Dynamic Localization and the Coulomb Blockade in Quantum Dots under ac Pumping

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(Received 6 February 2004; published 30 July 2004)

We study conductance through a quantum dot under Coulomb blockade conditions in the presence of an external periodic perturbation. The stationary state is determined by the balance between the heating of the dot electrons by the perturbation and cooling by electron exchange with the cold contacts. We show that the Coulomb blockade peak can have a peculiar shape if heating is affected by dynamic localization, which can be an experimental signature of this effect.

DOI: 10.1103/PhysRevLett.93.056804 PACS numbers: 73.21.La, 73.20.Fz, 73.23.–b, 78.67.Hc

Introduction.—Experimental observation of dynamic localization (DL) in trapped ultracold atoms in the field of a modulated laser standing wave [1] provided a solid ground for the preceding extensive theoretical studies of the kicked quantum rotor [2,3]. In a recent publication [4] we have shown that an analogous suppression of the energy absorption is possible for a solid-state system—a chaotic quantum dot under an ac excitation, e.g., such as those used in the experiments of Ref. [5], which makes the question about the possibility of observation of DL in a quantum dot highly relevant. If one wishes to detect this effect by transport measurements, the Coulomb blockade regime [6,7] is the most suitable, since it is in this regime that the transport is sensitive to the internal state of the dot. For an open dot, when electron-electron interaction can be neglected, the conductance is insensitive to the electron energy distribution in the dot [8,9].

Under nonequilibrium conditions, the (effective) electronic temperature T of the dot is determined by the balance between heating by the ac perturbation and cooling due to various mechanisms. At sufficiently low T cooling is dominated by simple electronic exchange between the dot and the contacts (the latter are assumed to be maintained at a constant low temperature determined by the cryostat), while the energy exchange with the phonon subsystem is negligible [10]. In this case, as the gate voltage is detuned from the Coulomb blockade peak, the cooling rate decreases, leading to an increase of T. The dynamic localization manifests itself in suppression of heating, which becomes T dependent. This makes the shape of the Coulomb blockade peak sensitive to dynamic localization.

Heating.—First, consider the standard picture of heating by an ac perturbation. Let the single-electron mean level spacing δ in the dot be small enough. Then, if an external time-dependent periodic perturbation with the frequency ω is applied, the total electronic energy E in the dot (counted from that of the ground state) grows linearly with time as described by the Fermi golden rule: $E(t) = \Gamma\omega^2 t/\delta \equiv W_0 t$. The probability per unit time Γ of each single-electron transition measures the ac power [11]. The criterion of validity of the Fermi golden rule is $\delta \ll \Gamma$, and $\Gamma \ll \omega$ is also assumed ($\hbar = 1$).

This classical picture is valid only if each act of photon absorption by an electron is independent of the previous ones; however, for a discrete (though dense) energy spectrum this turns out not to be the case. After many transitions the absorption rate decreases due to accumulation of the quantum interference correction [2–4], so that after a time $t_* \sim \Gamma/\delta^2$ the absorption is completely suppressed. This effect is known as the dynamic localization in energy space; the effective electronic temperature $T_* \sim \Gamma\omega/\delta$ (the characteristic spread of the electron distribution function) reached by the time t_* plays the role of the localization length. Note that DL has nothing to do with the saturation of absorption by a pumped two-level system since in our case the spectrum is unbounded.

The considerations of Ref. [4] were based on a random matrix theory description of the single-particle properties of the dot, valid provided that all energy scales in the problem are small compared to the Thouless energy E_{Th} (defined by the order of magnitude as the inverse of the time required for an electron to travel across the dot and thus to randomize its motion due to scattering off the dot boundaries). For the dot to be in the Coulomb blockade regime, the effective temperature should be also smaller than the dot Coulomb charging energy E_c. In the following, the hierarchy of scales $\delta \ll \Gamma \ll \omega \ll T_* \ll E_{\mathrm{Th}}, E_c$ is assumed (for a typical 2D GaAs dot $\delta \sim 1$ μeV, $E_{\mathrm{Th}} \sim 100$ μeV ~ 1 K [5]; the stronger the inequality $\delta \ll E_{\mathrm{Th}}$ is satisfied, the better).

Being an interference effect, DL requires perfect quantum coherence. Electron interaction and/or connection to leads causes electron *dephasing*. In the presence of *weak* dephasing with the rate $\gamma_\phi \ll 1/t_*$, there is a residual absorption with the rate determined by γ_ϕ:

$$W_{\mathrm{in}} \sim W_0 \gamma_\phi t_* = T_*^2 \gamma_\phi/\delta. \tag{1}$$

If the dephasing is too strong, $\gamma_\phi \gtrsim 1/t_*$, the dynamic localization is destroyed and $W_{\mathrm{in}} \sim W_0$.

The expression (1) was justified in Ref. [12], for the dephasing due to electron-electron collisions provided that the effective electronic temperature $T > T_*$. The main condition of its applicability is that dephasing should be a sequence of distinct phase-destroying events with average rate γ_ϕ, rather than phase diffusion, so that the dephasing rate roughly coincides with the quasiparticle relaxation rate: $\gamma_\phi \sim \gamma_{\rm qp}$. This is certainly correct for the case of electron escape to the contacts, since in this case the electron is effectively replaced by another one with an absolutely random phase. This is also true for electron-phonon collisions and electron-electron collisions in a quantum dot, since the typical energy transfer during a collision is of the order of the (effective) electronic temperature in the dot, which is large: $T > T_* \gg 1/t_*, \gamma_{\rm qp}$ ($T_* \gg 1/t_*$ due to $\Gamma, \omega \gg \delta$). Note that this condition is the reason why no exponentially small factor such as $e^{-T_*/T}$ or $e^{-t_*^{-1}/T}$ arises in Eq. (1), in contrast to hopping conductivity in disordered systems at low temperatures.

Once the condition $\gamma_\phi \sim \gamma_{\rm qp}$ is verified, the following consideration can be applied. As the collisions are rare ($\gamma_{\rm qp} t_* \ll 1$), the electrons spend most of the time in the states localized in the energy space, having definite phase relationships. When at some moment the phase of some electron is destroyed, its wave packet starts spreading along the energy axis. It localizes again after the time $\sim t_*$, in the meantime spreading by $\sim T_*$. Thus, the ac driven dynamics following the collision leads to a change of the total electronic energy of $\sim T_*$ per collision. The sign of this change is, however, arbitrary, because a periodic perturbation can equally cause transitions up and down the spectrum. Only the presence of the filled Fermi sea below (i.e., an energy gradient of the electronic distribution function) makes absorption the preferred direction, which means that if the electronic temperature $T \gg T_*$, the energy absorbed per collision is on the average $\sim T_*^2/T$ rather than T_*. The effective number of electrons that can participate in a collision is $\sim T/\delta$ (due to the degenerate Fermi statistics). During the time interval $\sim 1/\gamma_{\rm qp}$, each electron participates in one collision, so the total number of collisions per unit time is $\sim (T/\delta)\gamma_{\rm qp}$. This gives the energy absorption rate $W_{\rm in} \sim (T_*^2/T)(T/\delta)\gamma_{\rm qp}$, which is exactly Eq. (1).

The same can be seen from an alternative argument. After each collision the electron spends the time $\sim t_*$ absorbing the energy from the microwave field; then it stops to absorb (the DL occurs) and waits for the next event (provided that $t_* \ll 1/\gamma_\phi$). Thus the absorption rate of the whole system is given by the simple weighted average: $W_{\rm in} \sim W_0 \gamma_\phi t_*$, which is again Eq. (1).

As γ_ϕ, generally speaking, depends on the electronic temperature, so does the absorption rate in the DL regime (Fig. 1). The quasiparticle relaxation rate due to electron collisions in a quantum dot was calculated by Sivan, Imry, and Aronov [13]:

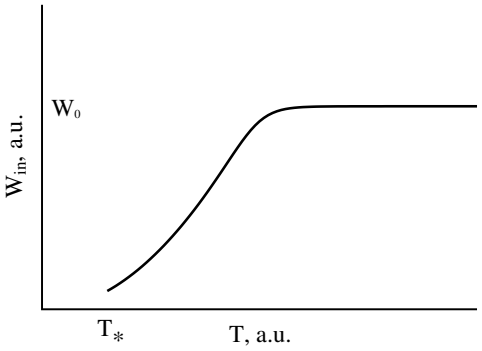

FIG. 1. A schematic view of the dependence of the absorption rate $W_{\rm in}$ on the effective electronic temperature in the dot: in the dynamic localization regime the absorption is due to dephasing, so it is temperature dependent; when the temperature becomes high enough, the dephasing destroys the DL. $T < T_*$ cannot be realized in the regime of strong DL.

$$\gamma_{e-e}(T) \sim \delta\,(T/E_{\rm Th})^2. \tag{2}$$

The derivation of this expression implies the effective continuity of the many-particle spectrum, which imposes a condition $T_* \gg \sqrt{E_{\rm Th}\delta/\ln(E_{\rm Th}/\delta)}$ [14]. Obviously, for the dynamic localization to have any chance to develop, the condition $\gamma_{e-e}(T_*)t_* \ll 1$ should also be satisfied.

Sequential tunneling.—We characterize the coupling of the dot to two contacts by electron escape rates γ_1, γ_2 ($\gamma_1 + \gamma_2 \equiv \gamma \ll \delta$). Let U be the energy cost of adding an electron to the dot, proportional to the gate voltage (at the exact degeneracy point $U = 0$, corresponding to the very top of the Coulomb blockade peak). When U is not far from the degeneracy point, the main contribution to the conductance comes from the leading order of the perturbation theory in the dot-contact coupling. For characteristic temperatures $T \gg \delta$, one can describe the system by the rate equations of Kulik and Shekhter [15].

It is quite straightforward to consider these equations for a general (nonequilibrium) electron energy distribution function in the dot. As an estimate for the electronic distribution function we use the Fermi-Dirac form with some temperature T. We also assume T to be much higher than the temperature of the contacts T_0 (T_0 can be made as low as ~ 10 mK [16]), which is true if the pumping power is high enough. Then a simple calculation gives the following expressions for the dot conductance G (in units $e^2/2\pi\hbar$), "renormalized" electron escape rate $\gamma_{\rm esc}$, and the cooling rate $W_{\rm out}$ [we denote $x \equiv U/(2T)$, $G_0 \equiv \gamma_1\gamma_2/(\gamma\delta)$]:

$$G(U) = G_0\left[1 - \frac{x\tanh x}{\ln(2\cosh x)}\right], \tag{3}$$

$$\gamma_{\rm esc}(U) = \frac{\gamma}{2}\left[1 - \frac{|x|}{\ln(2\cosh x)}\right], \quad (4)$$

$$W_{\rm out}(U) = \frac{\gamma}{\delta}T^2\left[\frac{\pi^2}{12} - x^2 + \frac{2x\int_0^x y\tanh y dy}{\ln(2\cosh x)}\right]. \quad (5)$$

Stationary state.—For each given U the temperature T of the stationary state is found from the energy balance equation $W_{\rm out} = W_{\rm in}$, where $W_{\rm out}$ is given by Eq. (5), $W_{\rm in}$ by Eq. (1), and the dephasing rate by Eqs. (2) and (4). Substituting T into Eq. (3), one obtains the shape of the Coulomb blockade peak.

Suppose for a moment that dephasing is dominated by electron-electron collisions, while cooling is dominated by the escape to the contacts [10]. One can notice the following property of Eqs. (3) and (5): G/G_0 and $W_{\rm out}(U)/[(\gamma/\delta)T^2]$ are functions of $x \equiv U/(2T)$ only. This allows us to write a relation

$$\frac{W_{\rm out}}{T^2} = (\gamma/\delta)\mathcal{W}(G/G_0). \quad (6)$$

The function $\mathcal{W}(G/G_0)$ is plotted in Fig. 2 and with logarithmic precision we have $(\gamma/\delta)\mathcal{W}(G/G_0) \sim G$. Thus the physical meaning of Eq. (6) is similar to the Wiedemann-Franz law.

The energy balance condition takes the form

$$W_{\rm in} \sim T_*^2[\gamma_\phi(T)/\delta] = W_{\rm out} = (\gamma/\delta)T^2\mathcal{W}(G/G_0), \quad (7)$$

or $G \sim (\gamma/\delta)\mathcal{W}(G/G_0) = [\gamma_\phi(T)/\delta](T_*/T)^2$. Remarkably, for $\gamma_\phi(T)$ given by Eq. (2) U and T drop out and the solution of this equation for G is independent of U, leading to a flat plateau $G \sim (T_*/E_{\rm Th})^2$ on the Coulomb blockade curve $G(U)$. Note that the plateau conductance must be smaller than the peak conductance $G_0 \sim \gamma/\delta$. Therefore, the solution exists only if

$$\gamma/\delta \gtrsim (T_*/E_{\rm Th})^2. \quad (8)$$

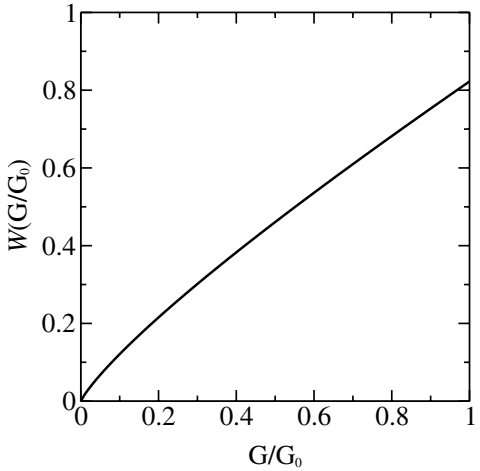

FIG. 2. The function $\mathcal{W}(G/G_0)$ defined in Eq. (6).

Now consider the top of the peak, $U = 0$. Including the dephasing due to both escape and electron-electron collisions, we can write the energy balance condition as

$$(\gamma/\delta)T^2 \sim (\gamma/\delta)T_*^2 + (T_*/E_{\rm Th})^2T^2. \quad (9)$$

Here the left-hand side represents the cooling rate in the peak and the two terms on the right-hand side come from the dephasing due to escape and electron collisions, respectively. Because of the condition (8), the second term is smaller than the left-hand side, so the only way to satisfy the equation is to have $T(U = 0) \sim T_*$. Thus, for the dynamic localization to be possible the dephasing in the very peak of the Coulomb blockade *must* be dominated by escape. One can extract the temperature of the stationary state at $U = 0$ measuring the curvature of the peak and study its dependence on control parameters: intensity Γ and coupling to the contacts γ.

As U is detuned from the peak, the dot becomes effectively more closed. Thus, the crossover from the peak to the plateau occurs when the two mechanisms are equally efficient. With logarithmic precision this happens at

$$T \sim T_* \qquad U \sim U_{\rm min} \sim T_*. \quad (10)$$

The plateau ends when the temperature of the dot becomes so large that the dynamic localization is destroyed by dephasing and $W_{\rm in} \sim W_0$. Obviously, this happens when the plateau $G \sim (T_*/E_{\rm Th})^2$ hits the curve $G_D(U) \sim (W_0/U^2)\ln(G_0U^2/W_0)$ determined from the condition of classical (Ohmic) absorption $W_0 = W_{\rm out}$. With logarithmic precision this happens at

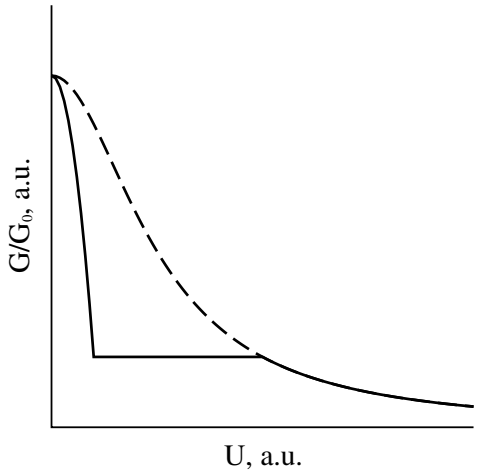

FIG. 3. A sketch of the Coulomb blockade peak shape in the DL regime (solid line): at small $U < U_{\rm min}$ the dephasing is dominated by the electron escape (peak); at larger U it is dominated by the electron-electron collisions (plateau); and finally, at $U > U_{\rm max}$ the cooling is insufficient, the dynamic localization is destroyed, and the dot is in the Ohmic regime. The Ohmic curve is also shown for reference by the dashed line.

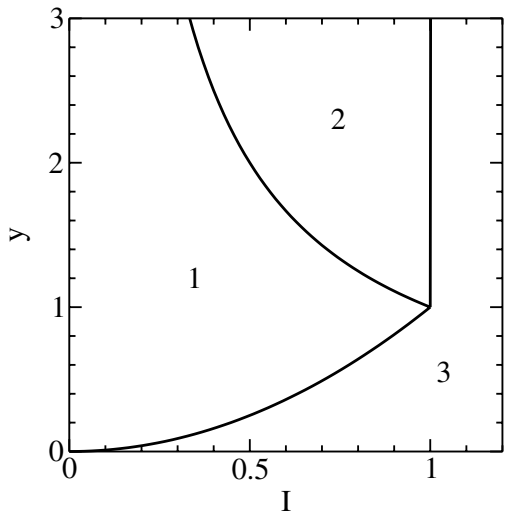

FIG. 4. A schematic view of the phase diagram in terms of the dimensionless intensity and escape rate ($I - y$ plane), without taking into account cooling and dephasing due to phonons. The top of the Coulomb blockade peak corresponds to DL only in region 1; the flat plateau in the tails exists both in regions 1 and 2; in region 3 DL is absent.

$$U \sim U_{\max} \sim E_{Th}\sqrt{\delta/\Gamma}\,. \tag{11}$$

The resulting shape of the Coulomb blockade peak is drawn schematically in Fig. 3.

It is convenient to introduce two dimensionless parameters, corresponding to two experimentally controllable parameters Γ (ac power) and γ (tunneling into leads):

$$I \equiv \frac{\Gamma}{\delta}\left(\frac{\omega}{E_{\rm Th}}\right)^{2/3}, \qquad y \equiv \frac{\gamma}{\delta}\left(\frac{\omega}{E_{\rm Th}}\right)^{-2/3}. \tag{12}$$

The necessary condition for dynamic localization $\gamma_{e-e}(T_*)t_* \ll 1$ becomes $I \ll 1$; the condition (8) is $y \gg I^2$. The top of the peak will correspond to DL regime if $\gamma t_* \ll 1$ or $Iy \ll 1$ [17]. The resulting "phase diagram" is shown in Fig. 4.

Conclusions.—We have studied electronic conduction through a quantum dot in the Coulomb blockade regime under an external periodic perturbation. In contrast to the well-studied equilibrium case, the electronic temperature of the ac driven dot is determined by the balance between heating by the perturbation and cooling due to electron exchange with cold contacts. The cooling rate thus depends on the gate voltage, and so does the dot temperature. As the gate voltage is detuned away from the peak, the cooling rate decreases, and the temperature increases. In the strong dynamic localization regime the heating rate is determined by dephasing, as the usual linear absorption is blocked by quantum interference. The most peculiar situation is realized when the dephasing is due to electron-electron collisions: in this case the Coulomb blockade peak has a flat shoulder, where the conductance does not depend on the gate voltage. Such a shape could be an experimental signature of the dynamic localization effect.

We are grateful to Yu. M. Galperin, B. N. Narozhny, C. M. Marcus, and B. L. Altshuler for helpful discussions.

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[11] The transition rate is proportional to the power of the microwave and can be estimated as $\Gamma \sim (e\mathcal{E}L)^2/E_{\rm Th}$, where $\mathcal{E}$ is the amplitude of the electric field in the dot, L is the dot size, and $E_{\rm Th}$ is the Thouless energy. If the screening length $1/\kappa < L$, one should substitute $\mathcal{E}L \to \mathcal{E}_{\rm ext}/\kappa$. Finally, instead of using the microwave, one can change the dot shape modulating the gate voltage.
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