# New Limits on Planck Scale Lorentz Violation in QED

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Constraints on possible Lorentz symmetry violation (LV) of order $E/M_{\text{Planck}}$ for electrons and photons in the framework of effective field theory (EFT) are discussed. Using (i) the report of polarized MeV emission from GRB021206 and (ii) the absence of vacuum Čerenkov radiation from synchrotron electrons in the Crab Nebula, we improve previous bounds by $10^{-10}$ and $10^{-2}$, respectively. We also show that the LV parameters for positrons and electrons are different, discuss electron helicity decay, and investigate how prior constraints are modified by the relations between LV parameters implied by EFT.

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The past few years have witnessed a rapid development of powerful constraints on some types of Lorentz symmetry violation (LV) that have been suggested by quantum gravity scenarios. While no current suggestion of LV is firm enough to be considered a prediction, there is nevertheless great interest in the possibility of LV induced by Planck scale physics since it offers the hope of an observational window into quantum gravity. To date, no LV phenomena have been observed (although the ultrahigh energy cosmic ray events detected by the Akeno Giant Air Shower Array, could possibly turn out to be harbingers of LV physics [1]). The absence of LV provides important constraints on viable quantum gravity theories. Moreover, these constraints are interesting in their own right as they extend the domain where relativity has been tested far beyond its previous frontiers.

The primary purpose of this Letter is to further strengthen the bounds on LV of order $E/M_P$ for photons and electrons, where $M_P = (\hbar c^5/G)^{1/2} = 1.22 \times 10^{19}$ GeV is the Planck energy, the presumed energy scale of quantum gravity. We use the reported observation [2] of polarized gamma rays from the gamma ray burst GRB021206 to improve the birefringence constraint by 10 orders of magnitude. [The results of [2] have been challenged [3] and defended [4]. If the polarization turns out to be weaker, then the birefringence constraint from GRB021206 is weakened (or eliminated).] By consideration of the vacuum Čerenkov process for the electrons producing the highest frequency synchrotron radiation from the Crab Nebula, we improve on the old birefringence constraint by 2 orders of magnitude.

A secondary purpose is to revisit previous constraints in light of the effective field theory (EFT) analysis of [5], some of which are strengthened and some weakened or limited in applicability. We show that EFT implies that the LV parameters for positrons are opposite (in two senses) compared to electrons, and we discuss a new LV process of "helicity decay," in which an electron of one helicity decays to a state with the opposite helicity. Finally, we pull together the strongest constraints to date and present them in a logarithmic plot that allows their nature and relative strength to be easily compared to previous work.

We adopt the framework of effective field theory as developed, e.g., in [1,5,6], focusing on the electron-photon sector since this involves no other particles and there are many observations allowing a number of independent constraints to be combined. We assume rotational symmetry is preserved in a preferred frame, which is taken to coincide with that of the cosmic microwave background radiation, and consider only LV suppressed by one power of the ratio $E/M_{\text{Planck}}$, which arises from mass dimension five operators in the Lagrangian. (We, thus, assume that lower mass dimension LV operators are suppressed by a symmetry or other mechanism; otherwise they would be expected to dominate [5,7].)

Under these assumptions, the most general photon and electron dispersion relations are [5]

$$E^2 = p^2 \pm \xi\, p^3/M \qquad \text{photons,} \tag{1}$$

$$E^2 = m^2 + p^2 + \eta_{R,L}\, p^3/M \qquad \text{electrons,} \tag{2}$$

where $\xi$, $\eta_R$, and $\eta_L$ are independent dimensionless parameters, and $M = 10^{19}$ GeV is factored out rather than the Planck mass $M_P = 1.22M$ for computational convenience. We adopt units with $\hbar = 1$ and the low energy speed of light $c = 1$. The sign in the photon dispersion relation (1) corresponds to the helicity (i.e., right or left circular polarization), while the labels $R$ and $L$ in the electron dispersion relation (2) apply for positive and negative electron helicity, respectively (see below for more details). The bound $|\eta_L - \eta_R| \leq 4$ [5] is provided by measurements of spin-polarized torsion pendulum frequency [8].

*New birefringence constraint.*—The dispersion relation (1) implies that electromagnetic waves of opposite

  

helicity have different phase velocities, which leads to a rotation of linear polarization direction through the angle

$$\theta(t) = [\omega_+(k) - \omega_-(k)]t/2 = \xi k^2 t/2M \qquad (3)$$

for a plane wave with wave vector $k$. Observations of polarized radiation from distant sources can, hence, be used to place an upper bound on $\xi$.

The best previous bound, $|\xi| \lesssim 2 \times 10^{-4}$, was obtained by Gleiser and Kozameh [9], using the observed 10% polarization of ultraviolet light from a distant galaxy. (See also [10,11] for similar birefringence bounds in the context of different types of Lorentz symmetry breaking.)

Recently, the prompt emission from the gamma ray burst GRB021206 was observed using the RHESSI detector [12]. A linear polarization of $80\% \pm 20\%$ was reported [2]. (This claim has been challenged [3] and defended [4].) During the 5 s of emission, the intensity varied strongly on a time scale of small fractions of a second consistently across the spectral window 0.15–2 MeV. The data [13] indicate a major contribution to the flux comes from photons significantly distributed over at least the energy range 0.1–0.5 MeV.

The constraint arises from the fact that, if the angle of polarization rotation (3) were to differ by more than $\pi/2$ over the range 0.1–0.3 MeV (and, hence, by more than $3\pi/2$ over the range 0.1–0.5 MeV), the instantaneous polarization at the detector would fluctuate sufficiently for the net polarization of the signal to be suppressed well below the observed value. (A stronger constraint could clearly be obtained by taking into account more precisely the spectral characteristics of the signal and detector.) The difference in rotation angles for wave vectors $k_1$ and $k_2$ is

$$\Delta\theta = \xi(k_2^2 - k_1^2)d/2M, \qquad (4)$$

where we have replaced the time $t$ by the distance $d$ from the source to the detector (divided by the speed of light).

While the distance to GRB021206 is unknown, it is well known that most cosmological bursts have redshifts in the range 1–2 corresponding to distances of greater than 1 Gpc. Using the distance distribution derived in Ref. [14], we conservatively take the minimum distance to this burst as 0.5 Gpc, corresponding to a redshift of $\sim$0.1. This then yields the constraint

$$|\xi| < 5.0 \times 10^{-15}/d_{0.5}. \qquad (5)$$

where $d_{0.5}$ is the distance to the burst in units of 0.5 Gpc.

*New Čerenkov-synchrotron constraint.*—In a region of the LV parameter space, there is an energy threshold for a free electron to emit a photon in a process called vacuum Čerenkov radiation. The threshold can occur either with emission of a soft photon or a hard photon depending on the parameters [15,16]. In the soft photon case, the threshold is $E_{\rm th} = (m^2M/2\eta)^{1/3} \approx 11\ {\rm TeV}/\eta^{1/3}$, from which it follows that the strength of the constraint on $\eta$ scales as the inverse cube of the electron energy, and that energies of the order of 10 TeV for the electron are required in order to put constraints of order unity on the LV parameters [15,16].

Electrons of energy up to 50 TeV are inferred via the observation of 50 TeV gamma rays from the Crab Nebula which are explained by inverse Compton (IC) scattering. Since the Čerenkov rate is orders of magnitude higher than the IC scattering rate, the Čerenkov process must not occur for these electrons [1,15]. This yields a constraint on $\eta$ of the order of $(10\ {\rm TeV}/50\ {\rm TeV})^3 \sim 10^{-2}$. Neither photon helicity should be emitted, so the absolute value $|\xi|$ is bounded, which strengthens the IC Čerenkov constraint. On the other hand, it could be that only one electron helicity produces the IC photons and the other loses energy by vacuum Čerenkov radiation. Hence, we can infer only that at least one of $\eta_R$ and $\eta_L$ satisfies the bound.

A complementary constraint was derived in [17] by making use of the very high energy electrons that produce the highest frequency synchrotron radiation in the Crab Nebula. For negative values of $\eta$, the electron has a maximal group velocity less than the speed of light; hence, there is a maximal synchrotron frequency that can be produced regardless of the electron energy [17]. Observations of the Crab Nebula reveal synchrotron radiation at least out to 100 MeV (requiring electrons of energy 1500 TeV in the Lorentz invariant case), which implies that at least one of the two parameters $\eta_{R,L}$ must be greater than $-7 \times 10^{-8}$ (this constraint is independent of the value of $\xi$). We cannot constrain both $\eta$ parameters in this way since it could be that all the Crab synchrotron radiation is produced by electrons of one helicity. Hence, for the rest of this discussion let $\eta$ stand for whichever of the two $\eta$'s satisfies the synchrotron constraint.

This must be the same $\eta$ as satisfies the IC Čerenkov constraint discussed above, since otherwise the energy of these synchrotron electrons would be below 50 TeV rather than the Lorentz invariant value of 1500 TeV. The Crab spectrum is well accounted for with a single population of electrons responsible for both the synchrotron radiation and the IC $\gamma$ rays. If there were enough extra electrons to produce the observed synchrotron flux with 30 times less energy per electron, then the electrons of the other helicity which would be producing the IC $\gamma$ rays would be too numerous.

We now use the existence of these synchrotron producing electrons to improve on the vacuum Čerenkov constraint. For a given $\eta > 0$, some definite electron energy $E_{\rm synch}(\eta)$ must be present to produce the observed synchrotron radiation. (This is higher for negative $\eta$ and lower for positive $\eta$ than the Lorentz invariant value [17].) Values of $|\xi|$ for which the vacuum Čerenkov threshold is lower than $E_{\rm synch}(\eta)$ for either photon helicity can therefore be excluded. (This is always a hard photon threshold, since the soft photon threshold occurs when the electron group velocity reaches the low energy speed of light, whereas the velocity required to produce

any finite synchrotron frequency is smaller than this.) For negative $\eta$, the Čerenkov process occurs only when $\xi < \eta$ [15,16], so the excluded parameters lie in the region $|\xi| > -\eta$.

*Implications of EFT for prior constraints.*—Photon time of flight.—The Lorentz violating dispersion relation (1) implies that the group velocity of photons, $v_g = 1 \pm \xi p/M$, is energy dependent. This leads to an energy dependent dispersion in the arrival time at Earth for photons originating in a distant event [18,19], which was previously exploited for constraints [20–22]. The dispersion of the two polarizations is larger since the difference in group velocity is then $2|\xi|p/M$ rather than $\xi(p_2 - p_1)/M$, but the time of flight constraint remains many orders of magnitude weaker than the birefringence one from polarization rotation. In Fig. 1, we use the EFT improvement of the constraint of [21] which yields $|\xi| < 63$.

Photon decay and photon absorption.—The constraints from photon decay $\gamma \to e^+e^-$ and absorption $\gamma\gamma \to e^+e^-$ must be reanalyzed to take into account the different dispersion for the two photon helicities, and the different parameters for the two electron helicities, but there is a further complication: Both of these processes involve positrons in addition to electrons. Previous constraint derivations have assumed that these have the same dispersion, but that need not be the case [23]. We show below for the $O(E/M)$ corrections that it is indeed not so. Taking into account the above factors could not significantly improve the strength of the constraints (which is mainly determined by the energy of the photons). We indicate here only what the helicity dependence of the photon dispersion implies, neglecting the important role of differing parameters for electrons and positrons and their helicity states.

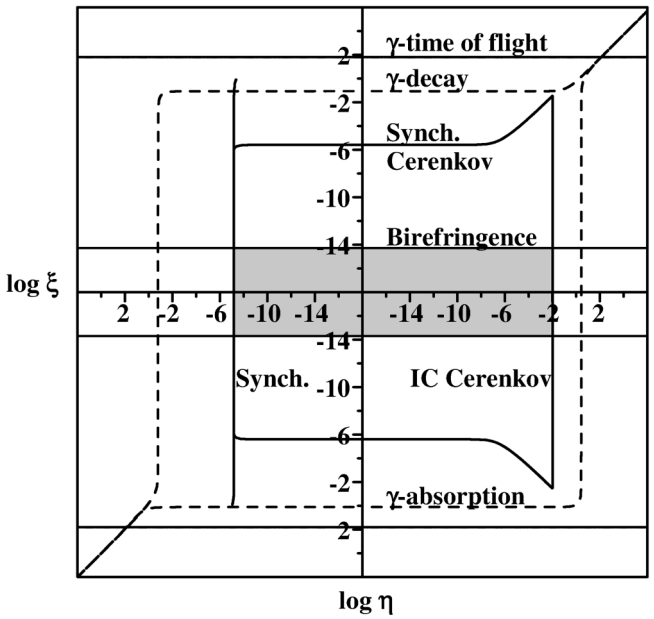


FIG. 1. Constraints on the photon ($\xi$) and electron ($\eta$) LV parameters. The birefringence constraint uses the observed polarization of MeV photons from GRB021206. The synchrotron and IC Čerenkov constraints use the observation of 0.1 GeV synchrotron and 50 TeV inverse Compton radiation from the Crab Nebula, respectively. For the origin of other constraints, see the text. For negative parameters the negative of the logarithm of the absolute value is plotted, and a region of width $10^{-18}$ is excised around each axis. The synchrotron and Čerenkov constraints are known to apply only for at least one $\eta_{R,L}$. The IC and synchrotron Čerenkov lines are truncated where they cross. Prior photon decay and absorption constraints are shown in dashed lines since they do not account for the EFT relations between the LV parameters.

The strongest limit on photon decay came from the highest energy photons known to propagate, which at the moment are the 50 TeV photons observed from the Crab Nebula [15,16]. Since their helicity is not measured, only those values of $|\xi|$ for which *both* helicities decay could be ruled out. The photon absorption constraint came from the fact that LV can shift the standard QED threshold for annihilation of multi-TeV $\gamma$ rays from nearby blazars such as Mkn 501 with the ambient infrared extragalactic photons [15,16,24–27]. LV depresses the rate of absorption of one photon helicity and increases it for the other. Although the polarization of the $\gamma$ rays is not measured, the possibility that one of the polarizations is essentially unabsorbed appears to be ruled out by the observations which show the predicted attenuation [27].

*Electron and positron dispersion.*—The Dirac equation in the Lorentz violating EFT including the dimension five operators can be written [5] as

$$[i\not{\partial} - m + (\eta_1 u + \eta_2 u\gamma^5)(u\cdot\partial)^2/M]\psi = 0, \qquad (6)$$

where $u^\alpha$ is the unit timelike four-vector that specifies the preferred frame. If we choose coordinates aligned with $u^\alpha$, so that $u^\alpha = \delta^\alpha_0$, an electron or positron mode of energy $E$ and momentum $p$ in the $x^3$ direction contributes to the field operator via $\exp[\mp i(Ex^0 - px^3)]\Upsilon$, where the upper sign here and below is for an electron and the lower for a positron, and $\Upsilon$ is the spinor. Inserting this in the deformed Dirac Eq. (6) yields

$$[\pm E\gamma^0 \mp p\gamma^3 - m - E^2(\eta_1\gamma^0 + \eta_2\gamma^0\gamma^5)/M]\Upsilon = 0. \qquad (7)$$

The helicity operator acting on $\Upsilon$ is $\pm(p_i/|p|)\Sigma^i$ [28], where $\Sigma^i = \gamma^5\gamma^0\gamma^i$. This is Hermitian and commutes with $\gamma^0$ times the operator in (7), which is also Hermitian. Hence, helicity remains a good quantum number in the presence of this Lorentz violation. Assuming without loss of generality that $p > 0$, a spinor for helicity $h$ therefore satisfies $\gamma^5\gamma^0\gamma^3\Upsilon = \pm h\Upsilon$, or, equivalently, $\gamma^0\gamma^5\Upsilon = \pm h\gamma^3\Upsilon$. For helicity eigenstates therefore

$$[(\pm E - \eta_1 E^2/M)\gamma^0 - (\pm p \pm h\eta_2 E^2/M)\gamma^3 - m]\Upsilon = 0. \qquad (8)$$

This has the form of the standard Dirac equation, with $E$ replaced by $\tilde{E} = \pm E - \eta_1 E^2/M$ and $p$ replaced by $\tilde{p} = \pm(p + h\eta_2 E^2/M)$. Hence, the dispersion relation is given by $\tilde{E}^2 = \tilde{p}_3^2 + m^2$. For $m \ll p \ll M$, this yields

$$E^2 = p^2 + m^2 + 2(\pm\eta_1 + h\eta_2)E^3/M. \quad (9)$$

With the definitions $\eta_R = 2(\eta_1 + \eta_2)$ and $\eta_L = 2(\eta_1 - \eta_2)$, the parameters in the dispersion relations for positive and negative helicity states, respectively, are thus $\eta_R$ and $\eta_L$ for electrons, and $-\eta_L$ and $-\eta_R$ for positrons.

*Possible new constraints from helicity decay.*—If $\eta_R$ and $\eta_L$ are unequal, say $\eta_R > \eta_L$, then a positive helicity electron can decay into a negative helicity electron and a photon, even when the LV parameters do not permit the vacuum Čerenkov effect. It can be shown (assuming $|\xi| \lesssim 10^{-3}$) that, for electrons of energy less than the transition energy $[m^2M/(\eta_R - \eta_L)]^{1/3}$, the lifetime of an electron susceptible to helicity decay is greater than $4\pi M/(\eta_R - \eta_L)e^2m^2$. At the limit of the best current bound $|\eta_L - \eta_R| < 4$, the transition energy is approximately 10 TeV and the lifetime for electrons below this energy is greater than $10^4$ seconds. This is long enough to preclude any terrestrial experiments from seeing the effect. The lifetime above the transition energy is instead bounded below by $E/e^2m^2$, which is $10^{-11}$ seconds for energies just above 10 TeV. The lifetime might therefore be short enough to provide new constraints.

Such a constraint might come from the Crab Nebula. Suppose that $\eta_L$ is below the synchrotron constraint (i.e., $\eta_L < -7 \times 10^{-8}$), so that $\eta_R$ must satisfy both the synchrotron and Čerenkov constraints as explained above. Then positive helicity electrons must have an energy of at least 50 TeV to produce the observed synchrotron radiation. These must not decay to negative helicity electrons (since those are unable to produce the synchrotron emission), which would require that the transition energy be greater than 50 TeV if the decay rate is fast enough. This would yield the constraint $\eta_R - \eta_L < 10^{-2}$.

*Combined constraints.*—The combined constraints are shown logarithmically in Fig. 1. The vast improvement in the birefringence constraint overwhelms the new synchrotron Čerenkov constraint, while the latter improves the previous birefringence constraint [9] by $10^2$. The allowed region is defined above and below by the birefringence bound $[O(10^{-14})]$, on the left by the synchrotron bound $[O(10^{-7})]$, and on the right by the IC Čerenkov bound $[O(10^{-2})]$. If the polarization of GRB021206 proves incorrect, the allowed region will expand vertically to the synchrotron Čerenkov lines. The combined constraints severely limit first order Planck suppressed LV, making any theory that predicts this type of LV very unlikely. The most useful improvements at this stage would be to strengthen the positive $\eta$ and $|\eta_R - \eta_L|$ bounds.

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*Note added in proof.*—If the charges producing the Crab nebula gamma rays consist of positrons as well as electrons, our earlier argument implies only that one of the four values $\pm\eta_{R,L}$ satisfies the combined synchrotron and Čerenkov constraints. We are investigating whether a more complete analysis of the effect on the synchrotron and IC spectra provides a stronger constraint.

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