Parton Structure of the Nucleon and Precision Determination of the Weinberg Angle in Neutrino Scattering

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A next-to-leading-order (NLO) calculation of neutrino cross sections, including power-suppressed mass terms, is used to evaluate the Paschos-Wolfenstein ratio, in order to better assess the validity and significance of the NuTeV anomaly. We study the shift of $\sin^2\theta_W$ obtained in calculations with parton distribution function sets that allow $s(x) \neq \bar{s}(x)$, enabled by recent neutrino dimuon data from CCFR and NuTeV. The extracted value of $\sin^2\theta_W$ is closely correlated with the strangeness asymmetry. Taken together with recent developments of possible isospin violation and electroweak effects, our results suggest that the new dimuon data, the Weinberg angle measurement, and other data sets used in global QCD parton structure analysis can all be consistent within the standard model. A full NLO analysis of the actual experimental measurement will help to clarify this issue further.

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Introduction.—An important open question in particle physics in recent years has been the significance of the "NuTeV anomaly"—a $3\,\sigma$ deviation of the measurement of $\sin^2\theta_W$ $(0.2277 \pm 0.0013 \pm 0.0009)$ reported in Ref. [1], from the world average of other measurements [2] (0.2227 ± 0.0004). Possible sources of the NuTeV anomaly, both within and beyond the standard model, have been examined in [3]. No consistent picture has yet emerged in spite of extensive literature [4–8] on this subject. The measurement in Ref. [1] was based on a correlated fit to the ratios $R_{\rm exp}^{\nu,\bar{\nu}}$ of "long" and "short" events [dominated by charged and neutral current (CC and NC) interactions, respectively] in sign-selected neutrino and antineutrino scattering on a (primarily) iron target at Fermilab. This procedure is closely related (but not identical) to measuring the Paschos-Wolfenstein (PW) ratio [9], which provides the theoretical underpinning of the analysis. Specifically, the Paschos-Wolfenstein ratio R^- is related to the Weinberg angle θ_W by

$$R^- \equiv \frac{\sigma_{\rm NC}^{\nu} - \sigma_{\rm NC}^{\bar{\nu}}}{\sigma_{\rm CC}^{\nu} - \sigma_{\rm CC}^{\bar{\nu}}} \simeq \frac{1}{2} - \sin^2\theta_W + \delta R_A^- + \delta R_{\rm QCD}^- + \delta R_{\rm EW}^-, \quad (1)$$

where the three correction terms are due to the nonisoscalarity of the target (δR_A^-), next-to-leading-order (NLO) and nonperturbative QCD effects ($\delta R_{\rm QCD}^-$), and higher-order electroweak effects ($\delta R_{\rm EW}^-$). Since R^- is a ratio of differences of cross sections, the correction terms are expected to be rather small. But at the accuracy required to test the consistency of the standard model, all the perturbative and nonperturbative corrections need to be quantified as precisely as possible.

In this paper, we focus on QCD corrections, which are generally recognized [4–8] to be the least well known. Let us write

$$\delta R_{\rm QCD}^- = \delta R_s^- + \delta R_I^- + \delta R_{\rm NLO}^-, \quad (2)$$

where the three terms on the right-hand side are due to possible strangeness asymmetry ($s^- = s - \bar{s} \neq 0$) and isospin violation ($u_{p,n} \neq d_{n,p}$) effects in the parton structure of the nucleon, and NLO ($O(\alpha_s)$) corrections, respectively [10]. The original NuTeV analysis was carried out at LO in QCD and assumed $\delta R_s^- = 0 = \delta R_I^-$. Our analysis is based on the recent NLO calculation of [11], together with new parton analyses that explicitly allow strangeness asymmetry ($\delta R_s^- \neq 0$) [12] and isospin violation ($\delta R_I^- \neq 0$) [13]. The actual calculation is carried out at the cross section level, i.e., using the first line of Eq. (1) rather than using the schematic linearized form given in the second lines of Eqs. (1) and (2). Our results provide more realistic estimates of the sizes and uncertainties of the QCD corrections and a new look at the significance of the "anomaly." (Compare also a recent

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reevaluation of the electroweak correction to the calculation of R^- [14].)

NLO calculation.—At sufficiently high neutrino energy, the total neutrino cross section

$$\sigma^\nu \equiv \sigma^{\nu N\to lX} = \int d^3p_l \frac{d^3\sigma^{\nu N\to lX}}{d^3p_l} \quad (3)$$

can be calculated in QCD perturbation theory—in contrast to charged lepton scattering, where the massless photon propagator leads to dominance of nonperturbative photoproduction events over deep inelastic scattering. The differential cross section in Eq. (3) factorizes into a sum of convolutions of parton distribution functions (PDFs) and partonic cross sections

$$d^3\sigma^{\nu N\to lX} = \sum_{f=q,g} f \otimes d^3\sigma^{\nu f\to lX}. \quad (4)$$

This calculation has been performed at NLO accuracy in Ref. [11]. The analysis included target and charm mass effects. These corrections are needed to obtain reliable results because there are non-negligible contributions from low Q values to the integral in Eq. (3) — e.g., about 5% from $Q^2 < 1$ GeV^2 for $\sigma^{\bar\nu}_{CC}$ and from $Q^2 < 2$ GeV^2 for σ^{ν}_{CC}. (For NC neutrino events, it is not possible to exclude the low-Q region by experimental kinematic cuts.) Other corrections included are the nonisoscalarity of the target material (iron), i.e., δR^-_A in (1); energy averaging over the neutrino and antineutrino flux spectra; and cuts in hadronic energy ($20 < yE_\nu < 180$ GeV for lepton inelasticity y) as used in the experimental analysis [1].

Reference [11] used previously available parton distributions [15,16], all of which assume isospin symmetry and $s = \bar s$ symmetry within the nucleon. The study confirmed the smallness [17] of the higher-order corrections to R^- in general. (The same conclusion is reached by the NLO and NNLO moment analyses of [3,6,18].) It was also shown that the nonmonochromatic neutrino and antineutrino beams, with different profiles, and typical cuts in the hadronic event energy do not alter $\delta R^-_{\rm NLO}$ substantially. In the next two sections, we will examine shifts of the NLO calculation due to recent advances in global QCD analysis of parton distributions that allow strangeness asymmetry and isospin violation.

In principle, the parton distribution functions in Eq. (4) should be those of nuclear targets. Our calculation is done as an incoherent sum of contributions from parton densities of unbound nucleons. This approximation is reasonable in that we calculate only relative shifts between $[S^-] = 0$ and $[S^-] \neq 0$ PDFs, where $[S^-]$ is defined in Eq. (7); similarly for isospin. In fact, experimental information on nuclear PDFs is relatively scarce, and nuclear PDFs account only for leading twist two ($\tau = 2$) effects. Higher twists, whether they relate to nuclear modifications or not, are generally difficult to handle consistently. By limiting ourselves to $\tau = 2$, our error estimates may be underestimates.

Strangeness asymmetry.—Because the strange quark mass m_s is comparable to $\Lambda_{\rm QCD}$, the strange quark PDF is a nonperturbative component of the nucleon bound state. Except for the strangeness number sum rule,

$$\int [s(x) - \bar s(x)]dx = 0, \quad (5)$$

there is no fundamental or approximate symmetry that relates the strange quark PDF $s(x)$ to the antiquark PDF $\bar s(x)$. Limits on $s^- \equiv s(x) - \bar s(x)$ can, therefore, only be derived from data (or perhaps eventually from a lattice QCD calculation). Until recently, s^- has been largely unknown and usually assumed to vanish. However, the recently published CCFR-NuTeV data on dimuon cross sections in νN and $\bar\nu N$ scattering yield a direct handle on $s(x)$ and $\bar s(x)$, and hence on s^- [12], because the dimuon data reflect semileptonic decays of the charm quark in $W^+ s \to c$ and $W^- \bar s \to \bar c$ events.

An asymmetric strange sea in the nucleon ($s^- \neq 0$) contributes to a correction term to R^- at LO [3]. If the scale dependence of the parton distributions is neglected, i.e., $f(x, Q) \simeq f(x)$, and in the approximation of overlooking experimental cuts, the total cross section in Eq. (3) is sensitive to the second Mellin moment integrals $\int dx x f(x)$ of the PDFs [3,6]. Making the further approximation of an isoscalar target, and in the limit of a negligible charm quark mass, a strange sea asymmetry contributes at LO as

$$\delta R^-_s \simeq -\left(\frac{1}{2} - \frac{7}{6}\sin^2\theta_{\rm W}\right)\frac{[S^-]}{[Q^-]}, \quad (6)$$

where the strangeness asymmetry is quantified by

$$[S^-] \equiv \int x[s(x) - \bar s(x)]dx, \quad (7)$$

and $[Q^-] = \int x[q(x) - \bar q(x)]dx$ with $q(x) = [u(x) + d(x)]/2$ represents the isoscalar up and down quark combination.

By including the dimuon data, and by exploring the full allowed parameter space in a global QCD analysis, Ref. [12] presents a general picture of the strangeness sector of the nucleon structure. The strong interplay between the existing experimental constraints and the global theoretical constraints, especially the sum rule (5), places useful limits on acceptable values of the strangeness asymmetry momentum integral $[S^-]$. The limit quoted in [12] is $-0.001 < [S^-] < +0.004$. A large negative $[S^-]$ is strongly disfavored by both dimuon and other inclusive data. The strict sum rule (5) implies that a nonzero $s^-(x)$ function must change sign at least once. Studies in [12] demonstrate that the exact value of $[S^-]$ is a volatile quantity. The best fit "B" is a solution where negative $s^-(x)$ at low x is compensated by positive $s^-(x)$

at large x; this leads to positivity of the second moment integral in Eq. (7). The same trend had previously been observed in a fit to inclusive neutrino scattering [4]. Also, this behavior was anticipated by a dynamical model [19] based on baryon-meson fluctuations of the nucleon light-cone wave function [20].

We quantify the impact of the PDFs of Ref. [12] on the Paschos-Wolfenstein relation in Eq. (1) by employing the NLO neutrino cross section calculations of Ref. [11]. The PDF sets A, B, C of Ref. [12] represent good fits within the allowed parton parameter space. They all have $s(x) \neq \bar{s}(x)$, and $[S^-] > 0$. In our calculations, we employ these PDFs consistently; i.e., we use the full set of PDFs, not just their strange quark distributions.

The shift in R^- due to strangeness asymmetry, δR_s^-, is obtained as the difference:

$$\delta R_s^- \equiv R^-_{\{A,B,C,B^+,B^-\}} - R^-_{\text{CTEQ6}}. \tag{8}$$

These are given in the last column of Table I, along with a summary of the underlying PDFs. We show not only the preferred fit values for the sets A, B, C but also results for fits B$^{\pm}$ that were obtained by using the Lagrange multiplier method to push the limits of the allowed $[S^-]$ value in both directions somewhat beyond the preferred range as described in [12]. The quality of the fits is indicated by the relative χ^2 values, which are normalized to the reference solution B. Thus, the values in row B are 1.0 (italicized) by definition. The three preferred sets A, B, C are comparable in quality; the extreme sets B$^+$ and B$^-$ are clearly disfavored.

For a given value of "measured" R^-, a shift of the theoretical prediction, such as δR_s^-, leads to a shift in the extracted $\sin^2\theta_W$ value according to [cf. Eq. (1)]

$$\delta(\sin^2\theta_W) = \delta R_s^-. \tag{9}$$

The results of our calculation (Table I), along with the range $-0.001 < [S^-] < 0.004$ of Ref. [12], which is based on more extensive studies than just the fits shown in Table I, lead us to estimate the range of δR_s^-, hence $\delta(\sin^2\theta_W)$, to be $-0.005 < \delta(\sin^2\theta_W) < +0.001$.

TABLE I. Shifts in R^-, calculated with PDF sets of Ref. [12] (with nonzero $[S^-]$) compared to the value with the CTEQ6M set ($[S^-] = 0$), are given in the last column. The quality of these new fits is gauged by the relative χ^2 values (normalized to that of the reference set "B") for the dimuon data set [22] and for the subset of the global data set which have some sensitivity to $s^-(x)$ (labeled "inclusive I"). See [12] for details.

Fit	$[S^-] \times 100$	χ^2_{dimuon}	$\chi^2_{\text{inclusiveI}}$	δR_s^-
B$^+$	0.540	1.30	0.98	−0.0065
A	0.312	1.02	0.97	−0.0037
B	0.160	*1.00*	*1.00*	−0.0019
C	0.103	1.01	1.03	−0.0012
B$^-$	−0.177	1.26	1.09	0.0023

We find that the shift in R^-, calculated as an average over ν and $\bar{\nu}$ energies according to their flux spectra, is relatively insensitive to the incident neutrino energy. The values of δR_s^- in Table I are also approximately unchanged when the cut on yE_ν is eliminated. These findings suggest that the incorporation of other detector effects [6,23], which make the analysis in Ref. [1] more involved than a direct measurement of R^-, will not significantly impact the importance of the $[S^-]$ contribution to $\sin^2\theta_W$. (To estimate the size of detector-dependent effects, we have calculated $\delta\sin^2\theta_W$ using the prescription of [23], summarized in the functional $\int F[\sin^2\theta_W, s - \bar{s}; x]dx$. The shifts are within 30% of those presented above.)

The shift in $\sin^2\theta_W$ corresponding to the central fit B bridges a substantial part of the original 3σ discrepancy between the NuTeV result and the world average of other measurements of $\sin^2\theta_W$. For PDF sets with a shift toward the negative end, such as -0.004, the discrepancy is reduced to less than 1σ. On the other hand, for PDF sets with a shift toward the positive end, such as $+0.001$, the discrepancy remains.

More input on $s^-(x) = s(x) - \bar{s}(x)$ would, of course, be helpful in pinning down the contribution of strangeness asymmetry to δR^-. Measurements of associated production of charmed jets and $W^{\pm}$ bosons at the Fermilab Tevatron, at the BNL Relativistic Heavy Ion Collider, or at the future CERN Large Hadron Collider, would increase our knowledge of $s(x)$ and $\bar{s}(x)$ (cf. [24]). It will help that the "valence" density $s^-(x)$ is more easily accessible than the predominantly singlet $s(x) + \bar{s}(x)$, which is concentrated at small x; however, the low expected statistics will make this measurement extremely challenging. In principle it seems also feasible to study $s(x) - \bar{s}(x)$ on the lattice [25]. Unfortunately, the most relevant moment $[S^-]$ does not correspond to a local operator and cannot be calculated on the lattice.

Possible isospin violation.—Isospin symmetry holds to a good approximation in low energy hadron spectroscopy and scattering, but it is not an exact symmetry. The level of accuracy of the usual assumption of isospin symmetry at the parton level, e.g., $u_p = d_n$ and $d_p = u_n$, is largely unknown. Isospin symmetry violation effects at the parton level contribute a shift of the PW ratio R^- by

$$\delta R_I^- \simeq -\left(\frac{1}{2} - \frac{7}{6}\sin^2\theta_W\right)\frac{[D_N^- - U_N^-]}{[Q^-]}, \tag{10}$$

where $N = (p + n)/2$ and, as before, [ ] denotes the second Mellin moment.

There have been model studies [7] that indicate δR_I^- could be large enough to have an effect on the interpretation of the NuTeV anomaly. However, it is preferable to quantify the allowed range of uncertainty of this effect directly and by model-independent global analysis of the differences. Unfortunately, there are few experimental constraints on these small differences.

Nonetheless, the MRST Collaboration [13] recently made a first attempt to separate proton and neutron PDFs where isospin for the valence quarks is broken by a function with a single parameter κ. Within physically reasonable limits, they find the overall χ^2 of the global fit to be rather insensitive to κ. By Eq. (10), the determination of $\sin^2\theta_W$ via the measurement of R^- is thus subject to a non-negligible uncertainty due to isospin violation.

To make this point more concrete, we have applied the candidate PDFs from [13] to our NLO calculation, in the same spirit as the study of strangeness asymmetry discussed above. We find that the range of allowed κ parameter given in [13], $-0.7 < \kappa < 0.7$, implies

$$-0.007 \lesssim \delta R_I^- \lesssim 0.007, \tag{11}$$

and the best fit value of $\kappa = -0.2$ corresponds to a shift of $\delta R_I^- = -0.0022$. A one-parameter functional form may not be general enough to pin down the *true* isospin violations of the parton structure. Nevertheless, the large range of δR_I^- in Eq. (11) indicates that a reasonable theoretical uncertainty due to isospin violation needs to be assigned to the determination of $\sin^2\theta_W$.

Conclusion.—The uncertainties in the parton structure of the nucleon that relate to R^- will not decrease substantially any time soon. The uncertainties in the theory that relates R^- to $\sin^2\theta_W$ are substantial on the scale of precision of the high statistics NuTeV data [1]. Within their bounds, the results of this study suggest that the new dimuon data, the Weinberg angle measurement, and other global data sets used in QCD parton structure analysis can all be consistent within the standard model of particle physics. The central value of the Weinberg angle measured in neutrino scattering will also depend on less speculative QCD and electroweak corrections [3,11,14,18] to the experimentally observable ν and $\bar{\nu}$ cross section ratios $R_{\rm exp}^{\nu,\bar{\nu}}$ [1] and a definitive statement will have to await a reanalysis by NuTeV.

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Note added.—After this manuscript was completed an investigation of the 3-loop perturbative strangeness asymmetry was presented in [26].

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