Interfacial Energy of the Superfluid ^{3}He A-B Phase Interface in the Zero-Temperature Limit

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We have measured the surface energy of the interface between the A and B phases of superfluid ^{3}He in the low temperature limit at zero pressure. Using a shaped magnetic field, we control the passage of the phase boundary through a small aperture. We obtain the interphase surface energy from the over- or undermagnetization required to force the interface through the aperture in both directions, yielding values of the surface tension and the interfacial contact angle. This is the first measurement of the interfacial energy in high magnetic fields and in the zero-temperature limit.

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The helium superfluids provide unique systems for probing macroscopic quantum behavior, especially at very low temperatures where the condensate is not masked by the "statistical noise" of thermal excitations. Superfluid ^{3}He, having several distinct phases, allows us to study the phase boundary between two condensates with different symmetries. The superfluid ^{3}He A-B boundary is the most-ordered phase boundary to which we have experimental access, and is analogous to an interface between two different quantum vacua [1].

The surface tension is a crucial quantity for understanding the superfluid structure in that it carries information not available from bulk thermodynamic measurements on the individual phases [2,3]. Surprisingly, how the order parameters transform across the phase interface, generating the interfacial energy, a most fundamental property of the interface, has hardly been studied experimentally. While superfluid ^{3}He is perhaps the best-understood ordered condensate system we have, and despite the quantity of theoretical work in this area, there is just one previous measurement at melting pressure [4].

Here we report the first measurement of the A-B phase interfacial energy at zero pressure in the $T \approx 0$ limit. We use a magnetic field to induce a transition from the B phase to the A phase and to stabilize and control the boundary between them.

A useful scale for the interfacial surface energy σ_{AB} comes from considering an interface of thickness ξ, the coherence length, with an energy density equal to the superfluid condensation energy density F_0, giving a surface energy of ξF_0. In reality, the order parameter must follow a complex trajectory from the axial A phase to the pseudoisotropic B phase via the planar state [5], giving a boundary thicker than ξ but with an energy density lower than F_0. Nevertheless, the value ξF_0 is used as a convenient scale for the interfacial surface tension.

Several calculations have been reported in the Ginzburg-Landau (GL) regime close to T_c at the polycritical point (PCP), where the A and B phases coexist in zero magnetic field but where strong coupling is significant. Cross [6], Kaul and Kleinert [7], and Schopohl [8] calculated the additional free energy associated with the smooth transformation from the A- to B-phase order parameters via nonequilibrium interface states. The three assume slightly different order parameter trajectories through the interface and derive surface energies of 1.1, 0.77, and $0.71\xi F_0$, respectively. Later, Thuneberg [9] with a full numerical solution of the Ginzburg-Landau equations on surfaces, found the surface tension to be $0.71\xi F_0$ at the PCP and calculated the surface tension for pressures below the PCP for a phase boundary stabilized by a magnetic field. At zero bar, the superfluid is thought to be in the weak-coupling limit for which GL theory predicts a second-order transition, and, hence, zero surface tension [9]. Outside the GL region, Ashida and Nagai [10] made microscopic calculations for the A-B transition under magnetic field at low pressures, predicting that the second order transition becomes first order below $0.8T_c$. While their calculated phase diagram agrees reasonably with experiment, they did not calculate the surface tension. However, since this derives from the superfluid condensation energy, it should map smoothly onto values in other parts of the phase diagram.

The only experimental value for the superfluid ^{3}He A-B interface surface tension comes from the old experiment of Osheroff and co-workers [4] at melting pressure (34 bars), from 0.5 to $0.8T_c$ in low magnetic field. Their value of $\sigma_{AB} \sim 0.7\xi F_0$ is held to be in reasonable agreement with the theoretical predictions even though made at higher pressures than the PCP and barely in the GL regime [7–9]. A measurement of the surface tension at very low temperatures, where there is a negligible normal fluid background and at low pressures where strong coupling corrections are small, should provide a more rigorous test of the "rigidity" of the order parameters and provide the impetus for further microscopic calculations.

The interfacial surface energy also governs the contact angle θ made by the interface at a solid wall, since the surface tension σ_{AB} and the wall surface energies, σ_{AW} and σ_{BW} for A and B phases, respectively, form a triangle of forces: $\sigma_{BW} - \sigma_{AW} = \sigma_{AB}\cos\theta$. The angle θ is thus

sensitive to the way the order parameter transforms between the A and B phases and between the two phases and a solid boundary. On glass at melting pressure and low field, Osheroff [11] measured 68°. Ginzburg-Landau calculations for a zero field at the polycritical point have produced contradictory results. Cross [6] calculated a contact angle in the range 60° to 80°, whereas Thuneberg [12] found 39° to 65°, the ranges giving limits for specular and diffuse quasiparticle scattering, respectively. It is thus not clear whether the measured value indicates that scattering at glass is definitely diffuse, as per Thuneberg, or somewhat specular, as per Cross. Measurements of θ on different surface morphologies should provide an additional test of theoretical calculations.

In the present experiment, we utilize the classical technique of measuring the interfacial energy by noting the free energy excess needed to "pop" the interface through a well characterized aperture [13]. We also determine the contact angle from the different "popping" behavior in the two directions ($A \rightarrow B$ or $B \rightarrow A$) (see below). We use techniques already developed [14] to manipulate the position of the A-B interface using a variable magnetic field profile. The equilibrium interface is stabilized at the critical field $B_{AB} \sim 340$ mT at 0 bar and low temperatures [15]. We move the interface by varying the field profile, and infer its velocity and relative position from the latent heat released by the motion.

The experiment, shown in Fig. 1, forms part of a Lancaster-style double nuclear-demagnetization stage [14]. The measurements are made in a tailpiece of the inner cell which constitutes a quasiparticle blackbody radiator [16,17] with a weak thermal link to the bulk liquid in the inner cell via the small radiator orifice.

The radiator body shown in Fig. 1 is made from a long sapphire tube, inner diameter 4.3 mm, closed at the bottom end. The top is closed by a thin lid of epoxy-impregnated paper containing the radiator orifice. To perform the experiment, we stabilize the A-B interface across the diameter of the tube (with A phase in the lower part) and move the equilibrium position up and down over a known length. The greater part of the required magnetic field is provided by a main solenoid with two smaller solenoids to adjust the field profile.

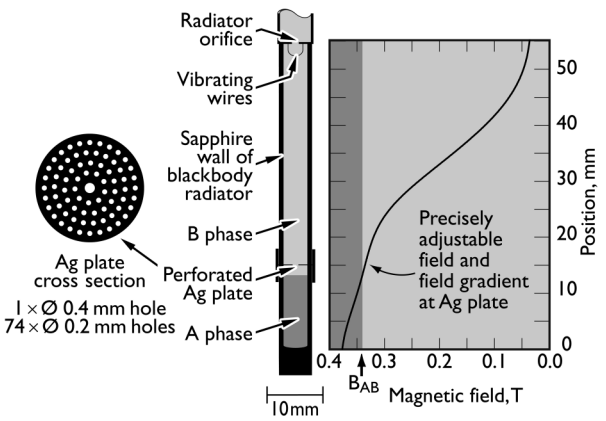

FIG. 1. The inner experimental cell and the field profile used to manipulate the A-B phase interface.

A 0.1 mm thick silver plate is glued between two sections of the sapphire tube 15.5 mm from the bottom. Drilled through the plate is a well-defined 0.400 mm aperture, positioned axially to minimize radial field gradients. To improve thermal contact between the upper and lower volumes of ^{3}He, the plate is also perforated by a further 74 0.2 mm diameter holes. These do not interfere with the measurement since the interface will emerge through the largest aperture first. After drilling, the aperture plate was polished to optical quality with alumina powder to reduce the possibility of surface-induced phase nucleations which might otherwise bypass the assumed emergence of the interface through the aperture.

At the top of the radiator are two vibrating wire resonators (VWRs), one acting as a heater and the other as a thermometer, operated in fields less than 80 mT to ensure that the surrounding B phase is undistorted. The frequency width Δf_2 of the VWR resonance varies as $\exp(-\Delta_B/k_BT)$, with Δ_B the B-phase energy gap, providing extremely sensitive thermometry [16]. When heat is supplied to the superfluid in the radiator, the temperature rises until the excitation flux leaving the orifice balances the power supplied. The RC-time constant for this process is 10–15 s with A phase in the lower part of the radiator. Residual heat leaks limit the base temperature of ~ 150 μK. We have used this technique to determine the latent heat of the transition [14], but here we are simply using the temperature changes to infer the relative motion of the interface.

To understand how the experiment proceeds, consider first the free movement of the A-B interface unconstrained by the perforated plate. As the field is increased, B phase is converted to A phase and the interface moves up the cell with a velocity $v = -\dot{B}/\nabla B$, where $\nabla B\,(<0)$ is the local field gradient and $\dot{B}$ is the ramp rate. The interface motion absorbs heat as $\dot{Q}_{AB} = LAv$, with L the volume latent heat and A the interface area, causing the radiator to cool. Conversely, when the field is reduced, the interface moves down the radiator producing a corresponding heating.

Figure 2 shows changes in temperature arising from the interface motion during a typical field cycle. We begin at high field with A phase in the bottom of the cell, B phase on top, and the interface above the silver plate as illustrated at the top of Fig. 2. As the main solenoid current is reduced, the equilibrium interface position moves down the cell, converting A to B phase and warming the cell with the release of latent heat. At X_1, the interface reaches the plate and is pinned there while the equilibrium interface position continues to move down the cell. The warming ceases and the temperature relaxes back towards the background level at a rate governed by the radiator time constant. However, before reaching the background level,

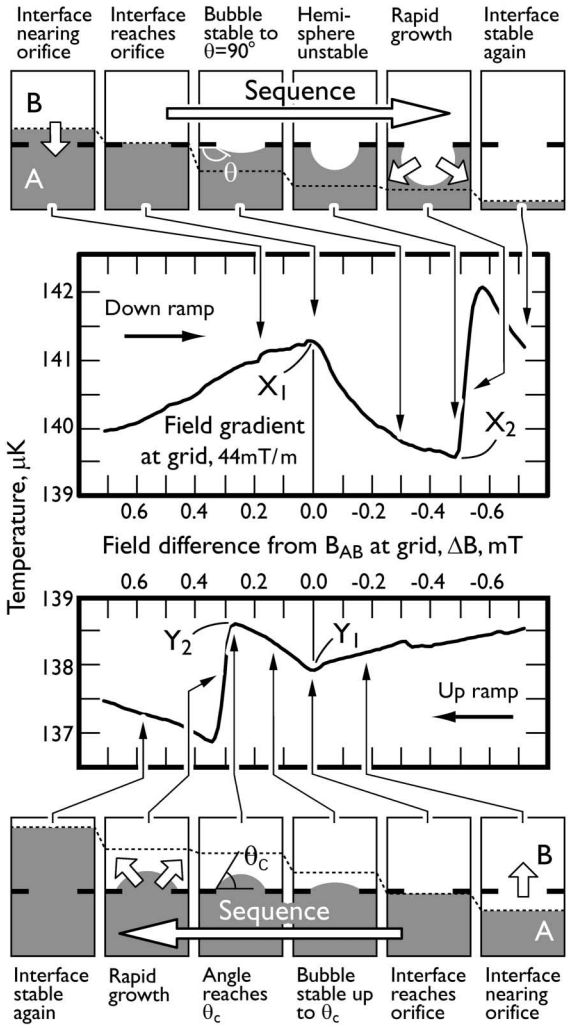

FIG. 2. The measured temperature as the interface moves through the cell during up and down ramps of the main solenoid current, versus the magnetic field at the grid relative to the equilibrium critical field B_{AB}. For labeled features see text. The horizontal dotted line marks the position of B_{AB}.

a sudden increase in warming at X_2 marks the point where the interface pops through the aperture moving rapidly to its bulk equilibrium position below the plate with the conversion of a large volume of A phase into B phase. The rounding of the features reflects the time constants of the VWR and associated electronics.

During the reverse up ramp, seen in the lower part of Fig. 2, we see cooling as latent heat is absorbed when B phase is converted to A phase. At point Y_1, the interface is again trapped by the plate, cooling ceases, and the cell temperature relaxes towards the background as the equilibrium interface position continues to move up the cell. At Y_2, the interface pops through the aperture, marked by a sudden increase in cooling.

Figure 3 shows the magnetic field difference at the aperture between the features X_1 and X_2 (solid circles), and Y_1 and Y_2 (empty circles) corrected for the Ag plate thickness, plotted against the field gradient at the perforated plate. In zero field gradient, the interface bulges out as a spherical cap whose radius of curvature, r, is related to the difference in volume free energy between the two phases, $\Delta G = 2\sigma_{AB}/r$. Here, $\Delta G = \frac{1}{2}\Delta\chi(B_{AB}^2 - B_i^2)$, where B_i is the field at the interface, and $\Delta\chi$ is the magnetic susceptibility difference between the A phase and the *magnetized* B phase, measured previously [14].

For the down ramps, the B phase is pulled through the aperture into the A phase. The interface pops when the bubble is hemispherical and $r = r_a$, the aperture radius. The surface tension can then be obtained from

$$\sigma_{AB} = (1/2)\Delta B B_{AB}\Delta\chi r_a, \tag{1}$$

where $\Delta B = B_{AB} - B_i \ll B_{AB}$ is the field difference between the features X_1 and X_2 as plotted in Fig. 3. An average of our zero field gradient data gives a value of $\sigma_{AB} = (3.03 \pm 0.28) \times 10^{-9}\ \mathrm{J\,m^{-2}} \equiv 0.38\xi F_0$. In finite field gradients, the field B_i at the interface is no longer uniform. The bubble shapes are no longer hemispherical and a larger field difference is required for the volume energy to overcome the surface energy. This we have analyzed by the method of Pozrikidis for axisymmetric drop shapes [18]. The calculated line shown for the down ramps has no free parameters and depends only on σ_{AB} (for details see [19]).

From a previous experiment [20] with a stack of glass tubes, we inferred from the capillary action a wall surface energy difference $\Delta\sigma_W = \sigma_{BW} - \sigma_{AW}$ of $(1.14 \pm 0.12) \times 10^{-9}\ \mathrm{J\,m^{-2}}$ for glass (a subsequent more accurate value than that quoted in [20]). Combining that result with the current measurement of σ_{AB}, we find the contact angle on glass to be $68° \pm 5°$. This is in startling agreement with the value of 68° obtained by Osheroff *et al.* [11] on glass at melting pressure. Given the large differences in condensation energy, coherence length, etc. between

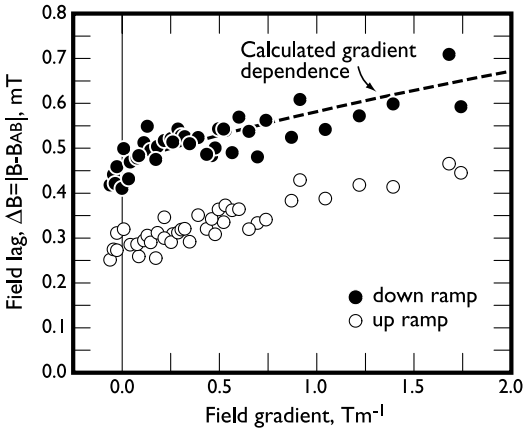

FIG. 3. The field-gradient dependence of the excess magnetic field needed to pop the interface through the central aperture. Filled points: B phase expanding into A phase. Empty points: vice versa. The dashed line shows the calculated gradient dependence with the surface tension σ_{AB} as a fitting parameter (see text).

our low pressure, low temperature regime and the high pressure, high temperature regime of the measurement of Osheroff *et al.*, we find it quite surprising that the two numbers are virtually identical.

Turning to the up ramps, for a given field gradient the value of ΔB needed to pop the A phase into the B phase is always smaller than that for B to A. There is no history dependence or stochasticity to indicate nucleation rather than popping [21]. We speculate that the difference arises from the preferential wetting by the A phase. While the A phase is bulging into the B phase, the inside angle between interface and plate surface is increasing from zero. However, it does not need to reach 90° to become unstable. As soon as the interface reaches the equilibrium contact angle θ_c with the plate surface, the interface is free to spread across the surface. For zero gradient, the "popping condition" for the A phase to emerge through the aperture is thus given by replacing σ_{AB} in (1) with $\sigma_{AB}\sin\theta_c$. Averaging the zero field gradient data for the up ramps in Fig. 3 thus gives an apparent contact angle on silver of $37° \pm 5°$.

It may seem surprising that the inferred contact angle on silver is much smaller than that for glass, given that theoretically the contact angle is expected to increase for a surface with more diffuse scattering. However, we should stress that the silver (polished with 0.3 μm powder) is rough on the scale of the coherence length. This may give an increase in the effective surface area/wall energies and, hence, a corresponding decrease in the apparent contact angle [22].

In conclusion, we have made the first measurements of the surface energy of an A-B interface in the low temperature, low pressure regime, finding σ_{AB} to be $(3.03 \pm 0.28) \times 10^{-9}\ \mathrm{J\,m^{-2}}$ at zero pressure. While such measurements reveal information about the condensates unavailable from bulk thermodynamic measurements [2,3], theoretical calculations exist only for the GL regime which give zero surface tension in the weak-coupling limit. Extending these calculations to the zero-temperature regime remains a major theoretical challenge. We have also measured the contact angle of the A-B interface on glass and an effective contact angle on polished silver. Surprisingly, our result is virtually identical to the one previous measurement at the opposite corner of the phase diagram. Although the contact angle, depending on surface properties, is a less fundamental quantity than σ_{AB}, we are nevertheless led to speculate that the contact angle on a flamed glass surface may be an intrinsic property of the condensates independent of pressure and temperature. We hope this work spurs further experimental and theoretical research in the field.

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