Nonlinearity and Disorder in Fiber Arrays

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(Received 23 December 2003; published 27 July 2004)

We experimentally investigate light propagation in a disordered two-dimensional array of mutually coupled optical fibers. In the linear case light either spreads in a diffusive manner or localizes at a few sites. For high excitation power diffusive spreading is arrested by the focusing nonlinearity, i.e., forming a discrete soliton. By contrast, fields, which are localized in the linear regime, can experience both spreading and contraction caused by the nonlinearity.

DOI: 10.1103/PhysRevLett.93.053901 PACS numbers: 42.81.Qb, 42.25.Fx, 63.20.Pw, 42.65.–k

Frequently, the properties and the mutual interaction among individual constituents of a complex system govern its dynamic behavior. Examples for such discrete systems appear in many areas of physics ranging from microscopic to macroscopic scales [1]. Periodic configurations or homogeneous lattices represent a particular specimen of discrete systems exhibiting a bounded spectrum of delocalized linear states. Already weak random deviations from the regularity obstruct the resonant interaction between different sites. Consequently, the excitation dynamics resembles a diffusion process thus slowing down the spreading. If the disorder is strong enough, a universal feature of random lattices takes action; viz., linear modes become localized [2,3], thus preventing the excitation from spreading. In a similar manner nonlinearity may detune some constituents from the rest of the lattice, leading to the formation of discrete breathers or solitons [4]. Therefore, it is interesting to study the combined action of nonlinearity and disorder on the excitation dynamics in discrete systems. The interplay of disorder, nonlinearity, and band structure of the spectrum may evoke interesting effects, such as, e.g., the appearance of a fractal set of so-called intraband discrete breathers inside the linear spectrum or a nonlinearly induced increase of mobility [5,6].

Ensembles of weakly coupled waveguides or fibers represent a typical discrete lattice. The existence of nonlinearly induced localized states, frequently called discrete solitons, was predicted [7] and experimentally confirmed in 1D [8] and 2D waveguide arrays [9]. Furthermore, the interaction of discrete solitons with linear defects [10] and nonlinearly induced delocalization at defects [11] has been experimentally studied in 1D arrays.

The aim of this Letter is to experimentally investigate the peculiarities of light propagation in slightly irregular 2D arrays of evanescently coupled fibers (see Fig. 1). The main emphasis is on this competition between nonlinearity and disorder. Besides the fundamental interest, systems of coupled fibers have a large application relevance in the context of fiber lasers and amplifiers [12,13].

The fiber sample was manufactured starting from a single preform of a telecommunication fiber with a partially removed cladding. This preform was drawn down to a diameter of 500 μm and cut into 400 identical pieces of 40 cm length, which were stacked into a glass tube to form the hexagonal array. The assembled array preform was finally drawn to a length of $>$100 m and a diameter of 320 μm, resulting in an average core-to-core spacing of 12.1 μm. A mean core diameter of 6.9 μm and an index contrast of $\delta n = 0.003\,84$ assured single-mode waveguiding at the wavelength of 800 nm. Some disorder was induced during the fabrication process. While evaluating microscope images of the front facet (see Fig. 1), we measured a variation of the core diameter of $\approx$180 nm and of the core spacing of $\approx$930 nm across the array.

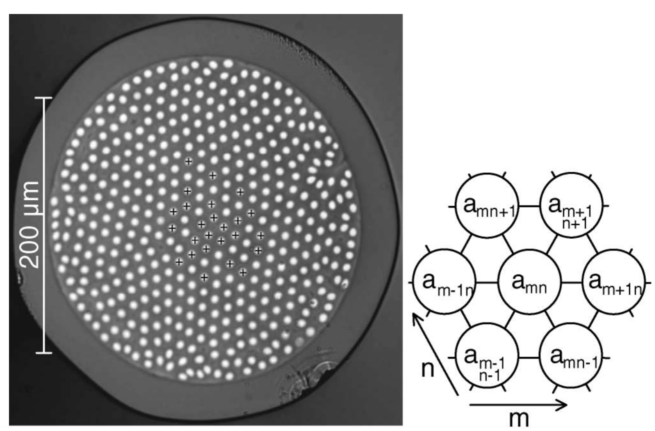

FIG. 1. Microscope image of the fiber array with various evaluated input positions marked.

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Although the symmetry of the waveguide lattice remained essentially hexagonal, it resembled more an amorphous solid than a perfect crystal. We found that the transverse structure of the array did not change over length scales of at least 5 m. Eventually, we used less than 60 cm of the fiber cut into pieces ranging from 12 to 200 mm length. Hence, we may assume that all samples have the same transverse structure, which is constant along the propagation direction. Comparing the output fields obtained for different sample lengths allows monitoring the field propagation in the disordered lattice.

In all measurements we excited a single waveguide of the fiber arrays with an amplified Ti:sapphire laser emitting pulses of 145 fs duration (FWHM) at $\lambda = 800$ nm. Both fiber facets were monitored by charge coupled device cameras to control the field distribution at the output and the position of the exciting beam simultaneously. It was carefully checked that the field did not reach the array boundaries.

To model the field propagation, the light dynamics on the hexagonal lattice (see Fig. 1) can be conveniently described by coupled mode equations as

$$\left[i\frac{d}{dz} + \beta_{mn} + \chi_{\mathrm{eff}}^{(3)}|a_{mn}|^2\right]a_{mn} + c_{m,n}^{(1)}a_{m+1,n} + c_{m-1,n}^{(1)}a_{m-1,n} + c_{m,n}^{(2)}a_{m,n+1} + c_{m,n-1}^{(2)}a_{m,n-1} + c_{m,n}^{(3)}a_{m+1,n+1} + c_{m-1,n-1}^{(3)}a_{m-1,n-1} = 0. \quad (1)$$

The amplitude $a_{m,n}$ of the modal field in the core with indices m and n evolves along z, where the evanescent coupling between adjacent guides induces the transverse dynamics. For high light intensities the local wave number is increased by the focusing nonlinearity of fused silica ($\chi_{\mathrm{eff}}^{(3)} = 5.5\ \mathrm{km}^{-1}\,\mathrm{W}^{-1}$). If we neglect nonlinearity and disorder, i.e., considering a homogeneous hexagonal lattice with $\chi_{\mathrm{eff}}^{(3)} = 0$, $c_{m,n}^{(k)} = c_0$, and $\beta_{m,n} = \beta_0$, the propagation constants of plane waves would be situated in a band ranging from $\beta_0 - 3c_0$ to $\beta_0 + 6c_0$. However, in our case the optical parameters of the array vary stochastically. Based on the reported characterization of the fiber array, we calculated that the coupling constant fluctuates around a mean value of $c_0 = 46\ \mathrm{m}^{-1}$ as $c_{m,n}^{(k)} = c_0 + \delta c_{m,n}^{(k)}$with $\langle \delta c_{m,n}^{(k)}\rangle = 0$ and $\sqrt{\langle(\delta c_{m,n}^{(k)})^2\rangle} = 26.7\ \mathrm{m}^{-1}$. In our simulations we assumed a Gaussian distribution of $\delta c_{m,n}^{(k)}$, which was truncated to ensure $c_{m,n}^{(k)} > 0$. Similarly, the stochastic variation of the core diameter across the fiber array translated into a Gaussian distribution of the propagation constants of the individual waveguides as $\beta_{m,n} = \beta_0 + \delta\beta_{m,n}$ with $\langle\delta\beta_{mn}\rangle = 0$ and $\sqrt{\langle(\delta\beta_{m,n})^2\rangle} = 140\ \mathrm{m}^{-1}$. Although fluctuations of the propagation constant are 5 orders of magnitude smaller than the respective mean value, they still have a huge impact on the field evolution. While a mean propagation constant $\beta_0 \approx 11.4 \times 10^6\ \mathrm{m}^{-1}$ can be removed from Eq. (1) by a phase transformation, the variations $\delta\beta_{mn}$ cannot. These variations are comparable to ($\sqrt{\langle(\delta\beta_{m,n})^2\rangle} \approx 3c_0$) but slightly smaller than the width of the band of the corresponding homogeneous array ($9c_0$). Hence, we deal with a system of pronounced, but still weak disorder. Correspondingly, the numerical analysis of the eigenmodes of a finite disordered array reveals that most modes acquire extended states with wave numbers within the band of the corresponding homogeneous array. Obviously, the strength of disorder does not suffice to induce complete localization. Although some localized states exist, most of the linear waves spread across the entire array.

To simplify the numerical analysis we have neglected all temporal effects in Eq. (1). This is justified since even for the longest sample (200 mm) group velocity dispersion spreads a linear pulse by a factor of 1.5 only. The primary effect of the pulsed excitation is a mere rescaling of the power levels compared with continuous wave simulations. In addition, results belonging to slightly different power levels are superimposed resulting in a smoother nonlinear response.

We compared the results of numerical simulations and experimental measurements by evaluating the averaged mean square width of respective field distributions. Furthermore, the simulated field evolution along the input guide was Fourier transformed to get the spectral content of the respective field as

$$u(\beta) = \frac{1}{L_{\max}}\int_0^{L_{\max}} dz\, a_{00}(z)\exp(-i\beta z), \quad (2)$$

where $L_{\max}$ is the total propagation length and a_{00} the field at the input waveguide.

We started our experiments by studying the linear light propagation in the fiber for the excitation of single waveguides at different positions [for a typical example see Fig. 2(a)]. Even for short pieces and input sites within apparently regular areas (see Fig. 1) the output field distribution is heavily distorted and the hexagonal symmetry of the lattice is not reproduced in the observed diffraction patterns. However, in contrast to disordered 1D lattices light spreading (or discrete diffraction) is not arrested by occasionally vanishing coupling and transverse light transport occurs. Nevertheless, the discrete diffraction process is considerably affected. In a perfectly homogeneous fiber array the width of a single waveguide excitation would grow linearly with the distance (see dashed line in Fig. 3) determined by the spreading of the fastest linear waves [14]. But in our distorted fiber array fields propagating in different waveguides run out of phase after about $2/\sqrt{\langle(\delta\beta_{m,n})^2\rangle} = 14$ mm (see where the curves separate in Fig. 3). Even though transverse coherence is lost, most of the light continues to spread in a diffusive way as expected in a random system and as known from conventional scattering. However, part of the light is trapped in certain areas of the array. Thus, for a few input positions the light remains localized near the

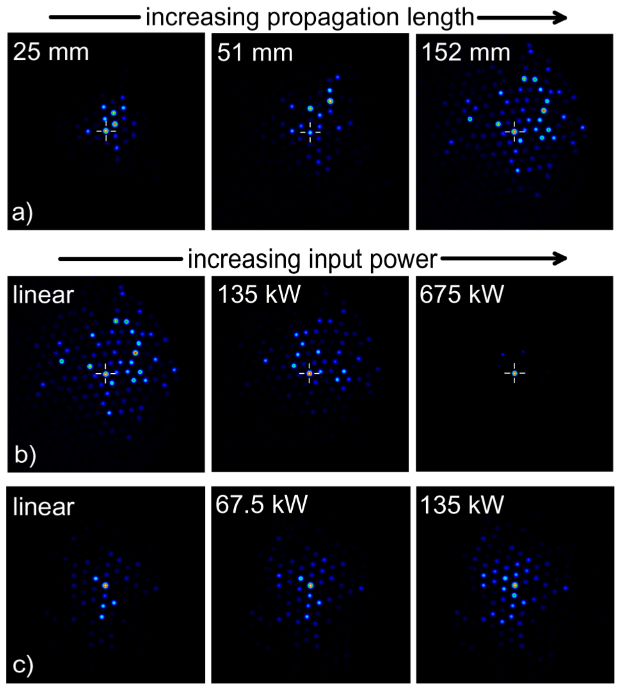

FIG. 2 (color online). Intensity distribution at the output facet of the array. (a) Linear diffraction for small input power; (b) nonlinear localization and formation of a discrete soliton for the same position of the excitation as in (a); (c) nonlinear delocalization for another input position [fiber length: (b) 150 mm, (c) 100 mm].

excited guide and does not spread at all, no matter how long it propagates down the fiber array. This is no surprise, because the strength of fluctuations compares to the width of the unperturbed band. Hence, one expects to find localized states with wave numbers either above or below the band. By varying the excitation site both localized and delocalized modes have been excited.

Now we investigate the influence of nonlinearity at higher input powers. The present fiber array offers the opportunity to study the effect of a focusing cubic nonlinearity on both linearly localized and delocalized states in a 2D random lattice. In what follows we compare typical experimentally determined field distributions with respective numerical simulations in a random lattice for different input positions. For linearly delocalized states the rather irregular field evolution changes considerably, already for power levels around 10 kW. This power is much less than the excitation threshold of a discrete soliton in a corresponding regular array (here about 100 kW). Because randomness confines transverse light propagation to a few possible paths a nonlinear phase shift can accumulate before interference with other trajectories occurs. Hence, the structure acts like a sensitive nonlinear interferometer with a sharp nonlinear response [see Figs. 2(b) and 2(c)]. Resulting irregular changes are averaged out, if only the evolution of the mean square width of the field distribution is considered as in Fig. 4. Furthermore, linearly delocalized states usually expand slightly with increasing power before forming a localized discrete soliton and eventually collapsing into a single guide, a feature that is well reproduced by respective numerical simulations [compare Figs. 4 and 5(a)]. Because of the focusing character of the nonlinearity ultimate localization into a single guide is always connected with the appearance of a discrete soliton with a wave number above the band. This scenario is similar to that of soliton formation in homogeneous arrays [see Figs. 5(b) and 5(d)]. However, intermediate localization at states with wave numbers inside the band seems to occur [see Fig. 5(d)]. This localization inside the band might be explained by the formation of intraband discrete breathers [5].

While for delocalized states this nonlinear localization scenario is rather general, localized states can evolve in two distinctively different ways. We found some linearly localized states to first expand and later contract for increasing input powers, while others contracted immediately (see Fig. 4). Again numerical simulations can explain the different behavior. If the excited linearly

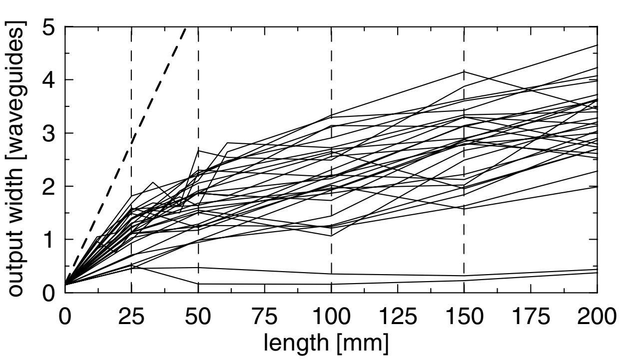

FIG. 3. Experimentally determined evolution of the width of a single waveguide excitation for different input positions (measurements taken at a length of 25, 50, 100, 150, and 200 mm; dashed line: simulation for a corresponding homogeneous array).

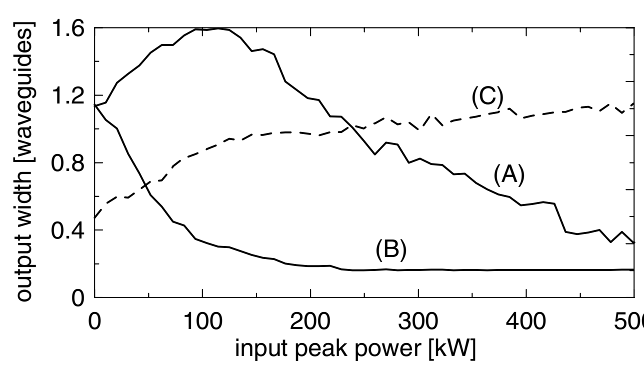

FIG. 4. Experimentally determined width of the field distribution vs input peak power for the excitation of (*A*) a linearly delocalized state with a wave number in the band and of linearly localized states with wave numbers above (*B*) and below (*C*) the band [fiber length: (*A*) 25 mm, (*B*) 25 mm, (*C*) 50 mm].

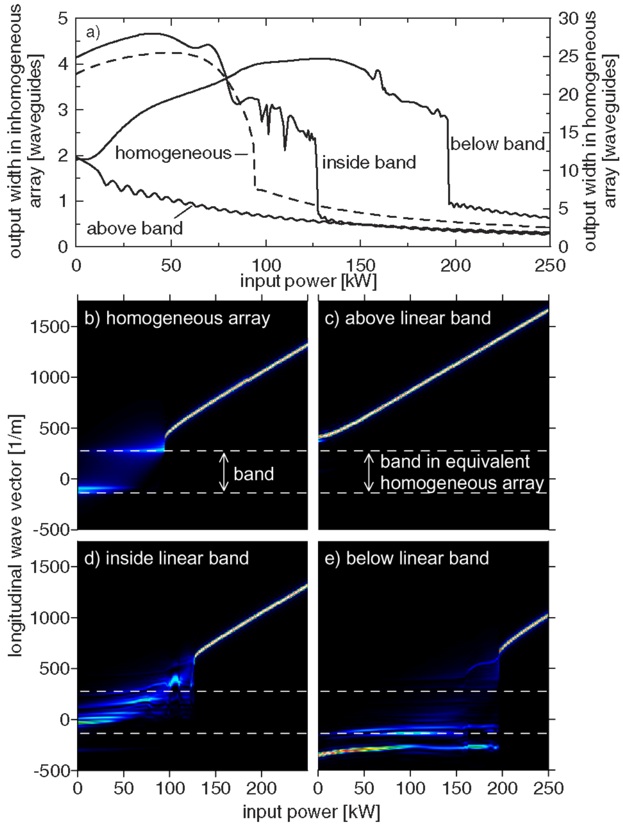

FIG. 5 (color online). (a) Numerically determined field width vs input power for the excitation at several input positions with a linearly localized state above the band, a linearly delocalized state inside the band or a localized state below the band and for comparison in a corresponding homogeneous array. (b)–(e) Wave number of the field in the input guide vs input power obtained by Fourier transform (2).

localized state possesses a wave number in the upper part of the band (of a comparable undistorted system) the output contracts monotonously for increasing input power since the focusing nonlinearity results in an increase of the wave number of the excited wave, i.e., shifting the excited waves out of the linear spectrum [see Fig. 5(c)]. By contrast, if the initial wave number of the localized state is situated at the lower edge of the linear band the same nonlinearity can shift the wave number into the band. Thus, a partial nonlinearly induced delocalization is observed if resonance transitions to other states occur [see Fig. 2(c)]. Fourier analysis shows the excited state to perform a sequence of abrupt jumps to higher wave numbers. Again, this can be interpreted as the excitation of a sequence of intraband discrete breathers [see Fig. 5(e)]. However, for even higher input powers eventually the upper edge of the band is reached and a localized state is formed, which smoothly contracts for a further increase of the in-coupled power.

Numerical simulations of large, but finite arrays indicate nonlinear localization into a single guide to occur for very high input powers, no matter which site of the simulated arrays is excited. However, in our experiments the available input power levels were limited and often a few cores remained illuminated besides the input guide even at high power. In particular, for those input positions where power induced delocalization can be observed, final localization was often incomplete. This is intuitively clear because the nonlinear wave number shift had to exceed the width of the band. This is also in agreement with the numerical simulations, where linearly localized states with wave numbers below the band required the highest power to finally contract [see Fig. 5(e)].

In conclusion, we have experimentally investigated a weakly disordered 2D fiber array. For low power excitations we found the light either to spread in a diffusive way or to localize at the input waveguide. For increasing input power levels, initially delocalized states contracted while finally forming discrete solitons for very high powers. Localized states with linear wave numbers above the band of propagating waves remained almost unchanged for increasing powers. They only further contract into a single guide. By contrast, bound states with a wave number below the band initially delocalize for increasing powers although the nonlinearity is focusing. While crossing the band the field seems to excite a sequence of intraband discrete breathers before eventually forming a discrete soliton.

The authors acknowledge support by the Deutsche Forschungsgemeinschaft (FG532).

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