Realization of a Photonic Controlled-NOT Gate Sufficient for Quantum Computation

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We report the first experimental demonstration of a quantum controlled-NOT gate for different photons, which is classically feed forwardable. In the experiment, we achieved this goal with only the use of linear optics, an entangled ancillary pair of photons, and postselection. The techniques developed in our experiment are of significant importance for quantum information processing with linear optics.

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Polarization-encoded qubits are well suited for information transmission in quantum information processing [1]. In recent years, the polarization state of single photons has been used to experimentally demonstrate quantum dense coding [2], quantum teleportation [3], and quantum cryptography [4–6]. However, due to the difficulty of achieving quantum logic operations between independent photons, the application of photon states has been limited primarily to the field of quantum communication. More precisely, the two-qubit gates suitable for quantum computation generically require strong interactions between individual photons, implying the need for massive, reversible nonlinearities well beyond those presently available for photons, as opposed to other physical systems [7].

Remarkably, Knill, Laflamme, and Milburn (KLM) [8] found a way to circumvent this problem and implement efficient quantum computation using only linear optics, photodetectors, and single-photon sources. In effect they showed that *measurement induced nonlinearity* was sufficient to obtain efficient quantum computation.

The logic schemes KLM proposed were not, however, economical in their use of optical components or ancillary photons. Various groups have been working on reducing the complexity of these gates while improving their theoretical efficiency (see, e.g., Koashi *et al.* [9]). In an exciting recent development, Nielsen [10] has shown that efficient linear optical quantum computation is, in fact, possible without the elaborate teleportation and Z-measurement error correction steps in KLM. This is achieved by the creation of linear optical versions of the cluster states of Raussendorf and Briegel [11]. Building on Nielsen's idea, it can be shown [12] that the gate we have performed, combined with single-photon measurements, is universal for linear optical quantum computing.

A crucial requirement of both KLM's and Nielsen's constructions is *classical feed forwardability.* Specifically, it must in principle be possible to detect when the gate has succeeded by measurement of ancilla photons in some appropriate state. This information can then be fed forward in such a way as to condition future operations on the photon modes.

Recently [13–15] destructive linear optical gate operations have been realized. As they necessarily destroy the output state, such schemes are not classically feed forwardable. In this Letter we report the first realization of a controlled-NOT (CNOT) gate that operates on two polarization qubits carried by independent photons and that satisfies the feed-forwardability criterion. Moreover, when combined with single qubit Hadamard rotations to perform a controlled-sign gate (so as to build the cluster states of [11] via Nielsen's method), this gate also satisfies the criterion that when it fails the qubits can be projected out in the computational basis.

A CNOT gate flips the second (target) bit if and only if the first one (control) has the logical value 1 and the control bit remains unaffected. The scheme we use to achieve the CNOT gate was first proposed in [16] by Pittman *et al.* and is shown in Fig. 1. This scheme performs a CNOT operation on the input photons in spatial modes a_1 and a_2; the output qubits are contained in spatial modes b_1 and b_2. The ancilla photons in the spatial modes a_3 and a_4 are in the maximally entangled Bell state

$$|\psi_{a_3a_4}\rangle = \frac{1}{\sqrt{2}}(|H\rangle_{a_3}|H\rangle_{a_4} + |V\rangle_{a_3}|V\rangle_{a_4}). \tag{1}$$

In the following H (a horizontally polarized photon) and V (a vertically polarized one) denote our logical 0 and 1. The scheme works in those cases where one and only one photon is found in each of the modes b_3, b_4, with a theoretical probability of $1/4$. When both photons are H polarized, no further transformation is necessary on the output state. As this is sufficient for a proof of principle demonstration, we operate the scheme only in this passive operation, whose success rate is reduced to $1/16$. Experimentally this success rate is further reduced by a factor of about 0, 79 by the limited fidelity. The scheme combines two simpler gates, namely, the destructive CNOT and the quantum encoder. The first gate can be seen in the lower part of Fig. 1 and is constituted by a polarizing beam splitter (PBS2) rotated by 45°(the rotation is represented by the circle drawn inside the symbol of the

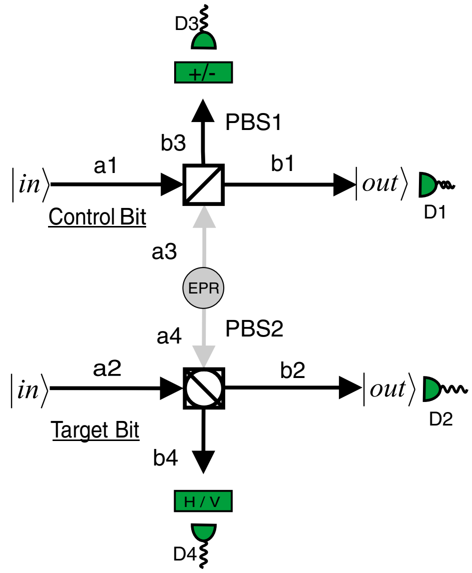

FIG. 1 (color online). The scheme to obtain a photonic realization of a CNOT gate with two independent qubits. The qubits are encoded in the polarization of the photons. The scheme makes use of linear optical components, polarization entanglement, and postselection. When one and only one photon is detected at the polarization sensitive detectors in the spatial modes b_3 and b_4 and in the polarization H, the scheme works as a CNOT gate.

PBS), which works as a destructive CNOT gate on the polarization qubits, as was experimentally demonstrated in [17]. The upper part, comprising the entangled state and the PBS1, is meant to encode the control bit in the two channels a_4 and b_1. The photons in the spatial modes a_3 and a_4 are in the maximally entangled Bell state (1).

Because of the behavior of our polarizing beam splitter that transmits horizontally polarized photons and reflects vertically polarized ones, the successful detection at the port b_3 of the state $|+\rangle$ (the symbols $+, -$ stand for $H + V$ and $H - V$) postselects the following transformation of the arbitrary input state in a_1

$$\alpha|H\rangle_{a_1} + \beta|V\rangle_{a_1} \rightarrow \alpha|HH\rangle_{a_4b_1} + \beta|VV\rangle_{a_4b_1}.$$

Thus, we have the control bit encoded in a_4 and in b_1, the photon in a_4 is the control input to the destructive CNOT gate and is destroyed, while the second photon in b1 is the output control qubit.

For the gate to work properly, we want the most general input state

$$|\Psi_{a_1a_2}\rangle = |H\rangle_{a_1}(\alpha_1|H\rangle_{a_2} + \alpha_2|V\rangle_{a_2}) + |V\rangle_{a_1}(\alpha_3|H\rangle_{a_2} + \alpha_4|V\rangle_{a_2}) \quad (2)$$

to be converted to the output state

$$|\Psi_{a_1a_2}\rangle = |H\rangle_{a_1}(\alpha_1|H\rangle_{a_2} + \alpha_2|V\rangle_{a_2}) + |V\rangle_{a_1}(\alpha_3|V\rangle_{a_2} + \alpha_4|H\rangle_{a_2}). \quad (3)$$

Let us consider first the case where the control photon is in the logical zero (H polarization state). The control photon then travels undisturbed through the PBS, arriving in the spatial mode b_1. As required, the output photon is H polarized. In order for the scheme to work a photon needs to arrive also at the detector D_3 in b_3: given the input photon already in the mode b_1, this additional photon comes necessarily from the Einstein-Podolsky-Rosen pair and is H polarized as it is transmitted by the PBS1. We know that the photons in a_3 and a_4 are correlated (1), so the photon in a_4 is also in the horizontal polarization. Taking into account the $-45°$ rotation of the polarization on the paths a_2, a_4 operated by the half-wave plates, the input in the PBS2 is then the state $|--\rangle_{a_2a_4}(|+-\rangle_{a_2a_4})$ for a target photon H (V) polarized. This state gives rise, with a probability of 50%, to the state where two photons go through the PBS2 $(|HH\rangle \pm |VV\rangle)_{b_2b_4}$ which, after the additional rotation of the polarization and the subsequent change to the H/V basis (where the measurement is performed) acquires the form $(|HH\rangle + |VV\rangle)_{b_2b_4}[(|HV\rangle + |VH\rangle)_{b_2b_4}]$. The expected result in the mode b_2 $H(V)$ is found for the case where the photon in b_4 is horizontally polarized. We can see in a similar way that the gate works also for the cases where the control photon is vertically polarized or is polarized at 45°.

Our experimental setup is shown in Fig. 2. In order to produce the entangled pair of ancilla photons in modes a_3 and a_4, we use a type II spontaneous parametric down conversion (SPDC) process; this pair is responsible for the transmission of the quantum part of the information. We also need to produce the two input qubits in the modes a_1 and a_2 to feed into the gate. In our setup these input qubits are another SPDC pair, where photon number entanglement is used and two photons are simultaneously produced; the polarization entanglement is destroyed by letting the photons pass through appropriate polarizers. Thanks to these polarization filters, and to appropriate half-wave plates, any desired two-qubits input state can be prepared.

An ultraviolet pulsed laser, centered at a wavelength of 398 nm, with pulse duration 200 fs and a repetition rate of 76 MH, impinges on a beta barium borate (BBO) crystal [18] producing probabilistically the first pair in the spatial modes a_1 and a_2: these two photons are fed into the gate as the input qubits. The UV laser is then reflected back by the mirror M1 and, passing through the crystal a second time, produces the entangled ancilla pair in spatial modes a_3 and a_4. Half-wave plates and nonlinear crystals in the paths provide the necessary birefringence compensation, and the same half-wave plates are used to adjust the phase between the down converted photons (i.e., to produce the state ϕ^+) and to implement the CNOT gate.

We then superpose the two photons at Alice's (Bob's) side in the modes a_1, a_3 (a_2, a_4) at the polarizing beam splitter PBS1 (PBS2). Moving the mirror M1, mounted on a motorized translation stage, allows one to change the arrival time to make the photons as indistinguishable as possible. A further degree of freedom is afforded by

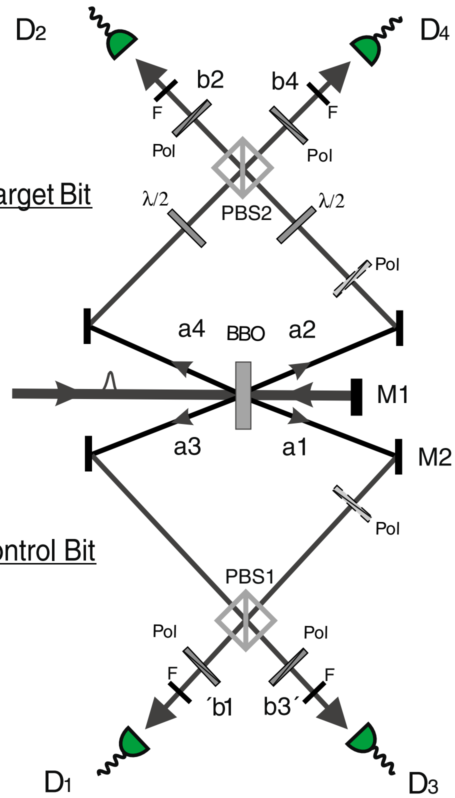

FIG. 2 (color online). The experimental setup. A type II spontaneous parametric down-conversion is used both to produce the ancilla pair (in the spatial modes a_3 and a_4) and to produce the two input qubits (in the spatial modes a_1 and a_2). In this case initial entanglement polarization is not desired, and it is destroyed by making the photons go through polarization filters that prepare the required input state. Half-wave plates have been placed in the photon paths in order to rotate the polarization; compensators are able to nullify the birefringence effects of the nonlinear crystal and of the polarizing beam splitters. Overlap of the wave packets at the PBSs is assured through spatial and spectral filtering.

the mirror M2, whose movement on a micrometrical translation stage corrects slight asymmetries in the arms of the setup. The indistinguishability between the overlapping photons is improved by introducing narrow bandwidth (3 nm) spectral filters at the outputs of the PBSs and monitoring the outgoing photons by fiber-coupled detectors. The single-mode fiber couplers guarantee good spatial overlap of the detected photons; the narrow bandwidth filters stretch the coherence time to about 700 fs—substantially larger than the pump pulse duration [19]. The temporal and spatial filtering process effectively erases any possibility of distinguishing the photon pairs and therefore leads to interference.

The scheme we have described allows the output photons to travel freely in space, so that they may be further used in quantum communication protocols, and this is achieved by detecting one and only one photon in modes b_3 and b_4. The fact that we do not yet have single-photon detectors for this wavelength at our disposal actually forces us to implement a fourfold coincidence detection to confirm that photons actually arrive in the output modes b_1 and b_2.

So far, we have analyzed only the ideal case where exactly one pair is produced at each passage of the UV light beam through the BBO crystal. In the real case, unwanted two-pair events contribute spuriously; their entity can be measured by blocking the paths a_1, a_2 in one case and measuring the fourfold coincidences due to two-pair production in the paths a_3, a_4. A similar measurement has to be performed on the other side. We coped with this noise by simply subtracting it from the measurements. Anyway, we note that the noise is not intrinsic in the setup and is only due to practical drawbacks. Indeed, an unbalancing method like the one used in [20] would allow one to increase the signal to noise ratio to any desired value.

One more detail that should be addressed here is the problem of birefringence. Each PBS introduces a small shift between the H and V components, thus deteriorating the overlap of the photon wave packets. This birefringence is responsible for the presence of unwanted terms at the output state. The nonlinear crystals put on the optical path are able to compensate for this as well, as noted elsewhere [21].

To experimentally demonstrate that the gate works, we first verify that we obtain the desired CNOT (appropriately conditioned) for the input qubits in states HH, HV, VH, and VV. In Fig. 3 we compare the count rates of all 16 possible combinations. We see, indeed, that the gate is working properly in this basis. Having verified this, we

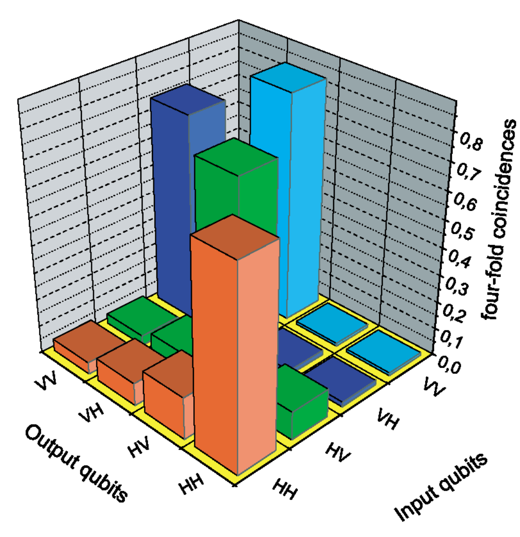

FIG. 3 (color). This graph shows that the scheme works, indeed, for the linear polarizations H,V. Fourfold coincidences for all the possible (16) combinations of inputs and outputs are shown. When the control qubit is in the logical value 0 (HH or HV), the gate works as the identity gate. In contrast, when the control qubit is in the logical value 1 (VH or VV), the gate works as a NOT gate, flipping the second input bit. Noise is due to the nonideal nature of the PBSs.

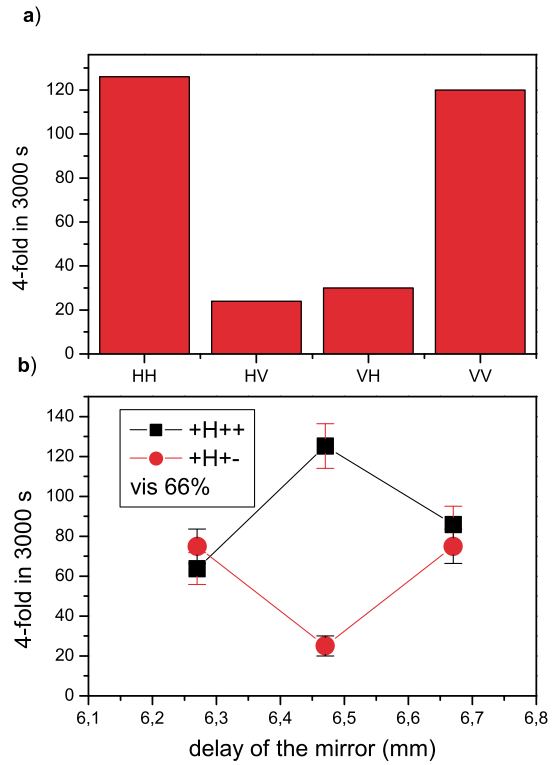

FIG. 4 (color). Demonstration of the ability of the CNOT gate to transform a separable state into an entangled state. In (a) the coincidence ratio between the different terms $HH, \ldots, VV$ is measured, proving the birefringence of the PBS has been sufficiently compensated; in (b) the superposition between HH and VV is proved to be coherent, by showing via the Ou-Hong Mandel dip at 45° that the desired $(H + V)$ state of the target bit emerges much more often than the spurious state $(H - V)$. The fidelity is of $81\% \pm 2\%$ in the first case and $77\% \pm 3\%$ for the second.

prove that the gate also works for a superposition of states. The special case where the control input is a 45° polarized photon and the target qubit is a H photon is very interesting: we expect that the state $|H + V\rangle_{a_1}|H\rangle_{a_2}$ evolves into the maximally entangled state $(|HH\rangle_{b_1b_2} + |VV\rangle_{b_1b_2})$. This shows the reason why CNOT gates are so important: they can transform separable states into entangled states and vice versa. We input the state $|+\rangle_{a_1}|H\rangle_{a_2}$; first we measure the count rates of the four combinations of the output polarization $(HH, \ldots, VV)$ and observe that the contributions from the terms HV and VH are negligible with a fidelity of 81%.

Then we prove that the output state is in a coherent superposition, which is done by a further polarization measurement. Going to the $|+\rangle, |-\rangle$ linear polarization basis, a Ou-Hong-Mandel interference measurement is possible; this is shown in Fig. 4.

To summarize, the above demonstrated realization of a feed-forwardable photonic CNOT gate uses only linear optics and entanglement. The nonlinearities required in such an interaction are obtained through projective measurement of the ancilla pair. Our result provides important progress in the direction of the realization of a quantum computer. The price we pay for a nondestructive scheme is the higher experimental sophistication, particularly the necessity to use high precision timing and coincidence techniques.

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