Black Hole Mass Decreasing due to Phantom Energy Accretion

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Solution for a stationary spherically symmetric accretion of the relativistic perfect fluid with an equation of state $p(\rho)$ onto the Schwarzschild black hole is presented. This solution is a generalization of Michel solution and applicable to the problem of dark energy accretion. It is shown that accretion of phantom energy is accompanied by the gradual decrease of the black hole mass. Masses of all black holes tend to zero in the phantom energy Universe approaching the Big Rip.

Observations of distant supernovae and cosmic microwave background anisotropy indicate in favor of the accelerating expansion of the Universe [1]. In the framework of a general relativity, it means that a considerable part of the Universe consists of the dark energy: the component with a positive energy density $\rho > 0$ and with a negative pressure $p < -(1/3)\rho$. This dark energy may be in the form of a vacuum energy (cosmological constant Λ) with $p = -\rho$, or a dynamically evolving scalar field with a negative pressure (quintessence [2] or k essence [3]). One of the peculiar features of the cosmological dark energy is a possibility of the Big Rip [4]: the infinite expansion of the Universe during a finite time. The Big Rip scenario is realized if dark energy is in the form of the phantom energy with $\rho + p < 0$. In this case, the cosmological phantom energy density grows at large times and disrupts finally all bounded objects up to subnuclear scale.

What would be the fate of black holes in the Universe filled with the phantom energy? Below we find the solution for a stationary accretion of the relativistic perfect fluid with an arbitrary equation of state $p(\rho)$ onto the Schwarzschild black hole. Using this solution, we show that the black hole mass diminishes by accretion of phantom energy. Masses of all black holes gradually tend to zero in the phantom energy Universe approaching the Big Rip. The diminishing of a black hole mass is caused by the violation of the energy domination condition $\rho + p \geq 0$ which is a principal assumption of the classical black hole "nondiminishing" theorems [5]. Another consequence of the existence of a phantom energy is a possibility of traversable wormholes [6].

The known analytic relativistic accretion solution onto the Schwarzschild black hole by Michel [7] (see also [8]) is not applied for a general case of a dark energy accretion. We find the requested generalization of Michel solution by modeling of the dark energy in the black hole gravitational field by the test perfect fluid with a negative pressure, an arbitrary equation of state $p(\rho)$, and energy-momentum tensor $T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}$, where ρ and p are the dark energy density and pressure correspondingly, and $u^\mu = dx^\mu/ds$ is a fluid four velocity

with $u^\mu u_\mu = 1$. The integration of the time component of the energy-momentum conservation law $T^{\mu\nu}_{;\nu} = 0$ gives the first integral of motion for the stationary spherically symmetric accretion (the relativistic Bernoulli or energy conservation equation):

$$(\rho + p)\left(1 - \frac{2}{x} + u^2\right)^{1/2} x^2 u = C_1, \qquad (1)$$

where $x = r/M$, $u = dr/ds$, r is a radial Schwarzschild coordinate, M is a black hole mass, and C_1 is a constant determined below. To obtain another integral of motion, we use the projection of the energy-momentum conservation law on the four-velocity $u_\mu T^{\mu\nu}_{;\nu} = 0$, which for a perfect fluid is

$$u^\mu \rho_{,\mu} + (\rho + p)u^\mu_{;\mu} = 0. \qquad (2)$$

The integration of (2) gives the second integral of motion (the energy flux equation):

$$ux^2 \exp\left[\int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = -A, \qquad (3)$$

where $u < 0$ in the case of inflow motion and a dimensionless constant $A > 0$. Note that the second integral of motion (3) is obtained without use of the particle conservation law. This is the principal generalization of the Michel approach [7]. From (1) and (3), one can easily obtain

$$(\rho + p)\left(1 - \frac{2}{x} + u^2\right)^{1/2} \exp\left[-\int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = C_2, \qquad (4)$$

where $C_2 = -C_1/A = \rho_\infty + p(\rho_\infty)$. From (3) and (4), one can find the relations for the fluid velocity $u_H = u(2M)$ and density $\rho_H = \rho(2M)$ at the black hole horizon $r = 2M$:

$$\frac{A}{4}\frac{\rho_H + p(\rho_H)}{\rho_\infty + p(\rho_\infty)} = \frac{A^2}{16u_H^2} = \exp\left[2\int_{\rho_\infty}^{\rho_H} \frac{d\rho'}{\rho' + p(\rho')}\right].$$

The constant A which determines the accretion flux is calculated by fixing parameters at a critical point. This

provides us with the continuity of the solution from the infinity to a horizon. Following Michel [7], we fix parameters of critical point $x = x_*$:

$$u_*^2 = \frac{1}{2x_*}, \qquad c_s^2(\rho_*) = \frac{u_*^2}{1 - 3u_*^2}, \qquad (5)$$

where $c_s^2(\rho) = \partial p / \partial \rho$ is the square of sound speed. Using (4) and (5), one can find the following relation:

$$\frac{\rho_* + p(\rho_*)}{\rho_\infty + p(\rho_\infty)}[1 + 3c_s^2(\rho_*)]^{-1/2} = \exp\left[\int_{\rho_\infty}^{\rho_*} \frac{d\rho'}{\rho' + p(\rho')}\right],$$

which determines the density at a critical point $\rho_* = \rho(x_*)$. Then, for a given ρ_*, with the help of (5), one can find x_* and u_*. Constant A is fixed by substituting of the calculated values in (3). Note that there is no critical point outside the black hole horizon ($x_* > 1$) for $c_s^2 < 0$ or $c_s^2 > 1$. This means that for unstable perfect fluid with $c_s^2 < 0$ or $c_s^2 > 1$ a dark energy flux onto the black hole depends on the initial conditions. This result has a simple physical interpretation: The accreting fluid has the critical point if its velocity increases from subsonic to transsonic values. In a fluid with a negative c_s^2 or with $c_s^2 > 1$, the fluid velocity never crosses such a point. It should be stressed, however, that fluids with $c_s^2 < 0$ are hydrodynamically unstable (see discussion in [9,10]).

Equations (3) and (4) along with the equation of state $p = p(\rho)$ describe the requested accretion flow onto the black hole. These equations are valid for perfect fluid with an arbitrary equation of state $p = p(\rho)$, in particular, for a gas with zero-rest-mass particles (thermal radiation), for a gas with nonzero-rest-mass particles, and for a dark energy. For a nonzero-rest-mass gas, Eqs. (3) and (4) are reduced to similar ones found by Michel [7].

The black hole mass changes at a rate $\dot{M} = -4\pi r^2 T_0^{\;r}$ due to the fluid accretion. With the help of (3) and (4), this can be expressed as

$$\dot{M} = 4\pi A M^2 [\rho_\infty + p(\rho_\infty)]. \qquad (6)$$

From this equation it is clear that the accretion of a phantom energy with $\rho_\infty + p(\rho_\infty) < 0$ is always accompanied with the diminishing of the black hole mass. This result is valid for any equation of state $p = p(\rho)$ with $p + \rho < 0$. If we neglect the cosmological evolution of ρ_∞, then from (6) we obtain

$$M = M_i\left(1 - \frac{t}{\tau}\right)^{-1}, \qquad (7)$$

where M_i is an initial mass of the black hole and a characteristic evolution time $\tau = \{4\pi A M_i[\rho_\infty + p(\rho_\infty)]\}^{-1}$.

As a particular fully solvable example, we consider the perfect fluid with a linear equation of state:

$$p = \alpha(\rho - \rho_0), \qquad \alpha = \text{const}, \qquad \rho_0 = \text{const}, \qquad (8)$$

where $\alpha = c_s^2$ and it is supposed that $\rho > 0$. Among others, this model includes radiation ($p = \rho/3$), ultrahard equation of state ($p = \rho$), and the simplest models of dark energy ($\rho_0 = 0$, $\alpha < 0$). The constant α is connected with an often used parameter $w = p/\rho$ by the relation $w = \alpha(\rho - \rho_0)/\rho$. The physically reasonable case corresponds to $0 < \alpha \leq 1$.

In the linear model (8), the radius of critical point $x_* = (1 + 3\alpha)/2\alpha$, velocity at critical point $u_*^2 = \alpha/(1 + 3\alpha)$, velocity at the horizon $u_H = -(A/4)^{-\alpha/(1-\alpha)}$, and constant A determining the energy flux in (6) is [11]

$$A = \frac{(1 + 3\alpha)^{(1+3\alpha)/2\alpha}}{4\alpha^{3/2}}. \qquad (9)$$

This relation is valid only for stable fluid with $0 < \alpha \leq 1$. For unstable fluid with $\alpha < 0$, the constant A is indeterminable by the condition of the solution continuity from critical point consideration. We suppose that in the unstable case the growth of instabilities in the accretion flow will cause the asymptotic growth of the accretion velocity up to the limiting speed of light at the horizon $u_H \rightarrow -1$. This helps to fix the value of constant A: $A = 4$.

For some particular choices of parameter α, the values $\rho(x)$ and $u(x)$ can be calculated analytically. For example, for $\alpha = 1/3$ the fluid density is given by

$$\rho = \frac{\rho_0}{4} + \left(\rho_\infty - \frac{\rho_0}{4}\right)\left[z + \frac{1}{3(1 - 2x^{-1})}\right]^2, \qquad (10)$$

where

$$z = \begin{cases} 2\sqrt{\frac{a}{3}}\cos\left(\frac{2\pi}{3} - \frac{\beta}{3}\right), & 2 \leq x \leq 3 \\ 2\sqrt{\frac{a}{3}}\cos\left(\frac{\beta}{3}\right), & x > 3, \end{cases}$$

$$\beta = \arccos\left[\frac{b}{2(a/3)^{3/2}}\right],$$

and

$$a = \frac{1}{3(1 - \frac{2}{x})^2}, \qquad b = \frac{2}{27(1 - \frac{2}{x})^3} - \frac{108}{(1 - \frac{2}{x})x^4}.$$

The density distribution for another physically interesting case $\alpha = 1$ is given by

$$\rho = \frac{\rho_0}{2} + \left(\rho_\infty - \frac{\rho_0}{2}\right)\left(1 + \frac{2}{x}\right)\left(1 + \frac{4}{x^2}\right). \qquad (11)$$

The corresponding radial fluid velocity $u = u(x)$ can be calculated by substituting of (10) or (11) into (1). For $\rho_0 = 0$, the solutions (10) and (11) describe correspondingly a thermal radiation and a fluid with ultrahard equation of state. In the case of $\rho_\infty < \alpha\rho_0/(1 + \alpha)$, the solutions (10) and (11) describe the phantom energy falling onto the black hole. For example, a phantom energy flow with parameters $\alpha = 1$ and $\rho_0 = (7/3)\rho_\infty$ results in a black hole mass diminishing with the rate $\dot{M} = -(8\pi/3)(2M)^2\rho_\infty$.

Now we turn to the problem of the black hole evolution in the Universe with the Big Rip when a scale factor $a(t)$ diverges at finite time [4]. For simplicity, we will take into account only dark energy and will disregard all other forms of energy. The Big Rip solution is realized for $\rho + p < 0$ and $\alpha < -1$. From the Friedmann equations for the linear equation of state model, one can obtain $|\rho + p| \propto a^{-3(1+\alpha)}$. Taking for simplicity $\rho_0 = 0$, we find the evolution of the density of a phantom energy in the Universe:

$$\rho_\infty = \rho_{\infty,i}\left(1 - \frac{t}{\tau}\right)^{-2}, \quad (12)$$

where

$$\tau^{-1} = -\frac{3(1 + \alpha)}{2}\left(\frac{8\pi G}{3}\rho_{\infty,i}\right)^{1/2}, \quad (13)$$

and $\rho_{\infty,i}$ is the initial density of the cosmological phantom energy and the initial moment of time is chosen so that the "doomsday" comes at time τ. From (6) using (12), we find the black hole mass evolution:

$$M = M_i\left(1 + \frac{M_i}{\dot{M}_0\tau}\frac{t}{\tau - t}\right)^{-1}, \quad (14)$$

where $\dot{M}_0 = (3/2)A^{-1}|1 + \alpha|$ and M_i is the initial mass of the black hole. For $\alpha = -2$ and a typical value of $A = 4$ (corresponding to $u_H = -1$), we have $\dot{M}_0 = 3/8$. In the limit $t \to \tau$ (i.e., near the Big Rip), the dependence of black hole mass on t becomes linear, $M \simeq \dot{M}_0(\tau - t)$. While t approaches τ, the rate of black hole mass decrease does not depend on both an initial black hole mass and the density of the phantom energy: $\dot{M} \simeq \dot{M}_0$. In other words, masses of all black holes in the Universe tend to be equal near the Big Rip. This means that the phantom energy accretion prevails over the Hawking radiation until the mass of black hole is the Planck mass [12].

In the remaining, let us confront our results with the calculations of (not phantom) scalar field accretion onto the black hole [13–16]. The dark energy is usually modeled by a scalar field ϕ with potential $V(\phi)$. The perfect fluid approach is more rough because for given "perfect fluid variables" ρ and p one cannot restore the "scalar field variables" ϕ and $\nabla\phi$. In spite of the pointed difference between a scalar field and a perfect fluid, we show below that our results are in very good agreement with the corresponding calculations of a scalar field accretion onto the black hole.

The Lagrangian of a scalar field is $L = K - V$, where K is a kinetic term of a scalar field ϕ and V is a potential. For the standard choice of a kinetic term $K = \phi_{;\mu}\phi^{;\mu}/2$, the energy flux is $T_{0r} = \phi_{,t}\phi_{,r}$. Jacobson [13] found the scalar field solution in Schwarzschild metric for the case of zero potential $V = 0$: $\phi = \dot{\phi}_\infty[t + 2M\ln(1 - 2M/r)]$, where $\dot{\phi}_\infty$ is the value of the scalar field at the infinity. In [15], it was shown that this solution remains valid also for

a rather general form of runaway potential $V(\phi)$. For this solution, we have $T_0^r = -(2M)^2\dot{\phi}_\infty^2/r^2$ and, correspondingly, $\dot{M} = 4\pi(2M)^2\dot{\phi}_\infty^2$.

It is interesting to note that the energy-momentum tensor constructed from Jacobson solution completely coincides with one for perfect fluid in the case of an ultrahard equation of state $p = \rho$ under the replacement $p_\infty \to \dot{\phi}_\infty^2/2$, $\rho_\infty \to \dot{\phi}_\infty^2/2$ [11]. Therefore the scalar field in Jacobson solution is a perfect fluid. In a view of this coincidence, it is easy to see the agreement of our result (6) for $\dot{M}$ in the case of $p = \rho$ and the corresponding result of [13,15].

To describe the phantom energy, the Lagrangian of a scalar field must have a negative kinetic term [4]; for example, $K = -\phi_{;\mu}\phi^{;\mu}/2$. In this case, the phantom energy flux onto a black hole has the opposite sign, $T_{0r} = -\phi_{,t}\phi_{,r}$, where ϕ is the solution of the same Klein-Gordon equation as in the case of standard scalar field, however, with the replacement $V \to -V$. For zero potential, this solution coincides with that obtained by Jacobson [13] for a scalar field with the positive kinetic term. A Lagrangian with a negative kinetic term and $V(\phi) = 0$ does not describe, however, the phantom energy. At the same time, the solution for scalar field with potential $V(\phi) = 0$ is the same as with a positive constant potential $V_0 = $ const, which can be chosen so that $\rho = -\dot{\phi}^2/2 + V_0 > 0$. In this case, the scalar field represents the required accreting phantom energy $\rho > 0$ and $p < -\rho$ and provides the decrease of black hole mass with the rate $\dot{M} = -4\pi(2M)^2\dot{\phi}_\infty^2$.

The simple example of phantom cosmology (without a Big Rip) is realized for a scalar field with the potential $V = m^2\phi^2/2$, where $m \sim 10^{-33}$ eV [17]. After short transition phase, this cosmological model tends to the asymptotic state with $H \simeq m\phi/3^{1/2}$ and $\dot{\phi} \simeq 2m/3^{1/2}$. In the Klein-Gordon equation, the m^2 term (with the mentioned replacement $V \to -V$) is comparable to other terms only at the cosmological horizon distance. This means that the Jacobson solution is valid for this case also. Calculating the corresponding energy flux, one can easily obtain $\dot{M} = -4\pi(2M)^2\dot{\phi}_\infty^2 = -64M^2m^2/3$. For $M_0 = M_\odot$ and $m = 10^{-33}$ eV, the effective time of black hole mass decrease is $\tau = (3/64)M^{-1}m^{-2} \sim 10^{32}$ yr.

The possible physical interpretation of a black hole mass diminishing is that accreting particles of a phantom scalar field have a total negative energy [18]. The similar negative energy particles are created in the Hawking radiation process and participate in the Penrose black hole rotation energy extraction mechanism. Formally saying the black hole mass decreasing in the process of the phantom energy accretion is due to the violation of the energy domination condition in phantom energy. It should be noted that the existence of the horizon is not crucial for the decrease of the black hole mass due to phantom energy accretion. The one-way-membrane property of the horizon merely makes this accretion inevitable and

irreversible. Any object or device would diminish its mass if it is capable of capturing phantom energy.

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