Decoherence in Superconducting Quantum Bits by Phonon Radiation

L. B. Ioffe,[1] V. B. Geshkenbein,[2,3] Ch. Helm,[2] and G. Blatter[2]
[1]Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854, USA
[2]Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland
[3]L. D. Landau Institute for Theoretical Physics RAS, 117940 Moscow, Russia
(Received 8 April 2004; published 27 July 2004)

We discuss a fundamental limitation for the coherent operation of superconducting quantum bits originating from phonon radiation generated in the Josephson junctions of the device. The time dependent superconducting phase across the junction produces an electric field that couples to the underlying crystal lattice via the piezoelectric effect. We determine the radiation resistance of the junction due to phonon emission and derive substantial decoherence rates for the quantum bits, which are compatible with quality factors measured in recent experiments.

DOI: 10.1103/PhysRevLett.93.057001 PACS numbers: 85.25.Cp, 03.65.Yz, 03.67.–a

Decoherence is the main adversary of the unitary time evolution governing the quantum systems that provide the hardware for a future quantum information technology. Solid state implementations of such "quantum hardware" based on superconducting structures have undergone an amazing development during the last years [1–7], thus underlying their potential for the construction of quantum information processors. Key elements in this type of hardware are the Josephson junctions with their dynamics driving the quantum fluctuations in these devices. In this Letter, we analyze the phonon radiation emitted from these Josephson junctions and determine the associated energy relaxation rate leading to the decoherence of the qubit's quantum state.

Superconducting qubits come in three main varieties: *charge qubits* [1,2,8,9] store the quantum information in the charge states of a small Cooper pair box, while superpositions of macroscopic ring currents with opposite circulation assume this role in *flux/phase qubits* [3–5,10–12]. Finally, *Josephson junction qubits* [6,7] store the information in the internal state of a current biased Josephson junction. The operation of these devices is governed by two energy scales, the Josephson energy $E_J = \hbar I_c/2e$ associated with the current flow and the capacitive energy $E_C = e^2/2C$ due to charging effects; here, I_c is the critical current and C denotes the capacitance. The information is stored in quantum states $|0\rangle$ and $|1\rangle$ residing at energies E_0 and E_1 separated by the operation frequency $\omega_{01} = (E_1 - E_0)/\hbar$, with $\hbar\omega_{01} = E_J$ the Josephson energy in the charge qubit, $\hbar\omega_{01} \equiv \Delta \sim \omega_p(E_J/E_C)^{1/4}\exp(-\sqrt{\alpha E_J/E_C})$ the coherence gap in the flux/phase qubit (the numerical α depends on the qubit design), and $\omega_{01} \equiv \omega_p \sim \sqrt{8E_CE_J}/\hbar$ the plasma frequency in the Josephson junction qubit.

A superposed state $|\Psi\rangle \propto [|0\rangle + \beta e^{-i\omega_{01}t}|1\rangle]$ induces oscillations in the phase φ across the junction. Josephson's relation $V = \hbar\dot{\varphi}/2e$ tells that an electric field $E = V/d$ appears in the junction insulator (of thickness d), which couples to the crystal lattice via the piezoelectric effect; see Fig. 1. The junction then acts as an antenna emitting phonons at the frequency ω_{01}: we show below that this radiation adds substantially to the energy relaxation rate γ_E and thus to the decoherence of the qubit [13].

For a pointlike junction with size L much smaller than the wavelength $\lambda_s = 2\pi c_s/\omega_{01}$ (c_s the sound velocity) the situation is analogous to the familiar dipole radiation in electromagnetism: the capacitive junction with contacts separated by d defines a dipole $D = dQ = dCV$ that radiates the power $\mathcal{P}_d = \omega^4D^2/3c^3 = V^2/2R_d$. This results in a radiation resistance R_d which is quantified through the ratio $R_Q/R_d = (\hbar c/6e^2)(\omega\sqrt{dC}/c)^4$ with $R_Q = \hbar/4e^2 \approx 1\ \mathrm{k}\Omega$ the quantum resistance.

The same junction dissipates energy through phonon emission: the electric field $E = V/d$ in the insulator couples to the displacement field $\mathbf{u}$ via the piezoelectric effect, producing a polarization density $\mathbf{P} = p(\nabla \cdot \mathbf{u})$ and

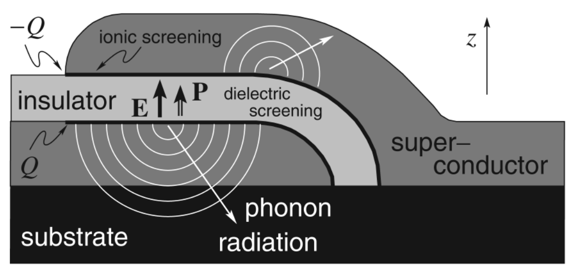

FIG. 1. Phonon radiation from a Josephson junction (schematic): The time dependent phase $\varphi(t)$ induces a voltage $V = \hbar\dot{\varphi}/2e \propto \cos(\omega_{01}t)$ across the junction generating charges $\pm Q = CV$. The electric field $\mathbf{E} = V\mathbf{e}_z/d$ polarizes the insulator (*dielectric screening*) and induces displacements $\mathbf{u}$ via the piezoelectric effect. In addition, displaced metallic ions in the superconductor contribute *ionic screening* of the surface charges $\pm Q$. The oscillating displacement field $\mathbf{u} \propto \cos(\omega_{01}t)$ produces phonon radiation into the substrate, which contributes to the energy relaxation or decoherence of the qubit.

 0031-9007/04/93(5)/057001(4)$22.50

resulting in the energy $\mathcal{E}_{\rm pe} = \mathbf{P}\cdot\mathbf{E}Ad \approx p(\nabla\cdot\mathbf{u})VA$, with $p \sim e/a^2 \sim 1\ \mathrm{A\,s/m^2}$ a typical piezoelectric constant, a the atomic distance, and Ad the junction volume. The associated force density $\mathbf{f} = -\partial_{\rm u}\mathcal{E}_{\rm pe} \approx (AeV/a^2)\nabla\delta(\mathbf{r})$ (approximated as a pointlike dipolar drive) acts on the elastic medium, $\mu\nabla^2\mathbf{u} \approx \mathbf{f}$ with μ an elastic constant, and produces a displacement field $\nabla\cdot\mathbf{u} \sim (eV/a^2\mu d)(Ad/r^3)$. The emitted power is given by the elastic energy accumulated within the near field zone $\lambda_{\rm s}^3$, $\mathcal{P} \sim \omega\mu(\nabla\cdot\mathbf{u})^2|_{\lambda_{\rm s}}\lambda_{\rm s}^3 \sim (e^2/a^4\mu)(dCV\omega^2)^2/c_{\rm s}^3$ with $e^2/a^4\mu \sim 1$ the ratio between the electrostatic and the elastic energy density. As a result, we find an equivalent inverse resistance $R_{\rm Q}/R_{\rm s} \sim (\hbar c_{\rm s}/e^2)(e^2/a^4\mu)\times(\omega\sqrt{dC}/c_{\rm s})^4$ due to sound radiation. With the sound velocity $c_{\rm s}$ much smaller than that of light (c), this phonon radiation produces a small resistance $R_{\rm s}$ and thus dissipates a large power at fixed driving voltage $V = \hbar\dot\varphi/2e$; in our analysis of recent devices we find quality factors comparable to those reported in the experiments [1–7]. This suggests that phonon radiation is ultimately limiting the performance of these qubits.

Numerous sources of decoherence have been analyzed [14–16], among which the coupling to the electromagnetic field has played the dominant role. While the decoherence due to photon radiation from the junction is small, the device has to be protected from external radiation sources, and appropriate measures such as filtering or embedding in electromagnetic cavities are being discussed [17]. The decoherence through phonon radiation [18] originates from within the junctions [19], and proper protection requires their optimization, e.g., through adequate choice of materials and geometry. While the piezoelectric effect is absent in bulk dielectrics with inversion symmetry, it is not possible to eliminate this coupling at the superconductor-insulator boundary where this symmetry is always broken; these surface-induced effects are sufficient to produce substantial decoherence.

Interestingly, decoherence through phonon radiation is absent in qubits with degenerate computational states [11,20,21], at least during idle time. Gate operations pushing the energy levels apart induce decoherence through phonon radiation during manipulation; keeping the operation frequency ω_{01} small reduces this residual radiation and results in an optimization task balancing decoherence due to phonon radiation and other sources.

In order to obtain a more accurate estimate for the decoherence rate due to phonon emission, we analyze the action $S = S_J + S_{\rm ph} + S_{\rm int}$ describing the Josephson junction coupled to the phonon bath. Here,

$$S_J = \int dt\left[\frac{\hbar^2}{4e^2}\,\frac{C}{2}\,\dot\varphi^2 - E_J(1-\cos\varphi)\right]$$

is the usual action for the junction with E_J the Josephson energy and $C = \varepsilon A/4\pi d$ the junction capacitance (ε is the dielectric constant of the insulator, $A = L^2$ is the junction area, and d its width), and the action for the phonon system reads [22]

$$S_{\rm ph} = \int dt\, d^3r\left[\frac{\rho}{2}\dot{\mathbf{u}}^2 - \frac{\lambda+\mu}{2}(\nabla\cdot\mathbf{u})^2 - \frac{\mu}{2}\sum_{i,j}(\partial_i u_j)^2\right],$$

with $\mathbf{u}(\mathbf{r})$ the displacement field, ρ the mass density, and λ, μ the Lamé coefficients; modeling the medium as an isotropic one simplifies the discussion. The interaction between the phase φ and the displacement $\mathbf{u}$ derives from the piezoelectric coupling $S_{\rm int} = \int dt\, d^3r\,\mathbf{P}\cdot\mathbf{E}$. The piezoelectric effect relates the polarization density $\mathbf{P}$ to the displacement field $\mathbf{u}$ via the piezoelectric tensor β_{ijk}, $P_i = \beta_{ijk}\partial_j u_k$. Here, $\mathbf{E} = (0, 0, \hbar\dot\varphi/2ed)$, and, using the simple ansatz $\beta_{3jk} = p\,\delta_{jk}$ with $p \sim e/a^2 \sim 1\ \mathrm{A\,s/m^2}$, a typical piezoelectric constant, we obtain

$$S_{\rm int} = g\int dt\,\dot\varphi\int d^3r\,\sigma(\mathbf{r})(\nabla\cdot\mathbf{u}) \tag{1}$$

with the coupling g and a shape function σ of the form $g \sim \hbar Ap/e$, $\sigma(\mathbf{r}) = \delta(z)\chi_A(\mathbf{R})/A$; χ_A is the characteristic function of the junction area (i.e., $\chi_A = 1$ on the superconductor-insulator interface and 0 else). In addition to this "bulk" term residing in the insulator, the two junction surfaces generate a coupling with $g_{\rm s} \sim \hbar(A\varepsilon/ed)p_{\rm s}$ and $\sigma(\mathbf{r}) = [\delta(z-d/2) - \delta(z+d/2)]\chi_A(\mathbf{R})/A$. This contribution is due to the lattice perturbation at the interface (generating a surface piezoelectric constant $p_{\rm s} \sim e/a$) and the penetration of the electric field into the metal, both extending over a distance of the order of an atomic separation a. The surface term is driven by the total charge Q and thus is proportional to the dielectric constant in the insulator when expressed through the voltage V, while the bulk term involves the bare electric field V/d; hence $g_{\rm s}/g \sim \varepsilon a/d$, and we expect both terms to be of similar magnitude.

The above action generates the dynamical equations

$$\frac{\hbar^2}{4e^2}C\ddot\varphi + E_J\sin\varphi + g\int d^3r\,\sigma(\mathbf{r})(\nabla\cdot\dot{\mathbf{u}}) = 0 \tag{2}$$

for the Josephson phase φ and

$$[\rho\ddot u_i - \mu\nabla^2 u_i] - (\lambda+\mu)\partial_i(\nabla\cdot\mathbf{u}) + g\dot\varphi\partial_i\sigma = 0 \tag{3}$$

for the displacement field $\mathbf{u}(\mathbf{r})$. In order to arrive at an effective dynamical equation for the phase variable φ, we express the phonon field $\mathbf{u}(\mathbf{r})$ through the driver $\propto\dot\varphi$ with the help of the appropriate Green's function $G_{ij}^{\rm ph}(\mathbf{r},\mathbf{r}';t-t')$ for the phonon system. The driven phonon field then takes the form

$$u_i(\mathbf{r},t) = -g\int dt'd^3r'\,\dot\varphi(t')G_{ij}^{\rm ph}(\mathbf{r},\mathbf{r}';t-t')\partial_j\sigma(\mathbf{r}') \tag{4}$$

and produces an additional dissipative term in the equation of motion for the junction,

$$\frac{\hbar C\ddot{\varphi}}{2e} + I_c \sin\varphi + \frac{\hbar\dot{\varphi}}{2eR} + \frac{\hbar C}{2e}\int dt'\kappa(t-t')\ddot{\varphi}(t') = 0. \tag{5}$$

The kernel κ describing the dissipative action of the phonons assumes the form

$$\kappa(t) = E_g \int d^3r\, d^3r' [\partial_i\sigma(\mathbf{r})]G^{\mathrm{ph}}_{ij}(\mathbf{r},\mathbf{r}';t)[\partial'_j\sigma(\mathbf{r}')] \tag{6}$$

with the coupling energy $E_g = (2eg/\hbar)^2/C$. Equation (8) has the form of a Kirchhoff law adding the current components across the junction, where the first and last terms add up to the displacement current accounting for the dielectric properties of the junction, the second term is the supercurrent and the third term is an additional dissipative shunting current, e.g., due to normal quasiparticles, added for the sake of completeness.

Equation (5) is conveniently written in the (real) frequency domain,

$$\frac{I_c}{2e}\varphi_\omega - i\omega R_{\mathrm{Q}}\left[\frac{1}{R} - i\omega(1+\kappa(\omega))C\right]\varphi_\omega = 0; \tag{7}$$

we have linearized the supercurrent term and have introduced the quantum resistance $R_{\mathrm{Q}} = \hbar/4e^2$; the quantity $1+\kappa(\omega)$ plays the role of an effective dielectric function. The phonon radiation κ adds another (parallel) impedance $Z_{\mathrm{s}}(\omega) = 1/i\omega\kappa(\omega)C$ to the usual resistively and capacitively shunted junction circuit. Its damping then involves the total shunt resistance $[R^{-1}+R_{\mathrm{s}}^{-1}]^{-1}$, with the radiation resistance R_{s} defined through $1/R_{\mathrm{s}} \equiv \mathrm{Re}[1/Z_{\mathrm{s}}] = \omega\kappa''C$ and $\kappa'' = \mathrm{Im}\,\kappa$. Assuming a high resistance shunt $R/R_{\mathrm{Q}} \to \infty$, we find that the plasma frequency picks up an imaginary part, $\omega_p + i\gamma_E/2 = \omega_p(1-i\kappa''/2)$, and we arrive at the quality factor

$$Q_J = \omega_p/\gamma_E = 1/\kappa''(\omega_p). \tag{8}$$

The result (8) quantifies the radiation-induced decoherence of the Josephson qubit; the corresponding expressions for the flux/phase and charge qubits read

$$Q_\Phi \approx E_C/\Delta\kappa''(\omega_\Delta), \qquad Q_q \approx E_J/E_C\kappa''(\omega_J). \tag{9}$$

These results follow from the estimate $E_{\mathrm{diss}} = \oint dt\, V^2/2R_{\mathrm{s}} \approx (R_{\mathrm{Q}}/R_{\mathrm{s}})\hbar\omega(\delta\varphi)^2$ for the energy dissipated during one cycle, where $\delta\varphi$ denotes the width of the quantum state. The ratio $\hbar\omega/E_{\mathrm{diss}}$ then provides the quality factor $Q \approx (R_{\mathrm{s}}/R_{\mathrm{Q}})/(\delta\varphi)^2$; for the Josephson qubit the phase is trapped in one valley, $(\delta\varphi)^2 \sim \sqrt{8E_C/E_J}$, and we recover (8). For a flux qubit the phase is delocalized between two valleys and hence $(\delta\varphi)^2 \approx \pi^2$; the quality factor Q_Φ then involves the coherence gap Δ. For the charge qubit the sharp variable is the dimensionless charge $q = Q/2e$ oscillating by unity; replacing in E_{diss} the phase with the charge variable we find the quality factor $Q \sim E_J/E_C\kappa''$.

Next, we determine the response function κ. For a precise calculation, various specific device elements, such as the junction material, its geometry, and its embedding, have to be taken into account. These various elements go into the determination of the Green's function G^{ph} describing the elastic properties of the device. Furthermore, the surface drive involves two sources of opposite sign placed a distance d apart, leading to a partial compensation in the displacement fields $\mathbf{u}$ within the superconductor. However, a hard insulator pinned at the junction edges effectively decouples the two source terms, and we concentrate on this situation. We proceed with the analysis of the generic case with one δ-function drive embedded within an infinite homogeneous and isotropic elastic medium. The calculation of the response function is conveniently done in Fourier space,

$$\kappa(\omega) = E_g \int \frac{d^3q}{(2\pi)^3}\sigma(-\mathbf{q})q_i[G^{\parallel}_{ij}(\omega,\mathbf{q})]q_j\sigma(\mathbf{q}), \tag{10}$$

and involves only the longitudinal part $G^{\parallel}_{ij}(\omega,\mathbf{q}) = (q_iq_j/q^2)[(2\mu+\lambda)q^2 - \rho\omega^2]^{-1}$ of the elastic Green's function G^{ph}. We assume a circular junction with an area of radius R_0 for convenience. For the imaginary part of the response function, we find the result

$$\kappa'' = \frac{E_g\omega}{2(2+\lambda/\mu)A\mu c_{\mathrm{s}}}\left[1 - \frac{c_s}{\omega R_0}J_1(2\omega R_0/c_{\mathrm{s}})\right], \tag{11}$$

with $c_{\mathrm{s}} = \sqrt{(2\mu+\lambda)/\rho}$ the (longitudinal) sound velocity and J_1 a Bessel function. Inserting the expression for the coupling energy $E_g = (2eg/\hbar)^2/C$, we can write the final result for the large junction $R_0 \gg c_{\mathrm{s}}/\omega$ in the form

$$\kappa'' = \frac{2\pi}{2+\lambda/\mu}\left(\frac{pa^2}{e}\right)^2 \frac{e^2/\varepsilon}{a^4\mu}\,\frac{\omega d}{c_{\mathrm{s}}}; \tag{12}$$

for a small junction with $R_0 \ll c_{\mathrm{s}}/\omega$ this result is reduced by the factor $(\omega R_0/c_{\mathrm{s}})^2/2$. The expression (12) applies to the bulk drive (1); the surface contribution is obtained with the substitution $p \to p_{\mathrm{s}}\varepsilon/d$. The dissipative response (12) defines the frequency dependent radiation resistance via $R_{\mathrm{Q}}/R_{\mathrm{s}} = (\hbar\omega/8E_C)\kappa'' \propto \omega^4$ for a pointlike radiation source and $\propto \omega^2$ for a planar source.

Let us estimate the damping coefficient κ'' for a typical $\mathrm{Al}/\mathrm{AlO}_x$ junction. Using standard parameters for Al ($\mu \approx 28$ GPa, $\lambda \approx 62$ GPa, $c_{\mathrm{s}} \approx 6.6\times10^5$ cm/s, $a \approx 2.5$ Å) and $\varepsilon \approx 8.5$, $d \approx 20$ Å for the insulator, we find that the ratio between the electrostatic and elastic energy densities assumes the value $e^2/\varepsilon a^4\mu \approx 0.25$. With typical operation frequencies in the range of $\nu_{01} \sim 10$ GHz we obtain the ratio $\omega_{01}d/c_{\mathrm{s}} \approx 1/50$, hence $\kappa'' \approx 10^{-2}\,(pa^2/e)^2$.

The parameter p is difficult to estimate. First, the bulk drive is expected to be small due to symmetry reasons. Second, we estimate the surface contribution arising from the field penetration into the metal. We make use of the electron-phonon interaction energy $\sqrt{\lambda}\varepsilon_{\mathrm{F}}\int d^3r\, n_{\mathrm{e}}(\nabla\cdot\mathbf{u})$,

TABLE I. Parameters and radiation-limited quality factors $Q_{\rm ph}$ for Josephson (J), flux/phase (Φ), and charge (q) qubits; the junction size L is measured in μm (we use $L^2 = \pi R_0^2$ and the expression for pointlike junctions for the flux/phase and charge qubits); energies and frequencies are measured in Hz.

Qubit	L	E_J	E_C	ν_{01}	$Q_{\rm ph}$
J [7]	10	10^{13}	3×10^6	7×10^9	$7\times10^3/\bar\lambda$
Φ [5]	0.2	3×10^{11}	7×10^9	6×10^9	$4\times10^4/\bar\lambda$
q [2]	0.1	2×10^{10}	1.5×10^{10}	1.5×10^{10}	$10^4/\bar\lambda$

where $\bar\lambda$ denotes the dimensionless electron-phonon coupling (of order unity in bulk Al) and $\varepsilon_{\rm F}$ is the Fermi energy. With the electronic density $n_{\rm e} = (Q/eA)\delta(z)\chi_A(\mathbf{R})$ accumulated at the superconductor-insulator interface, we find a surface piezoelectric coupling $p_{\rm s} = \sqrt{\bar\lambda}\varepsilon_{\rm F}/4\pi e$. Using parameters for Al ($\varepsilon_{\rm F} \approx 12$ eV, $e^2/a \approx 6$ eV) we obtain the estimate $p_{\rm s}a/e \approx 0.2\sqrt{\bar\lambda}$, hence $\kappa'' \approx 2\times10^{-4}\bar\lambda$. This result is in rough agreement with an experiment on $\rm Sn/SnO_x$ junctions [23], where the phonon radiation was found to produce a radiation resistance $R_{\rm Q}/R_{\rm s} \approx 2\times10^5$. Using appropriate parameters for the $\rm Sn/SnO_x$ system, we find a value $\kappa'' \approx 10^{-2}\bar\lambda$ ($e^2/\varepsilon a^4\mu \approx 0.1$, $p_{\rm s}a/e \approx 0.2\sqrt{\bar\lambda}$ for Sn and $\omega d/c_{\rm s} \approx 1$ in the experiment of Ref. [23]), and hence $R_{\rm Q}/R_{\rm s} \approx 3\times10^5\bar\lambda$. Furthermore, evidence for a substantial electric field penetration into the metal derives from the observation of an additional surface capacitance in experiments on $\rm Al/AlO_x$ thin film capacitors, where an interface capacitance corresponding to a surface layer of thickness $d_{\rm surf} \approx 3$ Å has been found [24] (the thickness $d_0 \approx 50$ Å in [24] corresponds to a penetration depth $d_{\rm surf} \approx d_0/2\varepsilon$). On the other hand, surface reconstruction may change the electron-phonon coupling $\bar\lambda$, hence an accurate value for $p_{\rm s}$ has to be found via an independent measurement of phonon radiation on $\rm Al/AlO_x$ junctions, e.g., using techniques as in Ref. [23].

An estimate of the quality factors for Josephson, flux/phase, and charge qubits has to account for the specific type of qubit and the junction parameters; Table I summarizes these results assuming parameters used in experiments. These numbers uncover a substantial decoherence due to phonon radiation and point to the importance of an independent and accurate measurement of the effective piezoelectric coupling in the $\rm Al/AlO_x$ system. Knowledge of this important device parameter will be helpful in the development of new junction designs exploiting geometric and elastic properties reducing the effect of phonon radiation. Experiments probing the energy relaxation γ_E (and its relative importance as compared to phase decoherence) may provide first indications confirming or refuting the presence and relevance of phonon radiation.

We acknowledge financial support from the Swiss National Foundation through the program MaNEP and the NSF Grant No. DMR-0210575 and the hospitality of the Center for Theoretical Studies at ETH.

[1] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
[2] D. Vion *et al.*, Science **296**, 886 (2002).
[3] J. R. Friedman, V. Patel, W. Chen, S. K. Tolpygo, and J. E. Lukens, Nature (London) **406**, 43 (2000).
[4] C. H. van der Wal *et al.*, Science **290**, 773 (2000).
[5] I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Science **299**, 1869 (2003).
[6] Y. Yu, S. Han, X. Chu, S. Chu, and Z. Wang, Science **296**, 889 (2002).
[7] J. M. Martinis, S. Nam, J. Aumentado, and C. Urbina, Phys. Rev. Lett. **89**, 117901 (2002).
[8] A. Shnirman, G. Schön, and Z. Hermon, Phys. Rev. Lett. **79**, 2371 (1997).
[9] D.V. Averin, Solid State Commun. **105**, 659 (1998).
[10] J. E. Mooij *et al.*, Science **285**, 1036 (1999).
[11] L. B. Ioffe, V. B. Geshkenbein, M.V. Feigel'man, A. L. Fauchère, and G. Blatter, Nature (London) **398**, 678 (1999).
[12] G. Blatter, V. B. Geshkenbein, and L. B. Ioffe, Phys. Rev. B **63**, 174511 (2001).
[13] Phonon radiation due to torsional deformations of macroscopic superconducting loops has been analyzed by E. M. Chudnovsky and A. B. Kuklov, Phys. Rev. B **67**, 064515 (2003).
[14] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[15] J. M. Martinis, S. Nam, J. Aumentado, K. M. Lang, and C. Urbina, Phys. Rev. B **67**, 094510 (2003).
[16] C. H. van der Wal, F. K. Wilhelm, C. J. P. M. Harmans, and J. E. Mooij, Eur. Phys. J. B **31**, 111 (2003).
[17] S. M. Girvin, R.-S. Huang, A. Blais, A. Wallraff, and R. J. Schoelkopf, cond-mat/0310670.
[18] Phonon radiation in semiconducting implementations of charge qubits has been analyzed by L. Fedichkin and A. Fedorov, Phys. Rev. A **69**, 032311 (2004).
[19] Another intrinsic source of decoherence are defects acting as two-level systems, which may be eliminated by optimized material growth, see R.W. Simmonds, K. M. Lang, D. A. Hite, D. P. Pappas, and J. M. Martinis, cond-mat/0402470 [Phys. Rev. Lett. (to be published)].
[20] L. B. Ioffe, M.V. Feigel'man, A. Ioselevich, D. Ivanov, M. Troyer, and G. Blatter, Nature (London) **415**, 503 (2002).
[21] M.V. Feigel'man, L. B. Ioffe, V. B. Geshkenbein, P. Dayal, and G. Blatter, Phys. Rev. Lett. **92**, 098301 (2004).
[22] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon Press, London, 1958).
[23] P. Berberich, R. Buemann, and H. Kinder, Phys. Rev. Lett. **49**, 1500 (1982).
[24] G. Singh-Bhalla, Xu Du, and A. F. Hebard, Appl. Phys. Lett. **83**, 2417 (2003).