Tunneling of Dipolar Spin Waves through a Region of Inhomogeneous Magnetic Field

S. O. Demokritov,* A. A. Serga,† A. André, V. E. Demidov, M. P. Kostylev, and B. Hillebrands
Fachbereich Physik and Focschungsschwerpunkt MINAS, Technische Universität Kaiserslautern, 67663 Kaiserslautern, Germany

A. N. Slavin
Department of Physics, Oakland University, Rochester, Michigan 48309, USA
(Received 16 November 2003; published 20 July 2004)

We show experimentally and by numerical simulations that spin waves propagating in a magnetic film can pass through a region of a magnetic field inhomogeneity or they can be reflected by the region depending on the sign of the inhomogeneity. If the reflecting region is narrow enough, spin-wave tunneling takes place. We investigate the tunneling mechanism and demonstrate that it has a magnetic dipole origin.

DOI: 10.1103/PhysRevLett.93.047201 PACS numbers: 75.30.Ds, 75.40.Gb

The effect of tunneling discovered in 1928 by Gamov and co-workers in quantum mechanics [1] is a striking manifestation of the wave nature of quantum-mechanical particles. This effect, however, manifests itself for waves of a different nature, and during the last decade tunneling of electromagnetic and acoustic waves through spatial regions where existence of these waves is prohibited attracted a lot of attention [2]. Spin waves can also demonstrate a tunneling effect, and since the frequency of spin waves depends on the applied magnetic field, a tunneling barrier for propagating spin waves can be created by a magnetic field inhomogeneity.

Propagation of spin waves in an inhomogeneous magnetic field was discussed for the first time in the 1960s [3–6]. It was Schlömann who first noticed a close similarity between propagation of exchange dominated spin waves and the motion a quantum-mechanical particle [3]. In fact, neglecting the magnetic dipole interaction and magnetic anisotropies the Landau-Lifshitz equation describing magnetic dynamics can be rewritten in the form of the stationary Schrödinger equation with the dynamic magnetization $m \propto \exp(i\omega t)$ being the analog of a wave function and the magnetic field playing the role of potential energy:

$$-\frac{2A}{M_S}\frac{\partial^2 m}{\partial z^2} + \left(H(z) - \frac{\omega}{\gamma}\right)m = 0, \quad (1)$$

where A is the exchange stiffness, M_S is the saturation magnetization, and γ is the gyromagnetic ratio of the medium. The dispersion relation for a plane spin wave $[m \propto \exp(iqz)]$ can then be written as

$$\omega = \Delta(z) + \frac{2\gamma A}{M_S}q^2, \quad (2)$$

where $\Delta(z) = \gamma H(z)$ is the gap of the spectrum, which is reminiscent to the dispersion of a particle in a potential field $U(z)$,

$$E = U(z) + \frac{\hbar^2}{2m}q^2. \quad (3)$$

Thus, if a spin wave of frequency ω enters a region where the field $H = H(z)$ (and the gap) varies, the wave keeps to propagate through the inhomogeneous field, albeit with changing wave vector, $q = q(z)$, to fulfill the dispersion law Eq. (2). However, if the value of the gap locally exceeds ω, there exists no real wave vector anymore to fulfill the dispersion law for this frequency. The wave is reflected from this region, which thus can be considered as a potential barrier. Recently it was shown that a strongly inhomogeneous internal field in magnetic a microstripe can cause such turning points within the stripe which reflect spin waves and thus create a spin-wave well [7–9].

A theoretical analysis of spin-wave reflection from a field inhomogeneity taking into account only the exchange interaction and neglecting the magnetic dipole interaction shows that the dynamic magnetization beyond the turning point is not zero: it just changes its dependence on z from sinusoidal $[m \propto \exp(iqz)]$ to exponential $[m \propto \exp(-\lambda z)]$ [3]. The spin waves tunnel through the barrier.

In this Letter we experimentally observe and investigate the effect of spin-wave tunneling. In contrast to previous studies [3,4,7] the magnetic dipole interaction dominantly determines the properties of spin waves under consideration. The character of the magnetic dipole interaction is nonlocal, and, as a consequence, the tunneling transmission coefficient depends nonexponentially on the barrier width.

The used experimental setup is schematically shown in Fig. 1. Microwave spin wave packets in an optically transparent yttrium-iron-garnet (YIG) film are generated by a strip-line antenna and are detected using the time- and space-resolved Brillouin light scattering (BLS) technique [10]. Both a homogeneous external field and the static magnetization are oriented in the plane of the film parallel to the propagation direction of the spin waves, z. In this case the dynamic magnetization components are m_x and m_y. Such an orientation of the field and the magnetization corresponds to the backward volume magnetostatic wave (BVMSW) geometry, characterized by a negative group velocity of the waves [11]. The microwave

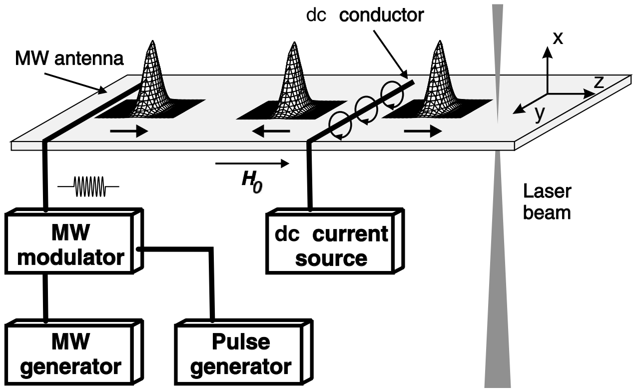

FIG. 1. Schematic layout of the Brillouin light scattering setup in the forward scattering geometry with space and time resolution for the study of spin-wave propagation through an inhomogeneous magnetic field. For a discussion of the components, see the main text.

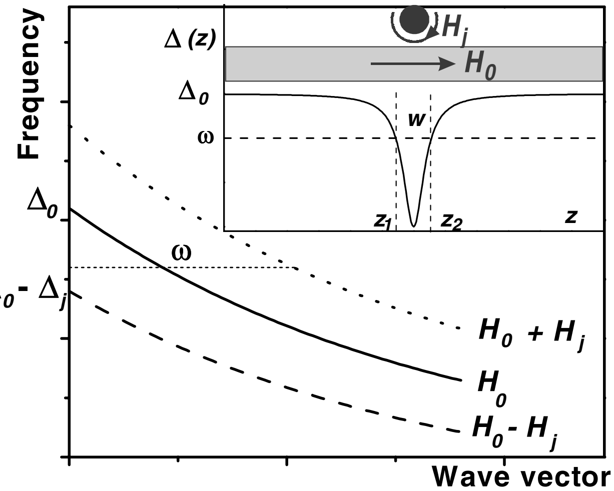

FIG. 2. Spectrum of BVMSW in a magnetic film. $\Delta(z)$ is the gap of the spectrum. The inset illustrates the profile of the gap, the creation of the turning points, and the forbidden interval due to the inhomogeneous field, caused by the dc conductor.

excitation part consists of a microwave generator and a modulator, which is controlled by a pulse generator (pulse length 10–30 ns) and connected to the antenna situated on the YIG film for spin-wave generation. BVMSW packets are generated with a carrier frequency $\omega = 2\pi \times 7.095$ GHz and a carrier wave vector $q = 210$–$220\ \mathrm{cm}^{-1}$, the value of q being determined by the dispersion relation of the spin wave in an external field of $H_0 = 1840$ Oe. A narrow conductor of 50 μm diameter mounted across the film carries a dc current. It is used to create a local inhomogeneous field, $H_j(z)$. Depending on the direction of the dc current, the total field (and, thus, the gap of the spin-wave spectrum) is locally either enhanced or reduced by the oersted field of the current up to a maximum field inhomogeneity of about 200 Oe.

The idea of the experiment is further illustrated in Fig. 2. As mentioned above, the frequency of the BVMSW decreases with increasing wave vector, and the allowed states are situated below the zero-wave-vector gap, Δ_0. Thus, to realize a scenario of spin-wave reflection from a field inhomogeneity, one should rather decrease, than increase the local field (gap). The inset of Fig. 2 illustrates the geometry of the field, the profile of the gap, and the creation of turning points, which are determined by the following condition: $\omega = \Delta[H(z) = H_0 - H_j(z)]$. As is seen from the inset of Fig. 2, this equation has two solutions, z_1 and z_2. The interval between the turning points is a prohibited region with no spin-wave state with ω. The width of the interval $w = z_2 - z_1$ is considered as the barrier width. On the other hand, an enhanced magnetic field does not essentially disturb the propagation: spin waves propagate through the inhomogeneity while increasing the wave vector according to the local field.

The propagation of spin-wave packets through the field inhomogeneity, measured using the space- and time-resolved Brillouin light scattering technique, is illustrated by Fig. 3 and the movies [12]. Two sequences of snapshots for different delay times are shown. Figure 3(a) [12] corresponds to the enhancement of the local field by the dc current, whereas the images shown in Fig. 3(b) [12] were obtained when the local field was reduced. Each snapshot displays the distribution of the spin-wave intensity (normalized to its maximum) as a function of the z coordinate.

Figure 3(a) [12] demonstrates no significant reflection of the spin waves. As is discussed above, a region with a slightly enhanced local field cannot contain a turning point for a BVMSW packet. The wave accommodates its wave vector according to the dispersion law and passes through the region of the field inhomogeneity almost unaffected [13].

On the other hand, the region of the reduced field inhibits propagation of spin waves as seen in Fig. 3(b) [12]. The spin-wave packet, however, is only partially reflected, and a certain part of it is transmitted through the barrier. The transmission and reflection probabilities depend on the barrier width and height, and the carrier wave vector of the wave. Indeed, this effect is reminiscent of the quasiclassical problem of a particle reflection and tunneling in quantum mechanics.

To understand the physics of the observed spin-wave tunneling the transmission coefficient, T, defined for the intensity of the wave, was measured as a function of the dc current. In agreement with the above consideration, T is close to unity until the current reaches a certain value, j_c, which corresponds to the creation of a turning point just below the dc conductor. Figure 4(a) demonstrates the normalized barrier transmission coefficient, $T_b = T(j)/T(j_c)$. As is seen in the inset of Fig. 2, it is possible to increase the distance between the turning points (i.e., the barrier width) by increasing the value of the dc current. The profile of the inhomogeneous field, the positions of the turning points and, accordingly, the width

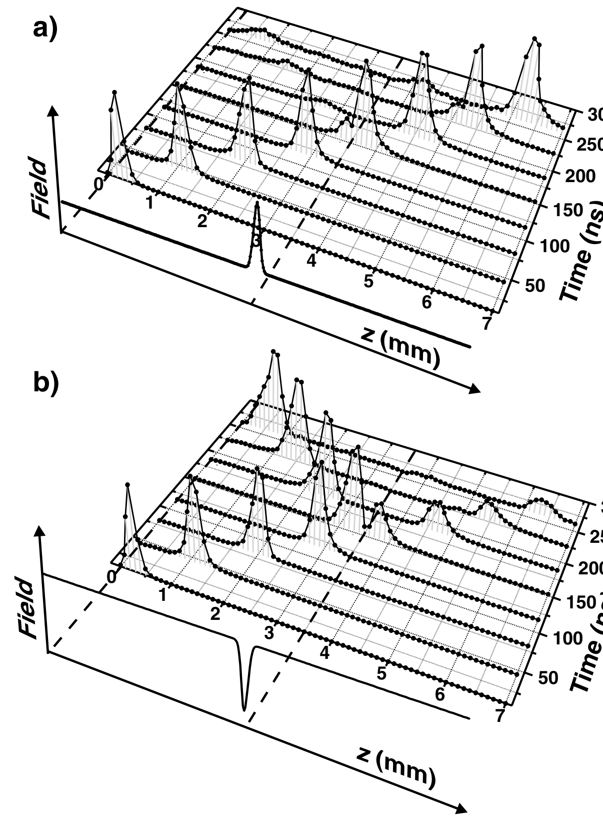

FIG. 3. Propagation of a spin-wave packet across a YIG film with local field inhomogeneity, observed by space- and time-resolved BLS. (a) Field/gap has a local maximum, creating a potential dip for the wave; (b) field/gap has a local minimum, creating a potential barrier for the wave. The maximum absolute value of H_j is the same for both cases and is 56 Oe. An animated version of this figure can be viewed in the online/PDF version of this Letter.

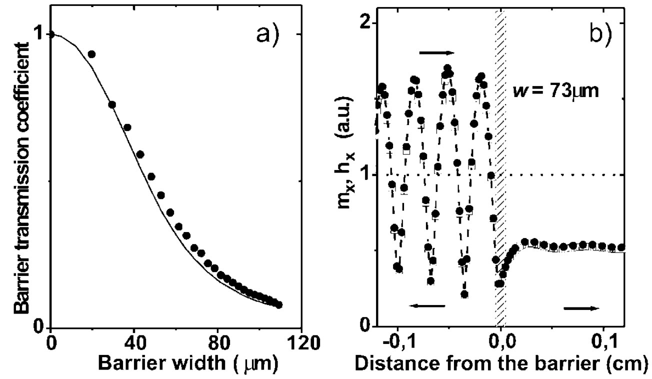

FIG. 4. (a) Barrier transmission coefficient as a function of the barrier width. Full circles: experimental values. Solid line: results of the numerical simulations using Eq. (5). (b) Profiles of the dynamic magnetization, m_x, (full circles) and dipole field, h_x, (open squares) in the wave, normalized to the values of the incident wave. The dashed line is a guide to the eye. Incident, reflected, and tunnelled waves are indicated by the arrows. The barrier is shown by the hatched area.

barrier, w, can be easily calculated for a given j in the experimental geometry. For the sake of clarity T_b is shown as a function of w. With increasing w the transmission coefficient decreases. From a comparison of the measured dependency $T_b(w)$ with the predictions of different theoretical models, the origin of the spin-wave tunneling can be clarified.

Despite the obvious similarity to the quantum mechanic problem, the physics of the spin-wave tunneling effect is more complex since the long-range magnetic dipole interaction important for magnetic systems must be taken into account.

If both magnetic dipole and exchange interactions are taken into account, the Landau-Lifshitz equation of motion for the variable magnetization (which, in particular, describes spin-wave processes in a magnetic film) can be reduced to an integro-differential equation (see, e.g., [14]), where a differential operator of second order describes the inhomogeneous exchange interaction [see Eq. (1)], while the integral operator (having the dipolar Green's function as a kernel) describes the nonlocal dipole-dipole interaction. For long-wavelength spin waves ($q \approx 10^2$–10^3 cm^{-1}) we neglect the small contribution of inhomogeneous exchange interaction and assume harmonic time dependence for the variable magnetization $\propto \exp(i\omega t)$. The resulting equation for dipolar spin waves turns out to be a purely integral equation. This equation written for the transverse component m_x of the variable magnetization in a magnetic film has the form

$$4\pi M_S \int_{-\infty}^{\infty} G_{xx}(z, z')m_x(z')dz' + \left(H(z) - \frac{\omega^2}{\gamma^2 H(z)}\right)m_x(z) = 0, \quad (4)$$

where $G_{xx}(z, z')$ is a component of the magnetostatic Green's function in the coordinate representation (z is the coordinate along the direction of wave propagation) derived for a magnetic film of a finite thickness in our previous work [see Eq. (3) in [15]].

The analytical solution of the integral Eq. (4) for any realistic profile of the inhomogeneous external magnetic field $H(z)$ is rather complex. Therefore it has been solved numerically for the experimental field profile shown in the inset of the Fig. 2 using the iterational convergence method. As a zero-order approximation for m_x the analytical solution of Eq. (4) for the spatially homogeneous static magnetic field $H(z) = H_0$ with an excitation source is used:

$$m_x(z) = \int_{-\infty}^{\infty} \frac{(\chi_0' - i\chi_0'')\exp(iqz)\langle h_x^s\rangle_q}{\{1 - \chi_0'[P(q) - 1]\} + i\chi_0''[P(q) - 1]} dq, \quad (5)$$

where $\chi_0 = 4\pi M_S H_0/[H_0^2 - (\omega/\gamma)^2]$ is the dynamic susceptibility at $H = H_0$, $P(q)$ is the dipole matrix element calculated in [16], and $\langle h_x^s\rangle_q$ is the spatial Fourier component of $h_x^s(z)$, the magnetic microwave field of the antenna which excites the propagating waves in the film. Wave damping was taken into account in the usual way [14], i.e., by adding an imaginary part to the static magnetic field: $H_0 \rightarrow H_0 + i\Delta H$, where ΔH is the linewidth of ferromagnetic resonance of the film.

The condition $\{1 - \chi_0'[P(q) - 1]\} = 0$ [see Eq. (5)] provides the dispersion equation for BVMSW. Since for large fields it can be satisfied for real values of q, the amplitude of the variable magnetization is relatively large. Contrary to that, for small H, where the above condition cannot be satisfied, the denominator in Eq. (5) becomes large, which leads to the decrease of magnitude of the dynamic magnetization.

A numerical algorithm providing fast convergence of the solution of Eq. (4) with the initial function $m_x(z)$ defined by Eq. (5) has been constructed. We used the fast Fourier transform algorithm for discrete values of z and q and then employed by the Runge-Kutta technique to solve the resulting system of differential equations.

The calculated dependence of the barrier transmission coefficient on the barrier width, $T_b(w)$, is shown in Fig. 4(a) by the solid line. Obviously, the obtained dependence is not exponential (contrary to the exchange case), due to the nonlocal character of the long-range magnetic dipole interaction. The quantitative agreement between the theory and the experiment is striking, especially if one takes into account that no fitting parameters were used in the numerical calculations. Such an agreement allows us to conclude that the observed effect can be interpreted as a tunneling effect of dipolar spin waves through the field inhomogeneity. It is important to mention that not only the dipole field, but also the dynamic magnetization tunnels through the barrier. This is illustrated by Fig. 4(b), where the calculated profiles of the dynamic magnetization and of the dipole field are presented. Both profiles are normalized by the values of the incident wave. It is clearly seen that the observed tunneling effect is, indeed, a tunneling of the dynamic magnetization and the dipole field. In fact, both values are connected by the local value of the dynamic susceptibility $\chi = 4\pi M_S H/[H^2 - (\omega/\gamma)^2]$ [11]. Since as it was mentioned above the value of the static field is just slightly changed by the field inhomogeneity, the relations between m_x and h_x are almost the same inside the barrier and far from it. A standing wave on the left side of the barrier is due to the interference between the incident and reflected waves. This interference is observed in the experiment if longer spin-wave pulses, as shown in Fig. 3 [12], are used.

In conclusion, we have demonstrated, both experimentally and by numerical simulation, that tunneling of dipolar spin-wave pulses takes place when it encounters a "well"-type localized inhomogeneity in the course of its propagation in a magnetic film. The dependence of the dynamic magnetization on the propagation coordinate, $m(z)$, inside the prohibited region is nonexponential, due to the nonlocal character of the dominating magnetic dipole interaction.

Support by the Deutsche Forschungsgemeinschaft, by the European Communities Human Potential programme under Contract No. HRPN-CT-2002-00318 ULTRASWITCH, by European Commission under Contract No. IST-2001-37334-NEXT, by NSF Grant No. DMR-0072017, and by the joint NSF-DAAD Grant No. INT-0128823 is gratefully acknowledged. M. K. is indebted to the DAAD for financial support.

*Electronic address: demokrit@physik.uni-kl.de
†Also at Department of Radiophysics, Taras Shevchenko National University of Kiev, Kiev, Ukraine.

[1] D. K. Roy, *Quantum Mechnical Tunneling and Its Application* (World Scientific, Singapore, 1986).
[2] A. Enders and G. Nimtz, Phys. Rev. E **48**, 632 (1993); Ph. Balcou and L. Dutriaux, Phys. Rev. Lett. **78**, 851 (1997); I. I. Smolyaninov *et al.*, Phys. Rev. Lett. **88**, 187402 (2002).
[3] E. Schlömann, in *Advances in Quantum Electronics*, edited by J. R. Singer (Columbia University Press, New York, 1961); E. Schlömann and R. I. Joseph, J. Appl. Phys. **35**, 167 (1964).
[4] J. R. Eshbach, Phys. Rev. Lett. **8**, 357 (1962); J. Appl. Phys. **34**, 1298 (1963).
[5] R.W Damon and H. van de Vaart, J. Appl. Phys. **36**, 3453 (1965); , **37**, 2445 (1966).
[6] A. V. Vashkovskii, A. V. Stal'makhov, and B. V. Tyulyukin, Tech. Phys. Lett. **14**, 1294 (1988).
[7] J. Jorzick, S. O. Demokritov, B. Hillebrands, M. Bailleul, C. Fermon, K. Y. Guslienko, A. N. Slavin, D. V. Berkov, and N. L. Gorn, Phys. Rev. Lett. **88**, 047204 (2002).
[8] J. P. Park, P. Earnes, D. M. Engebretson, J. Berezovsky, and P. A. Crowell, Phys. Rev. Lett. **89**, 277201 (2002).
[9] C. Bayer, S. O. Demokritov, B. Hillebrands, and A. N. Slavin, Appl. Phys. Lett. **82**, 607 (2003).
[10] S. O. Demokritov, B. Hillebrands, and A. N. Slavin, Phys. Rep. **348**, 441 (2001).
[11] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids **19**, 308 (1961).
[12] See EPAPS Document No. E-PRLTAO-93-007424 for auxiliary images. A direct link to this document may be found in the online article's HTML reference section. The document may also be reached via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html) or from ftp.aip.org in the directory /epaps/. See the EPAPS homepage for more information.
[13] An observed weak reflection is due to the nonadiabatical nature of the process; in fact, the above discussion implies that the wavelength of the wave is much smaller than the lateral scale of the field inhomogeneity. In the experiment, however, these two values are of the same order of magnitude.
[14] A. G. Gurevich and G. A. Melkov, *Magnetization Oscillations and Waves* (CRC Press, New York, 1996).
[15] K. Y. Guslienko, S. O. Demokritov, B. Hillebrands, and A. N. Slavin, Phys. Rev. B **66**, 132402 (2002).
[16] B. A. Kalinikos and A. N. Slavin, J. Phys. C **19**, 7013 (1986).