels reflects the energy dependence of the resonance phase shift [22]. The equivalence of the tunneling-Hamiltonian formulation and the scattering matrix approach are discussed in [23]. Theoretical proposals have considered the situation of two quantum dots in an AB interferometer and the possible coupling of the dots via phase coherent transport through the leads [24–26]. Coupled quantum dots in an AB interferometer may be used for detecting entanglement of spins [27].

Here we investigate the phase evolution of a system of two quantum dots with negligible mutual electrostatic interaction embedded in two arms of a four-terminal AB ring. Phase measurements in this unique system are desirable, because both arms of the ring including the two dots can be tuned individually, but the phase coherence of the entire system provides an inherent coupling mechanism [24] and makes transport through the dots interdependent. The arrangement enables us to control and analyze the effect of the interferometer reference arm on the measured phase. The transmission phase is studied when a single electron is added to either of the two or to both quantum dots, or interchanged between dots. At elevated magnetic fields the results agree qualitatively with theoretical expectations. Unexpectedly, the phase evolution is found to depend on the magnetic field and occasional phase lapses occur in certain field ranges. These observations highlight the need to consider nonlocal coherent effects in the entire system.

The sample is a Ga[Al]As heterostructure with a two-dimensional electron gas (2DEG) 37 nm below the surface. Oxide lines were written with atomic-force microscopy (AFM) lithography [28] as shown in Fig. 1(a). Lateral gate electrodes pg1–4 tune the conductance in each of the four ring quadrants. Measurements were carried out at 100 mK.

In the open regime each quadrant supports two to four modes. The AB effect is observed in a setup in which the current and voltage contacts are the same two terminals

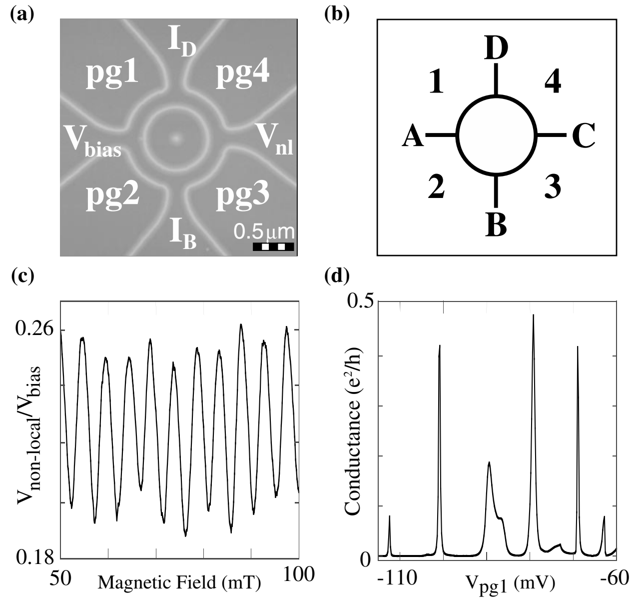

FIG. 1. (a) AFM micrograph of the ring structure. Bright oxide lines fabricated by AFM lithography lead to insulating barriers in the 2DEG. (b) Schematic arrangement of the four terminal ring. (c) AB oscillations in V_{nl}. (d) Conductance through a quantum dot induced in segment 1 as a function of V_{pg1}.

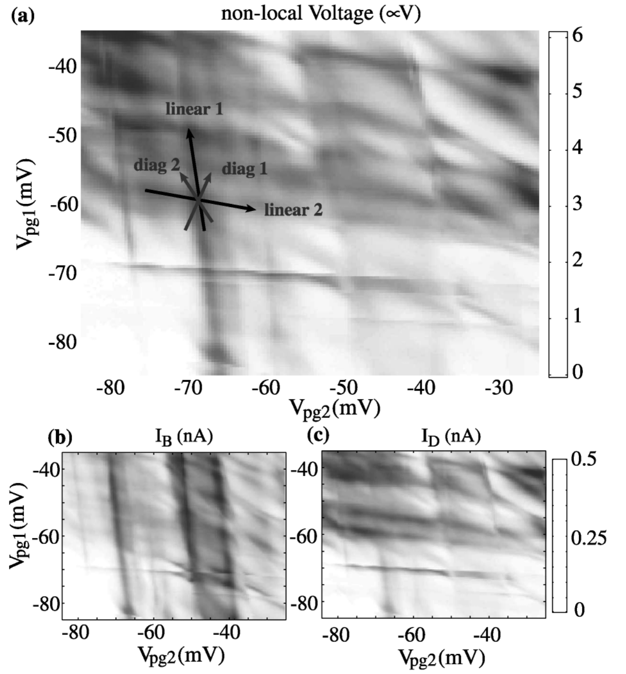

FIG. 2. Plots of (a) the nonlocal voltage V_{nl}, (b) the currents I_B and (c) I_D as functions, respectively, of V_{pg1} and V_{pg2} tuning the dots in segments 1 and 2.

[29]. The AB signal is maximized with a nonlocal setup: a bias voltage V_{bias} was applied to terminal A [Fig. 1(b)]. Contacts B and D were grounded via current-voltage converters measuring the currents I_B and I_D. Below we call V_{nl} nonlocal voltage since there is no net current flow through terminal C. Figure 1(c) shows AB oscillations in V_{nl} with period $\Delta B = 4.8$ mT consistent with the ring area $A = 0.85\ \mu m^2$.

If any segment is tuned close to pinch off, a Coulomb blockaded quantum dot is induced. As an example, Fig. 1(d) shows the conductance of segment one [A to D in Fig. 1(b)] as a function of V_{pg1} with segment three completely pinched off and segments two and four open. The quantum dot location can be estimated from the lever arms of gates pg1–4. The dots form within the segments and not at the openings to the contacts. From Coulomb blockade diamonds we find a typical charging energy of 1 meV, corresponding, in a disk capacitor model, to the area of a single segment. Details about the characterization of the ring can be found in Ref. [29].

Coulomb blockade is studied in the configuration of Fig. 1(a). We have chosen conductance peaks of one dot in segment one (dot 1) and one in segment two (dot 2). Segments three and four are open. The currents I_B and I_D are shown in Figs. 2(b) and 2(c) as functions, respectively, of V_{pg1} and V_{pg2} . The current I_B (I_D) exhibits mainly Coulomb maxima of dot 2 (dot 1), while those of the other dot are much weaker. The voltage V_{nl} in Fig. 2(a) displays conductance resonances of both dots. The lever arm of V_{pg1} (V_{pg2}) on dot 2 (dot 1) is 6 times smaller than for the direct gate voltage. We attribute stripes with an intermediate slope in all three quantities to the formation of standing waves between the quantum dots near lead A. This resonator is coupled in series to both dots, leading to a modulation of the dot currents in certain gate voltage ranges.

For transmission phase studies we measure V_{nl} as a function of magnetic field B. The results were obtained in a regime where the coupling to the leads was strong enough, but the dots were still well defined. Tuning V_{pg1} and V_{pg2} we follow the trace "linear 2" in Fig. 2(a), staying on a conductance maximum in dot 1 while stepping through a conductance peak in dot 2. As shown in Fig. 3(a), I_B shows the expected Coulomb peak of dot 2, while I_D changes little. At each gate voltage, AB oscillations are measured. The voltage V_{nl} in Fig. 3(b) shows an AB signal which is strongest in the range of the conductance maximum. Specific traces of V_{nl} versus B are depicted in Fig. 3(c). Vertical lines connect minima in the lowest trace with maxima in the uppermost trace indicating a phase shift of π.

The multiprobe conductance formula [1] gives $V_{nl} = -(T_{CA}/T_{CC})V_{bias}$. The transmission coefficients T_{ij} from probe j to i are named according to Fig. 1(b). Considering only h/e-periodic oscillations we assume $T_{CC} = -\sum_{i \neq C} T_{Ci} = \text{const}$. In this case AB oscillations in V_{nl} reflect the interference contribution to T_{CA} from which

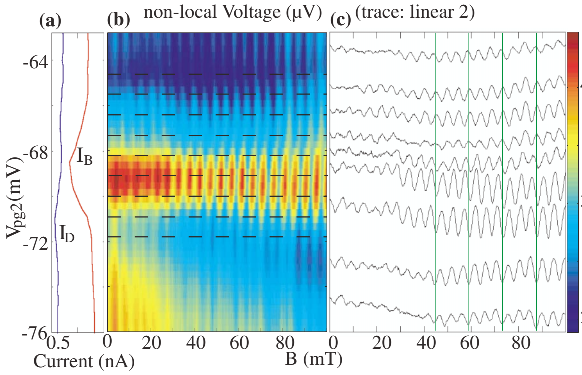

FIG. 3 (color online). AB oscillations in V_{nl} as a function of the plunger gate voltages tuned along trace linear 2. (a) The currents I_D and I_B at $B = 95$ mT. (b) A plot of V_{nl} as a function of plunger gates and magnetic field. (c) Cross sections along dashed lines in (b) giving V_{nl} vs B. The dashed lines mark constant values of B where a π phase shift of the AB oscillations can be seen, as gate voltages are tuned across the conductance peak in dot 2.

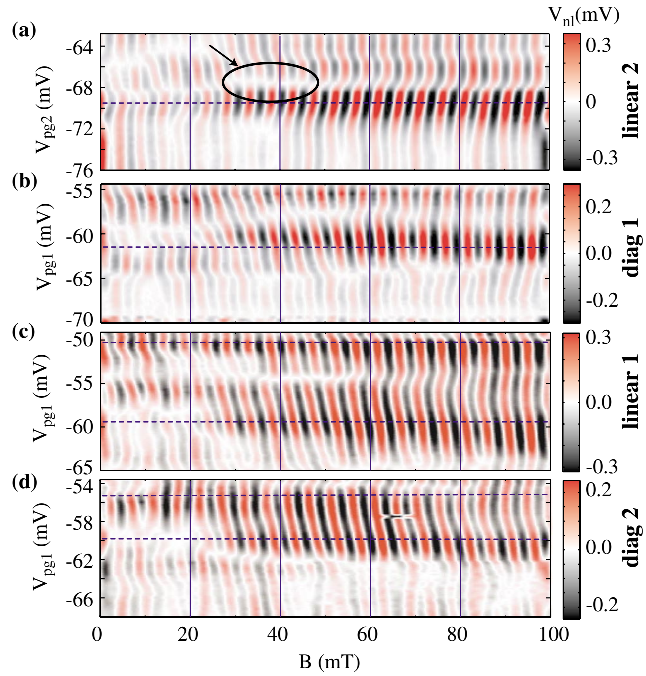

FIG. 4 (color online). AB oscillations along the different traces indicated in Fig. 2. (a) Sweep linear 2 (b) sweep diag 1 (c) sweep linear 1, and (d) sweep diag 2. Vertical lines are guides to the eye. Dashed horizontal lines indicate where maxima in the magnetic field averaged contribution to V_{nl} occur.

the phase difference between the two interfering transmission amplitudes can be directly read [4]. Phase changes seen in the currents do not directly reflect transmission phase differences of the dots. We therefore consider only AB oscillations in V_{nl}.

We have analyzed such phase shifts quantitatively for all four traces indicated by arrows in Fig. 2(a). The fast Fourier transform of each B sweep was multiplied with the filter function $f(\omega) = (\sigma\omega)^2/2\exp[1 - (\sigma\omega)^2/2]$ with $\sigma = h/(e\sqrt{2}\pi A)$ in order to obtain the pure h/e-periodic contribution. The inverse fast Fourier transform of the filtered data gives the oscillatory component of V_{nl} as a function of B plotted in Fig. 4. We have verified that this filtering procedure does not influence the phase of the AB oscillations by carefully comparing filtered data with raw data.

Figure 4(a) corresponds to trace linear 2 in Fig. 2(a). The dashed horizontal line indicates the maximum in V_{nl} averaged over B. The AB amplitude is strongest close to this line, as expected, but displays a pronounced B dependence; i.e., it is significantly larger for $B > 30$ mT than below. The phase shift can be directly read from the shift of AB maxima or minima as a function of V_{pg2}. The general trend is a shift to larger B with increasing V_{pg2}. The expected phase shift of π across the conductance resonance is found for $B > 50$ mT. However, there is a dip in the AB amplitude at $V_{pg2} \approx -68$ mV having a strong associated phase shift. At $B \approx 40$ mT and $V_{pg2} \approx -68$ mV [arrow in Fig. 4(a)] the AB amplitude is zero and a phase lapse of π occurs. It lies in the flank of the peak of the field averaged V_{nl} and at a point of zero AB amplitude, similar to the observations in Ref. [4] and in agreement with theory [15]. The novel aspect is the occurrence of the phase lapse in a limited magnetic field range, beyond which it disappears in favor of a continuous phase evolution of order π.

Along trace "diag 1" in Fig. 2(a) an electron is added to both dots and we expect zero phase shift. The experimental value in Fig. 4(b) is indeed only about $\pi/10$ for $B > 50$ mT. At lower fields phase lapses and related phase shifts occur similar to trace linear 2.

Measuring along trace "linear 1" [Fig. 4(c)] the gross trend of the phase shift differs in sign from that in Fig. 4(a), in agreement with the fact that we tune the dot in the other segment. In the coupling regime of the dots where AB oscillations are observable, Coulomb resonances can occur close to each other. It is the case for this sweep as indicated by the two dashed horizontal lines in Fig. 4(c). In such a case the question arises how the transmission phases of individual resonances combine to the observed phase shift. We find the phase accumulation across the first resonance just above $V_{pg1} = -60$ mV to be about $-\pi$ for B between 30 and 70 mT. At higher fields it weakens. At $V_{pg1} \approx -55$ mV and between -40 and -50 mT, phase lapses influencing the phase evolution at higher magnetic fields occur but cannot compensate the accumulated phase completely. At the top edge of the figure, the second resonance leads to a further phase accumulation in the same direction as the first.

Along trace "diag 2" [Fig. 4(d)] an electron is moved from dot 1 to dot 2. AB oscillation maxima shift in the same direction as in linear 1. However, for example, at

$B = 80$ mT, the phase accumulated between $V_{pg1} = -68$ and -56 mV is only slightly more than π rather than 2π. Again, the phase evolution is influenced by occasional phase lapses.

Summarizing, we have measured the AB phase in a four terminal quantum ring with two dots embedded in two different segments. The AB oscillations are suitable for phase measurements only at slightly elevated B. For $50 < B < 80$ mT, the expected phase shifts across Coulomb resonances are typically observed. Phase lapses occur occasionally at vanishing AB amplitude in finite magnetic field ranges at specific gate voltages. Outside the B ranges where phase lapses occur, the phase evolution can be strongly modified by their presence. Following Taniguchi *et al.* [15,16], the transmission amplitude describes a curve in the complex plane as a function of gate voltage. A phase lapse accompanied with a zero in transmission occurs when this curve runs through the origin. A change in B may shift this curve a little leading to a strong but continuous phase shift accompanied with small transmission. The appearance of phase lapses in finite B intervals impedes the definition of a magnetic field independent transmission phase. A conceivable origin of the phase lapses is the finite width of the ring segments accommodating several modes. In the B range investigated, the classical cyclotron radius is larger or comparable to the ring radius. A small influence of Lorentz force effects cannot be excluded at the highest fields.

Our samples differ from those in Ref. [4] by design and technological approach. We have adopted the general idea of reflecting walls for guiding the electrons around the ring. Compared to Ref. [4] the most noticeable feature of our structure is the second dot in the reference arm. Its addition gives individual control over the accumulated phases in both arms and therefore allows to play off the two transmission phases against each other. Since, e.g., in the measurement labeled linear 2, the reference dot is kept on a conductance resonance, i.e., in the position of maximum slope of the phase evolution, it is very sensitive to changes of its resonance. Should the reference dot be tuned slightly off resonance during the sweep linear 2, one would expect a pronounced influence on the measured phase evolution. The observation of a π phase change on trace linear 2 lets us conclude that the transmission phase through dot 1 is rather stable and the phase shift is dominated by dot 2. Our measurement results and their analysis extend the existing experimental work on phase lapses by showing (i) that the occurrence of phase lapses may depend on magnetic field and (ii) that phase lapses still have a strong influence on the phase evolution at magnetic fields outside the range of their occurrence.

In this experiment, the transmission phase was measured in a system with two Coulomb blockaded quantum dots embedded in a four-terminal quantum ring. The phase evolution could be determined with individual control over the electron occupancy in each dot. The measured transmission phase depends on the magnetic field range analyzed. Phase lapses can arise at certain magnetic fields, while at others they give way to a continuous phase evolution. In certain B intervals the phase behavior is in agreement with theoretical expectations, but it is more complex than anticipated. This suggests that, in addition to the dots, structural resonances play a role for the transmission phase of the system.

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