Probing the Speed of Light with Radio Waves at Extremely Low Frequencies

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The speed of light, a fundamental physical constant and thought to be independent of frequency, is tested here with naturally occurring radio waves in the atmosphere at extremely low frequencies. It is shown that the speed of light in the frequency range 5–50 Hz is known with an accuracy determined by perturbations of the ionospheric reflection height associated with space weather phenomena, which place an upper limit on the photon rest mass $m_\gamma \lesssim 4 \times 10^{-52}$ kg to date.

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Naturally occurring lightning discharges in the troposphere transmit radio waves at extremely low frequencies $\lesssim 50$ Hz [1], which propagate with little attenuation within the Earth's atmosphere [2], reflected between the conducting Earth and the time varying lower ionosphere at ~90–100 km height. The dispersion relation for the wave propagation at extremely low frequencies

$$\omega = kc\frac{1}{S} \tag{1}$$

is derived from Maxwell's equations [3–5] and connects the angular frequency ω to the wave number k, the speed of light c, and the radio wave propagation constant S, which describes ionospheric properties. The real part of the radio wave propagation constant

$$S = \sqrt{\frac{h_2}{h_1}} \tag{2}$$

is frequency dependent and describes the relative wave propagation velocity with the conduction boundary $h_1 \approx 50$ km, where the displacement and conduction current are on the same order of magnitude and the ionospheric height $h_2 \approx 100$ km, where the electromagnetic waves are reflected [3]. Since extraterrestrial ionization sources associated with space weather phenomena penetrate the atmosphere from the space above, the perturbed wave propagation constant is described during solar active conditions with a decrease Δh_2 of the ionospheric reflection height

$$S - \Delta S = \sqrt{\frac{h_2 - \Delta h_2}{h_1}} \approx S\left(1 - \frac{\Delta h_2}{2h_2}\right), \tag{3}$$

which relates the perturbed to the unperturbed wave propagation constant. The decrease of the ionospheric reflection height results in an increase $\Delta\omega$ of the frequency

$$\omega + \Delta\omega = kc\frac{1}{S - \Delta S}, \tag{4}$$

which is an observable and which can be inferred from magnetic field measurements of radio waves at extremely low frequencies [6].

The determination of ionospheric reflection height perturbations from observable frequency changes in Eq. (4) could provide in principle an unlimited accuracy. Yet, as the observed frequency change $\Delta\omega$ approaches zero, fundamental physical limitations $\delta\omega \lesssim \Delta\omega$ may be encountered, which are not associated with any ionospheric reflection height perturbation. These possible limitations are deviations from the wave number δk and deviations from the speed of light δc

$$\omega + \delta\omega = (k + \delta k)(c + \delta c)\frac{1}{S}. \tag{5}$$

It is sensible to place an upper limit on these fundamental limitations with the model derived from Maxwell's equations

$$(k + \delta k)(c + \delta c)\frac{1}{S} \lesssim kc\frac{1}{S - \Delta S} \tag{6}$$

by use of Eqs. (4) and (5) such that

$$\frac{\delta k}{k} + \frac{\delta c}{c} \lesssim \frac{\Delta h_2}{2h_2} \tag{7}$$

by use of Eqs. (3) and (6), and by neglecting quadratic terms of small deviations. In this way, the smallest ionospheric height perturbation Δh_2 in agreement with knowledge on the ionosphere provides an upper limit on deviations from the wave number and the speed of light.

In the absence of deviations from the wave number [$\delta k \to 0$ in Eq. (7)], an upper limit for the fractional accuracy of the speed of light is determined by a linear function of the ionospheric reflection height perturbation

$$\frac{\delta c}{c} \lesssim \frac{\Delta h_2}{2h_2}. \tag{8}$$

To illustrate this result, 12 year long magnetic field measurements of radio waves are used [6], which are recorded with circular loop antennas at Arrival Heights in the Antarctic since 1986 [7]. The mean fractional accuracy of the speed of light is computed from the difference between the observed wave propagation constant and theoretical modeling of the wave propagation constant from Maxwell's equations [6]. The resulting variability of the fractional accuracy of the speed of light with frequency can be constrained by theoretical modeling of a maximum ionospheric reflection height perturbation $\Delta h_2 \approx \pm 1.5$ km from Maxwell's equations, displayed for comparison in Fig. 1. This ionospheric reflection height perturbation determines the upper limit for the fractional accuracy of the speed of light in Eq. (8). It is hence obvious that the smallest observed ionospheric reflection height perturbation constrains the fractional accuracy of the speed of light inferred from radio waves at extremely low frequencies. Ionospheric reflection height variabilities $\sim$2.5 km associated with energetic charged particle precipitation [8], $\sim \pm 2$ km associated with the solar cycle and $\sim \pm 0.1$ km associated with a mean solar rotation period have been reported [6]. The best fractional accuracy of the speed of light at extremely low frequencies is then inferred from the solar rotation period and it is $\sim 5 \times 10^{-4}$ to date. The ratio of electric to magnetic units provides a fractional accuracy of the speed of light $\sim 1 \times 10^{-5}$ in a static approximation [9], while determinations from microwave frequencies up to the visible region and beyond are better than 1×10^{-11} [10–12]. Given the large differences between the quasistatic and dynamic fractional accuracies of the speed of light, it is not possible to exclude a frequency dependence of the speed of light.

The method proposed here provides the unique opportunity to improve the fractional accuracy of the speed of light by more subtle observations of ionospheric height perturbations and to translate the inferred fractional accuracy of the speed of light to an upper limit of the photon rest mass [13] by use of an appropriate dispersion relation [14]. In the absence of a fractional accuracy of the speed of light [$\delta c \rightarrow 0$ in Eq. (7)], the deviations of the wave number δk are constrained by ionospheric reflection height perturbations

$$\delta k \lesssim \frac{\omega}{c} \frac{\Delta h_2}{2\sqrt{h_1 h_2}} \tag{9}$$

by use of Eqs. (1), (2), and (7). The expression for δk is related to the inverse Compton wavelength

$$\mu = \frac{m_\gamma c}{\hbar} = \frac{\delta k}{\sqrt{g}}, \tag{10}$$

where m_γ is the photon rest mass, $\hbar$ the Planck constant, and $1/\sqrt{g} \approx 10$ the mass sensitivity coefficient derived from theoretical modeling of electromagnetic waves in a spherical cavity at extremely low frequencies [15]. The upper limit of the photon rest mass is then determined from the expression

$$m_\gamma \lesssim \frac{\hbar}{\sqrt{g}} \frac{\omega}{c^2} \frac{\Delta h_2}{2\sqrt{h_1 h_2}} \tag{11}$$

by use of Eqs. (9) and (10). As a consequence of the frequency dependence in Eq. (11), the smallest upper limit for the photon rest mass is determined by the smallest radio wave frequency [16] and yields $m_\gamma \lesssim 4 \times 10^{-52}$ kg at $\sim$8 Hz for an ionospheric reflection height perturbation of $\sim$0.1 km. This direct measurement of an upper limit for the photon rest mass is smaller than virtual photon rest mass determinations from the Earth's and Jupiter's magnetic fields [17–21], illustrated for comparison in Fig. 2, but larger than indirect determinations

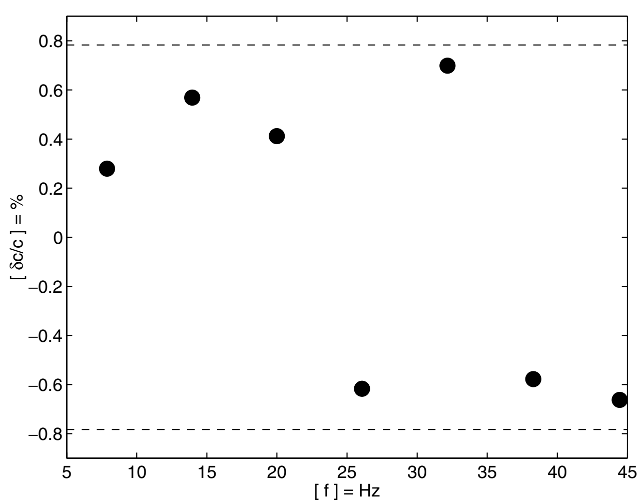

FIG. 1. The experimentally observed fractional accuracy of the speed of light (●) derived from theoretical modeling with Maxwell's equations can be constrained by a maximum ionospheric reflection height perturbation $\sim \pm 1.5$ km (dashed lines).

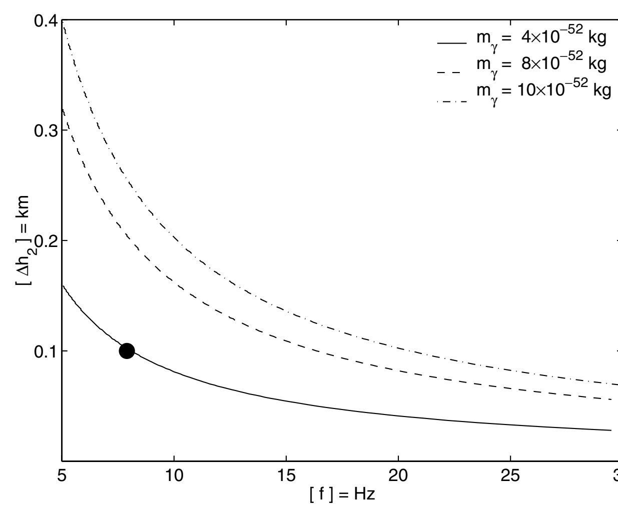

FIG. 2. The photon rest mass as a function of frequency and ionospheric reflection height perturbations (solid line). The smallest upper limit of the photon rest mass (●) is determined by the smallest radio wave frequency and compared to virtual photon rest mass determinations from the Earth's (dash-dotted line) and Jupiter's magnetic field (dashed line).

from the product of the photon rest mass squared and the estimated cosmic vector potential [22].

The two approximations in Eqs. (8) and (11) describe the limits for the fractional accuracy of the speed of light and the photon rest mass, respectively, and they are both limiting forms of Eq. (7). Since it is generally thought that a frequency dependence of the speed of light is a direct consequence of the photon rest mass [13,18], Eq. (7) represents are more stringent upper limit for both quantities, which may be connected to each other in a yet unexplored way. It is concluded that the detection of subtle ionospheric reflection height perturbations associated with space weather phenomena provides a unique opportunity to reduce both the uncertainty on the frequency dependence of the speed of light and the upper limit of the photon rest mass inferred from radio waves at extremely low frequencies.

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