

Model Independent Bound on the Unitarity Triangle from CP Violation in $B \rightarrow \pi^+ \pi^-$ and $B \rightarrow \psi K_S$

Gerhard Buchalla and A. Salim Safir

Ludwig-Maximilians-Universität München, Sektion Physik, Theresienstraße 37, D-80333 Munich, Germany

(Received 6 November 2003; published 6 July 2004)

We derive model-independent lower bounds on the CKM parameters $(1 - \bar{\rho})$ and $\bar{\eta}$ as functions of the mixing-induced CP asymmetry S in $B \rightarrow \pi^+ \pi^-$ and $\sin 2\beta$ from $B \rightarrow \psi K_S$. The bounds do not depend on specific results of theoretical calculations for the penguin contribution to $B \rightarrow \pi^+ \pi^-$. They require only the very conservative condition that a hadronic phase, which vanishes in the heavy-quark limit, does not exceed 90° in magnitude. The bounds are effective if $-\sin 2\beta \leq S \leq 1$. Dynamical calculations indicate that the limits on $\bar{\rho}$ and $\bar{\eta}$ are close to their actual values.

DOI: 10.1103/PhysRevLett.93.021801

PACS numbers: 12.15.Hh, 11.30.Er, 13.25.Hw

In the standard model CP violation is described by a single complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix of quark mixing. After the first observation of CP violation with $K_L \rightarrow \pi^+ \pi^-$ decays in 1964 [1], the standard model scenario has passed a crucial test with the recent discovery of a large CP asymmetry in $B \rightarrow \psi K_S$ decays [2,3] in agreement with earlier predictions [4]. Beyond this qualitative confirmation of standard model CP violation, efforts are now increasingly directed towards precision studies, which could reveal the existence of new physics. A large amount of experimental data continues to be collected at current and future B -meson facilities, both at dedicated e^+e^- colliders and at hadron machines.

To be useful for precision tests of weak interactions the observables must not depend on poorly understood hadronic quantities. Among the rather few observables that are essentially free of such theoretical uncertainties is the time-dependent CP asymmetry in $B \rightarrow \psi K_S$, which measures $\sin 2\beta$ (where β is one of the angles of the CKM unitarity triangle). Of comparable interest is the corresponding CP asymmetry in $B \rightarrow \pi^+ \pi^-$ decays, which we shall denote by S . However, a well-known problem here are penguin contributions in the decay amplitude, which introduce hadronic physics into the CP asymmetry and spoil a straightforward interpretation in terms of weak phases. A method to eliminate hadronic input based on an isospin analysis [5] appears to be very difficult for a practical implementation [6,7].

In this Letter we show how useful information on CKM parameters can be obtained from the mixing-induced CP asymmetries in $B \rightarrow \psi K_S$ and $B \rightarrow \pi^+ \pi^-$, in spite of the penguin contributions in the second case. We derive lower bounds on the unitarity triangle parameters $(1 - \bar{\rho})$ and $\bar{\eta}$ [8] that depend only on observables of CP violation in these channels.

The subject of extracting information on weak mixing angles from CP violation in $B \rightarrow \pi^+ \pi^-$ has already been discussed extensively in the literature [9–19]. We comment on the main features of our analysis that differ from

the approaches taken in previous work. In [9–19] important aspects of the problem have been discussed and suggestions were made to limit theoretical uncertainties. Constraints on CKM parameters have been obtained, usually in graphical form, depending on hadronic input from the penguin contribution. This input has been determined for instance with the help of SU(3) arguments and data from related $B \rightarrow \pi K$ decays [13,15,16] or from dynamical calculations [14]. In the present Letter we describe a new way of exploiting the information contained in the CP violation observables S and $\sin 2\beta$. The crucial elements are a definition of hadronic quantities r and ϕ independent of CKM parameters, the direct formulation of weak phases in terms of the basic Wolfenstein parameters $\bar{\rho}$, $\bar{\eta}$, the combined use of $B \rightarrow \pi^+ \pi^-$ and $B \rightarrow \psi K_S$ asymmetries, the resulting analytical determination of the unitarity triangle and the exact, explicit, and very simple dependence on the penguin parameters r and ϕ . This in turn greatly facilitates the analysis of theoretical uncertainties and gives, in combination with results based on the heavy-quark limit, direct determinations of the unitarity triangle, or CKM bounds with minimal hadronic input.

The time-dependent CP asymmetry in $B \rightarrow \pi^+ \pi^-$ decays is defined by

$$\begin{aligned} A_{CP}^{\pi\pi}(t) &= \frac{B(B(t) \rightarrow \pi^+ \pi^-) - B(\bar{B}(t) \rightarrow \pi^+ \pi^-)}{B(B(t) \rightarrow \pi^+ \pi^-) + B(\bar{B}(t) \rightarrow \pi^+ \pi^-)} \\ &= -S \sin(\Delta m_B t) + C \cos(\Delta m_B t), \end{aligned} \quad (1)$$

where

$$\begin{aligned} S &= \frac{2 \operatorname{Im} \xi}{1 + |\xi|^2}, \quad C = \frac{1 - |\xi|^2}{1 + |\xi|^2}, \\ \xi &= e^{-2i\beta} \frac{e^{-i\gamma} + P/T}{e^{+i\gamma} + P/T}. \end{aligned} \quad (2)$$

In terms of the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$ the CKM phase factors read

$$e^{\pm i\gamma} = \frac{\bar{\rho} \pm i\bar{\eta}}{\sqrt{\bar{\rho}^2 + \bar{\eta}^2}}, \quad (3)$$

$$e^{-2i\beta} = \frac{(1 - \bar{\rho})^2 - \bar{\eta}^2 - 2i\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2}.$$

The penguin-to-tree ratio P/T can be written as

$$\frac{P}{T} = \frac{re^{i\phi}}{\sqrt{\bar{\rho}^2 + \bar{\eta}^2}}. \quad (4)$$

The real parameters r and ϕ defined in this way are pure strong interaction quantities without further dependence on CKM variables.

For any given values of r and ϕ a measurement of S defines a curve in the $(\bar{\rho}, \bar{\eta})$ plane. Using the relations above this constraint is given by the equation

$$S = \frac{2\bar{\eta}[\bar{\rho}^2 + \bar{\eta}^2 - r^2 - \bar{\rho}(1 - r^2) + (\bar{\rho}^2 + \bar{\eta}^2 - 1)r\cos\phi]}{((1 - \bar{\rho})^2 + \bar{\eta}^2)(\bar{\rho}^2 + \bar{\eta}^2 + r^2 + 2r\bar{\rho}\cos\phi)}. \quad (5)$$

The current experimental results for S and C are (see Refs. [20,21])

$$S = \begin{cases} +0.02 \pm 0.34 \pm 0.05 & \text{(BABAR),} \\ -1.00 \pm 0.21 \pm 0.07 & \text{(Belle),} \end{cases} \quad (6)$$

$$C = \begin{cases} -0.30 \pm 0.25 \pm 0.04 & \text{(BABAR),} \\ -0.58 \pm 0.15 \pm 0.07 & \text{(Belle).} \end{cases}$$

A recent preliminary update from BABAR gives [3]

$$S = -0.40 \pm 0.22 \pm 0.03, \quad (7)$$

$$C = -0.19 \pm 0.19 \pm 0.05.$$

The present results are still somewhat inconclusive, but the accuracy should improve in the near future. In the remainder of this Letter we will focus our attention on S and will not need C . We note, however, that both quantities always fulfill the inequality $S^2 + C^2 \leq 1$. The implications of C will be considered elsewhere [22].

The hadronic parameters r and ϕ can be computed using QCD factorization in the heavy-quark limit [14,23]. A recent analysis gives [22]

$$r = 0.107 \pm 0.031, \quad \phi = 0.15 \pm 0.25, \quad (8)$$

where the error includes an estimate of potentially important power corrections. We quote these values in order to give an indication of the typical size of r and ϕ . However, we will not make use of the detailed prediction

$$\bar{\eta} = \frac{1}{(1 + \tau^2)S} [(1 + \tau S)(1 + r\cos\phi) - \sqrt{(1 - S^2)(1 + r\cos\phi)^2 - S(1 + \tau^2)(S + \sin 2\beta)r^2\sin^2\phi}]. \quad (14)$$

If $S \geq 0$, the coefficient of $r^2\sin^2\phi$ under the square root is negative; hence

(8). Instead, we allow for an arbitrary value $r \geq 0$ and shall only assume that the strong phase ϕ fulfills

$$-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}. \quad (9)$$

This is well justified since QCD factorization in the heavy-quark limit implies that the strong interaction phase ϕ is systematically suppressed, either by α_s , for perturbative, hard rescattering, or by Λ_{QCD}/m_b for soft corrections. We note that also the positive sign of r is justified by the heavy-quark limit [14]. In addition the same sign is obtained, independently, in the limit of QCD with a large number N of colors. In this case the result is similar to the one found using the phenomenological factorization of hadronic weak decay matrix elements, both giving $r > 0$. Uncalculated power corrections from the large tree-level transition $b \rightarrow u\bar{u}d$ cannot affect the penguin part, because they belong to a component of the decay amplitude with a different weak phase [see Eq. (2)]. The contributions to r from $b \rightarrow c\bar{c}d$ are suppressed since the charm quarks have to annihilate in a loop process and they are also subleading in $1/N$. The effect is computable in the heavy-quark limit [14], where it is a small contribution. We finally note that the condition $r\cos\phi > 0$ could be tested [22,24] by measuring CP violation in $B_s \rightarrow K^+K^-$, the U-spin counterpart of $B_d \rightarrow \pi^+\pi^-$ [25].

After these preliminaries we now turn to the constraints on the CKM parameters $\bar{\rho}$ and $\bar{\eta}$ that can be derived from CP violation in $B \rightarrow \psi K_S$ and $B \rightarrow \pi^+\pi^-$ decays. The angle β of the unitarity triangle is given by

$$\tau \equiv \cot\beta = \frac{\sin 2\beta}{1 - \sqrt{1 - \sin^2 2\beta}}, \quad (10)$$

where $\sin 2\beta$ is measured directly and with negligible theoretical uncertainty [6] from the mixing-induced CP asymmetry in $B \rightarrow \psi K_S$. A second solution with a positive sign in front of the square root in (10) is excluded using other constraints on the unitarity triangle, such as $|V_{ub}/V_{cb}|$ [7]. The current world average [3]

$$\sin 2\beta = 0.739 \pm 0.048 \quad (11)$$

implies

$$\tau = 2.26 \pm 0.22. \quad (12)$$

Given a value of τ , $\bar{\rho}$ is related to $\bar{\eta}$ by

$$\bar{\rho} = 1 - \tau\bar{\eta}. \quad (13)$$

The parameter $\bar{\rho}$ may thus be eliminated from S in (5), which can be solved for $\bar{\eta}$ to yield

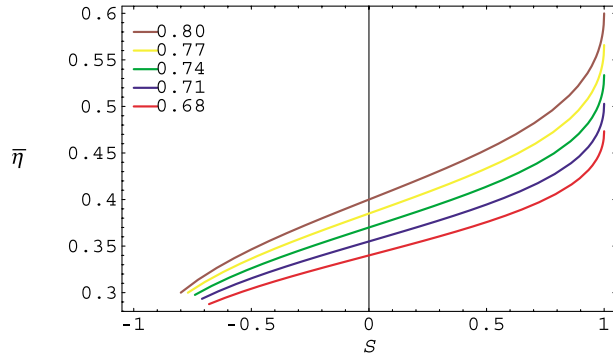


FIG. 1 (color online). Lower bound on $\bar{\eta}$ as a function of S for various values of $\sin 2\beta$ (increasing from bottom to top).

$$\bar{\eta} \geq \frac{1 + \tau S - \sqrt{1 - S^2}}{(1 + \tau^2)S} (1 + r \cos \phi). \quad (15)$$

If $-\sin 2\beta \leq S \leq 0$, the same bound is obtained so that (15) holds for the entire range $-\sin 2\beta \leq S \leq 1$. We note that this bound is still *exact* and requires no information on the phase ϕ . (The only condition is that $(1 + r \cos \phi)$ is positive, which is no restriction in practice.)

Assuming now (9), we have $1 + r \cos \phi \geq 1$ and

$$\bar{\eta} \geq \frac{1 + \tau S - \sqrt{1 - S^2}}{(1 + \tau^2)S} \quad \text{if} \quad -\sin 2\beta \leq S \leq 1. \quad (16)$$

This constraint is the main result of this Letter. We emphasize that the lower bound on $\bar{\eta}$ depends only on the observables τ and S and is essentially free of hadronic uncertainties. It holds in the standard model and it is effective under the condition that S will eventually be measured in the interval $[-\sin 2\beta, 1]$. Since both r and ϕ are expected to be quite small, we anticipate that the lower limit (16) is a fairly strong bound, close to the actual value of $\bar{\eta}$ itself. We also note that the lower bound (16) represents the solution for the unitarity triangle in the limit of vanishing penguin amplitude, $r = 0$. In other words, the model-independent bounds for $\bar{\eta}$ and $\bar{\rho}$ are simply obtained by ignoring penguin contributions and taking $S \equiv \sin 2\alpha$ when fixing the unitarity triangle from S and $\sin 2\beta$. (The term model independent is employed here in the sense of relying only on a qualitative condition, $r \cos \phi > 0$, which holds in the heavy-quark limit.)

Let us briefly comment on the second solution for $\bar{\eta}$, which has the minus sign in front of the square root in (14) replaced by a plus sign. For positive S this solution is always larger than (14) and the bound (16) is unaffected. For $-\sin 2\beta \leq S \leq 0$ the second solution gives a negative $\bar{\eta}$, which is excluded by independent information on the unitarity triangle [for instance from indirect CP violation in neutral kaons (ε_K)]. Because we have fixed the angle β , or τ , the lower bound on $\bar{\eta}$ is equivalent to an upper bound on $\bar{\rho} = 1 - \tau \bar{\eta}$. The constraint (16) may also be expressed as a lower bound on the angle γ

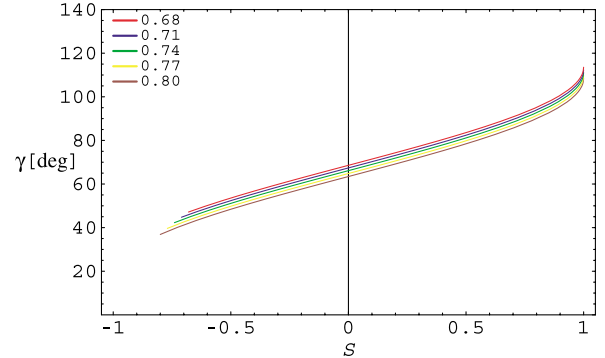


FIG. 2 (color online). Lower bound on γ as a function of S for various values of $\sin 2\beta$ (decreasing from bottom to top).

$$\gamma \geq \frac{\pi}{2} - \arctan \frac{S - \tau(1 - \sqrt{1 - S^2})}{\tau S + 1 - \sqrt{1 - S^2}} \quad (17)$$

or a lower bound on R_t

$$R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} \geq \frac{1 + \tau S - \sqrt{1 - S^2}}{\sqrt{1 + \tau^2} S}. \quad (18)$$

In Figs. 1 and 2 we represent, respectively, the lower bound on $\bar{\eta}$ and γ as a function of S for various values of $\sin 2\beta$. From Fig. 1 we observe that the lower bound on $\bar{\eta}$ becomes stronger as either S or $\sin 2\beta$ increase. The sensitivity to $\sin 2\beta$ is less pronounced for the bound on γ . Similarly to $\bar{\eta}$, the minimum allowed value for γ increases with S . A lower limit $\gamma = 90^\circ$ would be reached for $S = \sin 2\beta$.

Figure 3 illustrates the bound in the $(\bar{\rho}, \bar{\eta})$ plane. The standard unitarity triangle fit from [7] is indicated by the elliptical region.

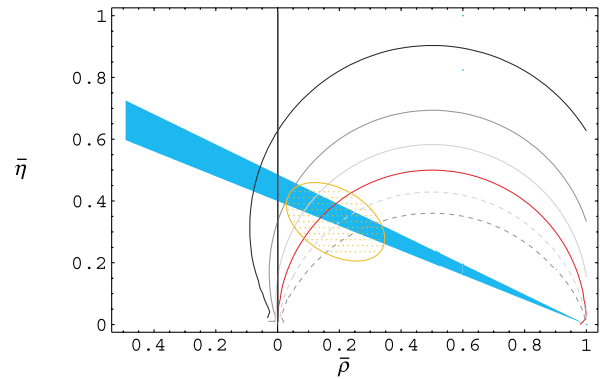


FIG. 3 (color online). Region in the $(\bar{\rho}, \bar{\eta})$ plane constrained by $\sin 2\beta = 0.739 \pm 0.048$ (shaded sector) and various possible values for S . The allowed area is the part of the shaded sector to the left of a given line defined by S . These lines correspond, from bottom to top, to $S = -0.6, -0.3, 0, 0.3, 0.6,$ and 0.9 . The bound becomes stronger with increasing S . The result of a standard unitarity triangle fit (dotted ellipse) is overlaid for comparison.

To summarize, we have demonstrated that theoretically clean information can be extracted from CP violation in $B \rightarrow \pi^+ \pi^-$ decays in the form of stringent limits on $\bar{\rho}$ and $\bar{\eta}$ even in the presence of penguin contributions, requiring only condition (9). In order to make an optimal use of these bounds, the quantities $\sin 2\beta$ and S should be measured as precisely as possible. We hope that the analysis suggested in this article will facilitate a transparent interpretation and an efficient use of future data on CP violation in $B \rightarrow \pi^+ \pi^-$. Further discussions on this and related topics will be given in a separate publication [22].

This work is supported in part by the Deutsche Forschungsgemeinschaft (DFG) under Contract No. BU 1391/1-2.

-
- [1] J.H. Christenson *et al.*, Phys. Rev. Lett. **13**, 138 (1964).
- [2] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **86**, 2515 (2001); Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **87**, 091802 (2001).
- [3] Heavy Flavor Averaging Group, <http://www.slac.stanford.edu/xorg/hfag/index.html>
- [4] A.B. Carter and A.I. Sanda, Phys. Rev. D **23**, 1567 (1981); I.I. Bigi and A.I. Sanda, Nucl. Phys. **B193**, 85 (1981).
- [5] M. Gronau and D. London, Phys. Rev. Lett. **65**, 3381 (1990).
- [6] BABAR Collaboration, P.F. Harrison *et al.*, "The BABAR Physics Book," SLAC-R-0504.
- [7] CKMFitter Working Group, <http://ckmfitter.in2p3.fr>
- [8] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983); A. J. Buras, M.E. Lautenbacher, and G. Ostermaier, Phys. Rev. D **50**, 3433 (1994).
- [9] J.P. Silva and L. Wolfenstein, Phys. Rev. D **49**, 1151 (1994).
- [10] Y. Grossman, Y. Nir, and M.P. Worah, Phys. Lett. B **407**, 307 (1997).
- [11] Y. Grossman and H.R. Quinn, Phys. Rev. D **58**, 017504 (1998).
- [12] J. Charles, Phys. Rev. D **59**, 054007 (1999).
- [13] R. Fleischer, Eur. Phys. J. C **16**, 87 (2000).
- [14] M. Beneke *et al.*, Nucl. Phys. **B606**, 245 (2001); M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333 (2003).
- [15] M. Gronau and J.L. Rosner, Phys. Rev. D **65**, 013004 (2002) [**65**, 079901(E) (2002)]; **65**, 093012 (2002).
- [16] M. Gronau and J.L. Rosner, Phys. Rev. D **66**, 053003 (2002) [**66**, 119901(E) (2002)].
- [17] C.D. Lu and Z.j. Xiao, Phys. Rev. D **66**, 074011 (2002); Z. j. Xiao, C. D. Lu, and L. Guo, hep-ph/0303070; T. Yoshikawa, Phys. Rev. D **68**, 017501 (2003).
- [18] R. Fleischer, G. Isidori, and J. Matias, J. High Energy Phys. **05** (2003) 053.
- [19] A. Höcker, H. Lacker, M. Pivk, and L. Roos, LAL 02-103, LPNHE 2002-15, [http://ckmfitter.in2p3.fr; eConf C0304052, \(2003\) WG418, hep-ph/0307013](http://ckmfitter.in2p3.fr;eConf C0304052,(2003)WG418,hep-ph/0307013).
- [20] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **89**, 281802 (2002).
- [21] Belle Collaboration, K. Abe *et al.*, hep-ex/0401029.
- [22] G. Buchalla and A. S. Safir (to be published).
- [23] M. Beneke *et al.*, Phys. Rev. Lett. **83**, 1914 (1999).
- [24] A. S. Safir (to be published).
- [25] I. Dunietz, FERMILAB-CONF-93-090-T, in Proceedings of the Summer Workshop on B Physics at Hadron Accelerators, Snowmass, CO, 1993; R. Fleischer, Phys. Lett. B **459**, 306 (1999).