Atomic Fermi-Bose Mixtures in Inhomogeneous and Random Lattices: From Fermi Glass to Quantum Spin Glass and Quantum Percolation

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We investigate strongly interacting atomic Fermi-Bose mixtures in inhomogeneous and random optical lattices. We derive an effective Hamiltonian for the system and discuss its low temperature physics. We demonstrate the possibility of controlling the interactions at local level in inhomogeneous but regular lattices. Such a control leads to the achievement of Fermi glass, quantum Fermi spin-glass, and quantum percolation regimes involving bare and/or composite fermions in random lattices.

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Fermi-Bose (FB) mixtures attract considerable interest in the physics of ultracold atomic and molecular gases, comparable with the interest in molecular Bose-Einstein condensation [1], or Bardeen-Cooper-Schrieffer transition [2] in ultracold Fermi mixtures. The reason for interest in FB systems is threefold. First, they are very fundamental systems without direct analogues in condensed matter. Second, these systems can be efficiently cooled using sympathetic cooling down to very low temperatures (tens of nK) [3–6]. Finally, their physics is extremely rich and not yet fully understood.

FB mixtures have been intensively studied in traps [7], but the experimental observation of the superfluid to Mott-insulator (MI) transition in bosonic gases [14], predicted in Ref. [15], has triggered the interest in the physics of FB mixtures in optical lattices [16]. Under appropriate conditions such mixtures are described by the Fermi-Bose Hubbard model (FBH) [17]. A particularly appealing feature of the FBH model is the possibility to produce novel quantum phases [18], fermion-boson induced superfluidity [19], and composite fermions, which for attractive (repulsive) interactions between fermions and bosons, are formed by a fermion and bosons (bosonic holes) as shown in [20] (see also [21,22]).

FB mixtures in the limit of strong atom-atom interactions (strong coupling regime) show a very rich variety of quantum phases in periodic optical lattices [21]. They include the mentioned composite fermions, and range from a normal Fermi liquid, a density wave, a superfluid liquid, to an insulator with fermionic domains. The phase diagram of the system has been determined in Ref. [23] by means of mean-field theory [24]. These studies have been generalized recently to inhomogeneous lattices [25] to include the effects of the lattice and of a possible trap potential. So far, only the case of strong interactions and vanishing hopping has been considered.

In the present Letter we study the low temperature physics of FB mixtures in optical lattices with local and random inhomogeneities in the strong interactions limit but including tunneling as a perturbation. We show that interactions and tunneling may be controlled at the local level in inhomogeneous lattices [26]. This control gives access to a wide variety of regimes and we derive the corresponding effective Hamiltonians. We then show how to achieve Fermi glass, fermionic spin-glass, and quantum percolation regimes involving bare and/or composite fermions in random lattices.

We consider a sample of ultracold bosonic and (polarized) fermionic atoms (e.g., ^{7}Li-^{6}Li or ^{87}Rb-^{40}K) trapped in an optical lattice. At low temperature, the atoms occupy only the lowest energy band and it is convenient to work in the corresponding Wannier basis [15]. Note that a fermion number N_{F} strictly smaller than the number of lattice sites N is required here. The Hamiltonian of the system reads [17,27]:

$$H_{\mathrm{FBH}} = -\sum_{\langle ij\rangle}(J_{\mathrm{B}} b_i^\dagger b_j + J_{\mathrm{F}} f_i^\dagger f_j + \mathrm{H.c.}) + \sum_i\left[\frac{1}{2}V n_i(n_i-1) + U n_i m_i - \mu_i^B n_i - \mu_i^{\mathrm{F}} m_i\right], \tag{1}$$

where b_j, f_j are the bosonic and fermionic annihilation operators, $n_i = b_i^\dagger b_i$ and $m_i = f_i^\dagger f_i$. The FBH model describes: (i) nearest neighbor (nn) boson (fermion) hopping, with an associated negative energy, $-J_{\mathrm{B}}$ $(-J_{\mathrm{F}})$; (ii) on-site repulsive boson-boson interactions with an energy V; (iii) on-site boson-fermion interactions with an energy U, which is positive (negative) for repulsive (attractive) interactions, (iv) and, finally, interactions with the optical potential, with energies μ_i^{B} and μ_i^{F}. In the following, we consider only the case $J_{\mathrm{B}} = J_{\mathrm{F}} = J$ and the regime of strong interactions $(V, U \gg J)$.

In a periodic optical lattice, $\mu_i^{\mathrm{B,F}}$ is simply the (bosons or fermions) chemical potential and is independent of the site i. It is, however, possible to add a laser field independent of the lattice to modify the depths of the optical

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potential wells in a site-dependent way [28]. In this case, the local potential depth has to be added to $\mu_i^{\mathrm{B,F}}$, which may now be inhomogeneous. If the added field is periodic and if the spatial period is commensurate with the lattice period, $\mu_i^{\mathrm{B,F}}$ is periodic; if the spatial periods are incommensurate, $\mu_i^{\mathrm{B,F}}$ is quasiperiodic. One can also add a random speckle field, so that $\mu_i^{\mathrm{B,F}}$ is random. Experimental techniques offer full possibilities to control such periodic, quasiperiodic, or disordered $\mu_i^{\mathrm{B,F}}$ [29]. Note, that the additional inhomogeneous potential might, but does not have to, act equally on both atomic species. Here, we study the case $\mu_i^{\mathrm{F}} = 0$, $\mu_i^{\mathrm{B}} = \mu_i V$.

In Ref. [21] we have used the method of degenerate second order perturbation theory to derive an effective Hamiltonian by projecting the wave function onto the multiply degenerate ground state of the system in the absence of tunneling. This can be extended to the present situation, where there are very many states with similar energies. It is thus reasonable to project the wave function on the manifold of "ground states." These states are local minima of energy, since at least some of hopping acts increase their energy by V or $|U|$.

Let us consider first $J = 0$, and the case $0 \leq \mu_i < 1$. In the absence of a fermion one expects one boson per site, i.e., $n_i = 1$ [30]. We shall consider here only the case of repulsive interactions, i.e., $\alpha = U/V > 0$.

It is useful to divide the sites into (i) A sites, for which $\mu_i - \alpha \geq 0$, and fermions do not push bosons out, and (ii) B sites, for which $\mu_i - \alpha < 0$, and the fermion pushes the boson out forming a composite fermion-bosonic hole. Energetically, the second situation is favorable, so for a given set of N^A of A sites, and $N^B = N - N^A$ of B sites, the fermions will first occupy the B sites until $N_{\mathrm{F}} = N^B$, and then they will start to occupy the A sites. We construct the corresponding projector operators P, $Q = 1 - P$, which depend on N^{A} and N_{F}. The operator P describes the projection onto the manifold of quasidegenerated states in which the fermions occupy the B sites stripped of bosons and some A sites only if $N_{\mathrm{F}} \geq N^B$. In this case there is a boson in any A site and if $N_{\mathrm{F}} < N^B$ there is also a boson in the B sites which do not contain fermions. We use second order time dependent perturbation theory [32], and project the Schrödinger equation, $i\hbar\partial_t|\psi(t)\rangle = (H_0 + H_1)|\psi(t)\rangle$, onto the manifold of states spanning P. The "zeroth-order" part H_0 contains the atomic interactions and terms proportional to the chemical potential and commutes with P. H_1 represents the tunneling terms. The effective equation for $|\psi_P\rangle = P|\psi\rangle$ reads then $i\hbar\partial_t|\psi_P(t)\rangle = H_{\mathrm{eff}}|\psi_P(t)\rangle$, where

$$\langle \mathrm{out}|H_{\mathrm{eff}}|\mathrm{in}\rangle = \langle \mathrm{out}|H_0|\mathrm{in}\rangle + \langle \mathrm{out}|PH_1P|\mathrm{in}\rangle - \frac{1}{2}\langle \mathrm{out}|PH_1Q\left(\frac{1}{H_0 - E_{\mathrm{in}}} + \frac{1}{H_0 - E_{\mathrm{out}}}\right) \times QH_1P|\mathrm{in}\rangle. \quad (2)$$

The effective Hamiltonian H_{eff} has the form

$$H_{\mathrm{eff}} = \sum_{\langle ij\rangle}[-(d_{ij}F_i^{\dagger}F_j + \mathrm{H.c.}) + K_{ij}M_iM_j] + \sum_i \tilde{\mu}_i M_i, \quad (3)$$

where F_i is the (composite) fermionic annihilation operator, and $M_i = F_i^{\dagger}F_i$. The hopping amplitudes d_{ij} and the nn couplings K_{ij} [which might be repulsive (>0) or attractive (<0)] are of the order of J^2/V. The couplings depend on α, $\tilde{\mu}_i$, $\tilde{\mu}_j$, and J, and have to be determined carefully for different cases, as discussed below. Note, however, that the hopping $i \rightarrow j$, or back causes the energy change $\pm(\Delta_{ij} = \mu_i - \mu_j)$ in units of V, i.e., is highly nonresonant and inefficient for $\Delta_{ij} \simeq 1$; it first leads to jump rates of order $O(J^4/V^3)$. Additionally, composite fermions may feel the local energy $\tilde{\mu}_i$.

I. All sites are of type B.—In this case we have a gas of composites flowing within the MI with one boson per site. The couplings are

$$d_{ij} = \frac{J^2}{V}\left(\frac{\alpha}{\alpha^2 - (\Delta_{ij})^2} + \frac{1}{\alpha}\right), \quad (4)$$

$$K_{ij} = -\frac{J^2}{V}\left(\frac{4}{1 - (\Delta_{ij})^2} - \frac{2}{\alpha} - \frac{2\alpha}{\alpha^2 - (\Delta_{ij})^2}\right). \quad (5)$$

The chemical potential $\tilde{\mu}_i/V \simeq \mu_i$ up to corrections of order $O(J^2/V)$. The hopping amplitudes d_{ij} are for this case always positive, although may vary quite significantly with disorder, especially when $\Delta_{ij} \simeq \alpha$. As shown in Fig. 1, for $\alpha > 1$, $K_{ij} \leq 0$ and we deal with attractive (although random) interactions. For $\alpha < 1$, but close to 1, K_{ij} might take positive or negative values for Δ_{ij} small or $\Delta_{ij} \simeq \alpha$. In this case *the qualitative character of interactions is controlled by inhomogeneity.*

The physics of the system depends on the relation between μ_i's and α. For small inhomogeneities, we may neglect the contributions of Δ_{ij} to $d_{ij} \simeq d$ and $K_{ij} \simeq K$, and keep only the leading disorder contribution in $\tilde{\mu}_i$. Note, that the latter contribution is relevant in 1D and 2D leading to Anderson localization of single particles [33].

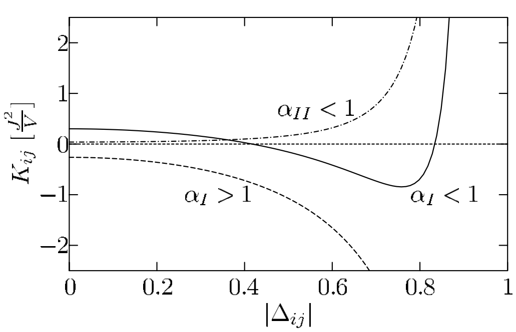

FIG. 1. Nearest neighbor couplings K_{ij} as a function of Δ_{ij}. Solid line: Coupling in case I, with $\alpha_{\mathrm{I}} = 0.93$. Dashed line: Same expression with $\alpha_{\mathrm{I}} = 1.07$. Dash-dotted line: Coupling in case II with $\alpha_{\mathrm{II}} = 0.03$.

When $K \ll d$ one will have a Fermi glass phase, i.e., Anderson localized (and many-body corrected) single particle states will be occupied according to the Fermi-Dirac rules [34]. For repulsive interactions and $K \gg d$, the ground state will be a Mott insulator for large enough filling factors. In particular, for filling factor $1/2$ a checkerboard phase is expected. For intermediate values of K/d delocalized metallic phases with enhanced persistent currents are possible [35]. Similarly, for attractive interactions ($K < 0$) and $|K| < d$ one expects competition between pairing of fermions and disorder; for $|K| \gg d$, the fermions will form a domain insulator.

Another interesting limit is when $|\Delta_{ij}| \simeq \alpha \simeq 1$. The tunneling becomes then nonresonant and negligible, while the couplings K_{ij} fluctuate strongly. We end up with the (fermionic) Ising spin glass model [36] described by the Edwards-Anderson model [37]:

$$H_{\text{E-A}} = \frac{1}{4}\sum_{\langle ij\rangle} K_{ij} s_i s_j + \sum_i \tilde{\mu}_i s_i/2, \tag{6}$$

with $s_i = 2M_i - 1 = \pm 1$. The above Hamiltonian is well approximated by a random one with Gaussian and independent distributions for $K_{ij}/4$ and $\tilde{\mu}_i/2$ with mean 0 (H), and variances K (h), respectively. In this limit the system may be used to study various open questions of spin-glass physics, concerning the nature of ordering (Parisi's [37] versus "droplet" picture [38]), broken symmetry and dynamics in classical (in the absence of hopping) and quantum (with small, but nevertheless present hopping) spin glasses [24,39]. The predictions of Parisi's mean field theory for the model (6) can be obtained by replacing the model by the corresponding Sherrington-Kirkpatrick model, and employing the standard method of replica trick [37]. The calculations differ from the standard ones in that the constraint of fixed mean number of fermions is applied, and one deals simultaneously with random couplings and "magnetic fields" $\tilde{\mu}_i$. Following the de Almeida and Thouless (AT) approach [40], we obtain the AT surface separating the stable paramagnetic state from the "true" spin-glass state, characterized by replica symmetry breaking, and ultrametrically arranged ground states. The paramagnetic state is stable for

$$\left(\frac{k_B T}{K}\right)^2 > \left\langle\left\langle \operatorname{sech}^4\left(\frac{x\sqrt{K^2 q + h^2} + H}{k_B T}\right)\right\rangle\right\rangle_x, \tag{7}$$

where $q = \langle\langle \tanh^2[(x\sqrt{K^2 q + h^2} + H)/k_B T]\rangle\rangle_x$, the constraint is $m = \langle\langle \tanh[(x\sqrt{K^2 q + h^2} + H)/k_B T]\rangle\rangle_x$, with $m = 2N_F/N - 1$ and $\langle\langle . \rangle\rangle_x$ denotes averaging over normally distributed random variable x which represents disorder within the replica method [37]. Note, that according to the predictions of the alternative "droplet" model [38], applied to (6), no AT surface is expected.

II. All sites are of type A.—In this case $\alpha < 1$, and we have a gas of bare fermions flowing over the MI with one boson per site. The coefficients are

$$d_{ij} = J,$$
$$K_{ij} = -\frac{J^2}{V}\left[\frac{8}{1-(\Delta_{ij})^2} - \frac{4(1+\alpha)}{(1+\alpha)^2-(\Delta_{ij})^2} - \frac{4(1-\alpha)}{(1-\alpha)^2-(\Delta_{ij})^2}\right], \tag{8}$$

and $\tilde{\mu}_i \simeq 0$ up to corrections of order $O(J^2/V)$. The couplings K_{ij} are positive, and for $\alpha \simeq 0$, $K_{ij} \simeq O(\alpha^2)$, and both the repulsive interactions, and disorder are very weak, leading to a Fermi liquid behavior at low T. For finite α, and $\Delta_{ij} \simeq 1 - \alpha$, however, the fluctuations of K_{ij} might be quite large. Note, that for $\alpha \simeq 1$, this will occur even for small disorder. Assuming for simplicity that K_{ij} take either very large, or zero value, we see that the physics of bond percolation [41] will play a role. The bonds will form a "weak" and "strong" clusters, each of which may be percolating. The fermions will hope freely in the "weak" cluster; only one fermion per bond will be allowed in the "strong" cluster.

III. Both N^A and N^B of order $N/2$—In this case the physics of site percolation [41] will be relevant. If $N_F \leq N^B$ the composite fermions will move within a cluster of B sites. When N^B is above the classical percolation threshold, this cluster will be percolating. The expressions Eqs. (4) and (5) will still be valid, except that they will connect only the B sites.

The physics of the system will be similar as in case I, but it will occur now on the percolating cluster. For small disorder, and $K \ll d$ the system will be a Fermi glass in which the interplay between the Anderson localization of single particles due to fluctuations of μ_i and quantum percolation effects (randomness of the B-sites cluster) will occur. For repulsive interactions and $K \gg d$, the ground state will be a Mott insulator on the cluster for large filling factors. It is an open question whether the delocalized metallic phases with enhanced persistent current of the kind discussed in Ref. [35] might exist in this case. Similarly, it is an open question whether for attractive interactions ($K < 0$) and $|K| < d$ pairing of (perhaps localized) fermions will take place. If $|K| \gg d$, we expect the fermions to form a domain insulator on the cluster.

In the "spin-glass" limit $\Delta_{ij} \simeq \alpha \simeq 1$, we deal with the Edwards-Anderson spin glass on the cluster. Such systems are of interest in condensed matter physics [42], and again questions connected to the nature of spin-glass ordering may be studied in this case.

When $N_F > N^B$, all B sites will be filled, and the physics will occur on the cluster of A sites. For $\alpha \simeq 0$, we shall deal with a gas with very weak repulsive interactions, and no significant disorder on the random cluster. This is an ideal test ground to study quantum percolation at low T. For finite α, and $\Delta_{ij} \simeq 1 - \alpha$, the interplay between the fluctuating repulsive K_{ij}'s and quantum percolation might be studied.

Summarizing, we have studied atomic Fermi-Bose mixtures in optical lattices in the strong interaction limit, and in the presence of an inhomogeneous, or random onsite potential. We have derived the effective Hamiltonian describing the low temperature physics of the system, and shown that an inhomogeneous potential may be efficiently used to control the nature and strength of (boson mediated) interactions in the system. Using a random potential, one is able to control the system in such a way that its physics corresponds to a whole variety of quantum disordered systems: Fermi glass, fermionic spin-glass, and quantum percolation systems.

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[1] S. Jochim *et al.*, Sci. Express **302**, 2101 (2003); M. Greiner, C. A. Regal, and D. S. Jin, Nature (London) **426**, 537 (2003); M.W. Zwierlein *et al.*, Phys. Rev. Lett. **91**, 250401 (2003).
[2] G.V. Shlyapnikov, in *Proceedings of the XVIII International Conference on Atomic Physics*, edited by H.R- Sadeghpour, D. E. Pritchard, and E. J. Heller (World Scientific, Singapore, 2002).
[3] A. G. Truscott *et al.*, Science **291**, 2570 (2001).
[4] F. Schreck *et al.*, Phys. Rev. Lett. **87**, 080403 (2001).
[5] Z. Hadzibabic *et al.*, Phys. Rev. Lett. **91**, 160401 (2003).
[6] G. Modugno *et al.*, Science **297**, 2240 (2002).
[7] The studies of trapped FB gases concerned, in particular, FB phase separation [8,9], the phase diagram [10], novel collective modes [9,11], Fermi-Fermi interactions mediated by bosons [9,12], the collapse of the Fermi cloud in the presence of attractive FB interactions [6], or the effects characteristic for 1D FB mixtures [13].
[8] K. Mølmer, Phys. Rev. Lett. **80**, 1804 (1998).
[9] M. J. Bijlsma, B. A. Heringa, and H. T. C. Stoof, Phys. Rev. A **61**, 053601 (2000); H. Pu, W. Zhang, M. Wilkens, and P. Meystre, Phys. Rev. Lett. **88**, 070408 (2002).
[10] R. Roth, Phys. Rev. A **66**, 013614 (2002).
[11] P. Capuzzi and E. S. Hernández, Phys. Rev. A **64**, 043607 (2001); X.-J. Liu and H. Hu, *ibid.* **67**, 023613 (2003).
[12] A. Albus *et al.*, Phys. Rev. A **65**, 053607 (2002); L. Viverit and S. Giorgini, *ibid.* **66**, 063604 (2002); H. Heiselberg, C. J. Pethick, H. Smith, and L. Viverit, Phys. Rev. Lett. **85**, 2418 (2000).
[13] K. K. Das, Phys. Rev. Lett. **90**, 170403 (2003); M. A. Cazalilla and A. F. Ho, *ibid.* **91**, 150403 (2003).
[14] M. Greiner *et al.*, Nature (London) **415**, 39 (2002).
[15] D. Jaksch *et al.*, Phys. Rev. Lett. **81**, 3108 (1998).
[16] G. Modugno *et al.*, Phys. Rev. A **68**, 011601(R) (2003).
[17] A. Albus, F. Illuminati, and J. Eisert, Phys. Rev. A **68**, 023606 (2003).
[18] H. P. Büchler and G. Blatter, Phys. Rev. Lett. **91**, 130404 (2003); R. Roth and K. Burnett, Phys. Rev. A **68**, 023604 (2003).
[19] F. Illuminati and A. Albus, cond-mat/0311248.
[20] A. B. Kuklov and B.V. Svistunov, Phys. Rev. Lett. **90**, 100401 (2003).
[21] M. Lewenstein, L. Santos, M. A. Baranov, and H. Fehrmann, Phys. Rev. Lett. **92**, 050401 (2004).
[22] This phenomenon, related to the appearance of counterflow superfluidity in Ref. [20], may occur also in the absence of the optical lattice, M. Yu. Kagan, D.V. Efremov, and A.V. Klaptsov, cond-mat/0209481.
[23] H. Fehrmann *et al.*, cond-mat/0307635.
[24] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, 1999).
[25] M. Cramer, J. Eisert, and F. Illuminati, cond-mat/0310705.
[26] For control in other context, see L.-M. Duan, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **91**, 090402 (2003).
[27] A. Auerbach, *Interacting Electrons and Quantum Magnetism* (Springer, New York, 1994).
[28] B. Damski *et al.*, Phys. Rev. Lett. **91**, 080403 (2003).
[29] L. Guidoni and P. Verkerk, Phys. Rev. A **57**, 1501(R) (1999); L. Guidoni, C. Triché, P. Verkerk, and G. Grynberg, Phys. Rev. Lett. **79**, 3363 (1997); P. Horak, J.-Y. Courtois, and G. Grynberg, Phys. Rev. A **58**, 3953 (1998).
[30] In current experiments [14], the exact commensurate filling factor 1 appears in the central region of the lattice due to the presence of a weak confining potential [31].
[31] G. Batrouni *et al.*, Phys. Rev. Lett. **89**, 117203 (2002).
[32] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions: Basic Processes and Applications* (Wiley, New York, 1992).
[33] E. Abrahams, P.W. Anderson, D. C. Licciardello, and T.V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).
[34] R. Freedman and J. A. Hertz, Phys. Rev. B **15**, 2384 (1977).
[35] G. Benenti, X. Waintal, and J.-L. Pichard, Phys. Rev. Lett. **83**, 1826 (1999); E. Gambeti-Césare *et al.*, Europhys. Lett. **60**, 120 (2002).
[36] R. Oppermann and D. Sherrington, Phys. Rev. B **67**, 245111 (2003).
[37] M. Mézard, G. Parisi, and M. A. Virasoro, *Spin Glass and Beyond* (World Scientific, Singapore, 1987).
[38] D. S. Fisher and D. A. Huse, Phys. Rev. Lett. **56**, 1601 (1986); A. J. Bray and M. A. Moore, *ibid.* **58**, 57 (1987); C. M. Newman and D. L. Stein, J. Phys. Condens. Matter **15**, R1319 (2003).
[39] A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001).
[40] J. R. L. de Almeida and D. J. Thouless, J. Phys. A **11**, 983 (1978).
[41] A. Aharony and D. Stauffer, *Introduction to Percolation Theory* (Taylor & Francis, London, 1994).
[42] P. H. R. Barbosa, E. P. Raposo, and M. D. Coutinho-Filho, Phys. Rev. Lett. **91**, 197207 (2003).