Phonon-Induced Decay of the Electron Spin in Quantum Dots

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(Received 28 October 2003; published 28 June 2004)

We study spin relaxation and decoherence in a GaAs quantum dot due to spin-orbit (SO) interaction. We derive an effective Hamiltonian which couples the electron spin to phonons or any other fluctuation of the dot potential. We show that the spin decoherence time T_2 is as large as the spin relaxation time T_1, under realistic conditions. For the Dresselhaus and Rashba SO couplings, we find that, in leading order, the effective B field can have only fluctuations transverse to the applied B field. As a result, $T_2 = 2T_1$ for arbitrarily large Zeeman splittings, in contrast to the naively expected case $T_2 \ll T_1$. We show that the spin decay is drastically suppressed for certain B-field directions and ratios of SO coupling constants. Finally, for the spin-phonon coupling, we show that $T_2 = 2T_1$ for all SO mechanisms in leading order in the electron-phonon interaction.

DOI: 10.1103/PhysRevLett.93.016601 PACS numbers: 72.25.Rb, 03.67.Lx, 73.21.La

Phase coherence of spin in quantum dots (QDs) is of central importance for spin-based quantum computation in the solid state [1,2]. Sufficiently long coherence times are needed for implementing quantum algorithms and error correction schemes. If the qubit is operated as a classical bit, its decay time is given by the spin relaxation time T_1, which is the time of a spin-flip process. For quantum computation, however, the spin decoherence time T_2—the lifetime of a coherent superposition of spin-up and spin-down states—must be sufficiently long. In semiconductor QDs, the spin coherence is limited by the dot *intrinsic* degrees of freedom, such as phonons, spins of nuclei, excitations on the Fermi surface (e.g., in metallic gates), fluctuating impurity states nearby the dot, electromagnetic fields, etc. It is well known (and experimentally verified) that the T_1 time of spin in QDs is extremely long, extending up to 100 μs. The decoherence time T_2, in its turn, is limited by both spin-flip and dephasing processes, and can be much smaller than T_1, although its upper bound is $T_2 \leq 2T_1$. Knowledge of the mechanisms of spin relaxation and decoherence in QDs can allow one to find regimes with the least spin decay.

Recently, the spin T_1 time in a one-electron GaAs QD was measured [3] by a pulsed relaxation measurement technique (PRMT) [4]. This technique was previously applied to detect triplet-to-singlet relaxation in a two-electron quantum dot [4], yielding a spin relaxation time of 200 μs. Application of PRMT to Zeeman sublevels became possible with resolving the Zeeman splitting in dc transport spectroscopy [3,5], which required a magnetic field $B > 5$ T. The results of Ref. [3] show that $T_1 > 50$ μs at $B = 7.5$ T and 14 T, with no indication of a B-field dependence. Experimental values for the spin T_2 time in a single QD are not available yet, but an ESR scheme for its measurement has been proposed [6]. The ensemble spin decoherence time T_2^* was measured in n-doped GaAs bulk semiconductors [7], demonstrating coherent spin precession over times exceeding $T_2^* \sim 100$ ns. This indicates that the decoherence time of a single spin is even larger, $T_2 \geq T_2^*$. However, the mechanisms of spin decoherence for extended and localized electrons are rather different (cf. Ref. [8]).

Different mechanisms of spin relaxation in QDs have been considered, such as spin-phonon coupling via spin-orbit (SO) [9] or hyperfine interaction [10], and spin-nuclear coupling [11–13]. The SO mechanisms yield no spin decay at $B = 0$, due to the Kramers degeneracy. Interestingly, for GaAs QDs, the orbital effect of B leads to no spin decay in lowest order in SO interaction [9,14]. This is due to the special form (linear in p) of the SO coupling in 2D. The leading order contribution is, thus, proportional to the Zeeman splitting and leads to long T_1 times in GaAs QDs varying strongly with B [9]. However, previous theories [9] do not apply to the high values of B used in recent experiments [3], and thus, no comparison could be made so far. As for the nuclear mechanism, the electron spin decay can be suppressed by applying a B field or by polarizing the nuclei [11,12].

In this Letter, we show that the spin T_2 time, caused by SO interaction in GaAs QDs, is as large as the spin T_1 time. We assume low temperatures, $T \ll \hbar\omega_0$, where $\hbar\omega_0$ is the dot size-quantization energy, and with no external noise in the applied B field. We, thus, argue that the lower bound $T_1 \geq 50$ μs established in Ref. [3] is, in fact, also a lower bound for T_2. Furthermore, we show that the spin decay can be reduced by a special choice of direction of **B**, if there is Rashba coupling.

The Hamiltonian describing the electron in a QD reads

$$H = H_d + H_Z + H_{SO} + U_{ph}, \tag{1}$$

$$H_d = \frac{p^2}{2m^*} + U(\mathbf{r}), \tag{2}$$

$$H_{SO} = \beta(-p_x\sigma_x + p_y\sigma_y) + \alpha(p_x\sigma_y - p_y\sigma_x), \tag{3}$$

$$H_Z = \tfrac{1}{2}g\mu_B\mathbf{B}\cdot\boldsymbol{\sigma}, \tag{4}$$

where $\mathbf{p} = -i\hbar\boldsymbol{\nabla} + (e/c)\mathbf{A}(\mathbf{r})$ is the electron 2D momentum, $U(\mathbf{r})$ is the lateral confining potential, with $\mathbf{r} = (x, y)$, and $\boldsymbol{\sigma}$ are the Pauli matrices. The axes x and y point along the main directions in the (001) plane of GaAs. The SO Hamiltonian (3) includes both the Dresselhaus SO coupling (β), due to the bulk inversion asymmetry of the GaAs lattice, and the Rashba SO coupling (α), due to asymmetry of the quantum well profile in the z direction. We consider here α and β as model parameters; for their microscopic derivation, see Ref. [15]. The magnetic field $\mathbf{B} = B(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ defines the spin quantization axis via the Zeeman term (4). The phonon potential is given by

$$U_{\rm ph}(\mathbf{r}) = \sum_{\mathbf{q}j} \frac{F(q_z)e^{i\mathbf{q}_\parallel \mathbf{r}}}{\sqrt{2\rho_c\omega_{qj}/\hbar}}(e\beta_{\mathbf{q}j} - iq\Xi_{\mathbf{q}j})(b^\dagger_{-\mathbf{q}j} + b_{\mathbf{q}j}),$$

where $b^\dagger_{\mathbf{q}j}$ creates an acoustic phonon with wave vector $\mathbf{q} = (\mathbf{q}_\parallel, q_z)$, branch index j, and dispersion ω_{qj}; ρ_c is the sample density [volume is set to unity in (5)]. Optical phonons play no role at the low energies considered here. The factor $F(q_z)$ in Eq. (5) equals unity for $|q_z| \ll d^{-1}$ and vanishes for $|q_z| \gg d^{-1}$, where d is the size of the quantum well along the z axis. We take into account both piezoelectric ($\beta_{\mathbf{q}j}$) and deformation potential ($\Xi_{\mathbf{q}j}$) kinds of electron-phonon interaction [16]. Next, we derive an effective Hamiltonian for the low temperature ($T \ll \hbar\omega_0$) spin dynamics, relaxation, and decoherence.

The electron spin couples to phonons due to the SO interaction (3). For typical GaAs QDs, the SO length $\lambda_{\rm SO} = \hbar/m^*\beta$ is much larger than the electron orbit size λ. The linear in the $\lambda/\lambda_{\rm SO}$ contribution to the spin-phonon coupling is due only to a finite Zeeman splitting [9,14]. We consider a B field, for which the spin-phonon coupling dominates the spin decay. For simplicity, we assume $m^*\beta^2 \ll g\mu_B B \ll \hbar\omega_0$. Using perturbation theory (or Schrieffer-Wolff transformation), we obtain [17] the effective Hamiltonian

$$H_{\rm eff} = \tfrac{1}{2}g\mu_B[\mathbf{B} + \delta\mathbf{B}(t)]\cdot\boldsymbol{\sigma}, \tag{5}$$

$$\delta\mathbf{B}(t) = 2\mathbf{B}\times\boldsymbol{\Omega}(t), \tag{6}$$

where $\boldsymbol{\Omega}(t) = \langle\psi|[(\hat{L}_d^{-1}\boldsymbol{\xi}), U_{\rm ph}(t)]|\psi\rangle$, $|\psi\rangle$ is the electron orbital wave function, $\hat{L}_d$ is the dot Liouvillean, $\hat{L}_d A = [H_d, A]$. The vector $\boldsymbol{\xi}$ has a simple form in the coordinate frame $x' = (x+y)/\sqrt{2}$, $y' = -(x-y)/\sqrt{2}$, $z' = z$ (see Fig. 1 inset), namely, $\boldsymbol{\xi} = (y'/\lambda_-, x'/\lambda_+, 0)$, where $1/\lambda_\pm = m^*(\beta \pm \alpha)/\hbar$. Equation (6) contains one of our main results: In first order in SO interaction, there can be only transverse fluctuations of the effective magnetic field, i.e., $\delta\mathbf{B}(t)\cdot\mathbf{B} = 0$ [18]. This statement holds true for spin coupling to any fluctuations, be it the noise of a gate voltage or coupling to particle-hole excitations in a Fermi sea. Next, we consider the decay of the electron spin, $\mathbf{S} = \boldsymbol{\sigma}/2$, governed by Eq. (5).

The phonons which are emitted or absorbed by the electron leave the dot during a time τ_c, $d/s \lesssim \tau_c \lesssim \lambda/s$, where s is the sound velocity. The electron spin decays over a much longer time in typical structures, and thus, undergoes many uncorrelated scattering events. Then, the spin obeys the Bloch equation [19]

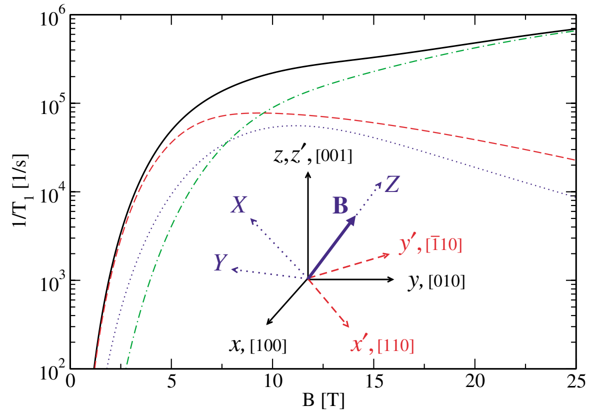

FIG. 1 (color online). Solid curve: The relaxation rate $1/T_1$ of Eq. (18) as a function of an in-plane B for a GaAs QD with $\hbar\omega_0 = 1.1$ meV, $d = 5$ nm, $\lambda_{\rm SO} = \hbar/m^*\beta = 1$ μm, and $\alpha = 0$. Dashed (dotted) curve: Contribution of the piezoelectric mechanism ($\overline{\beta}_{j\vartheta}$) with transverse (longitudinal) phonons. Dot-dashed curve: Contribution of the deformation potential mechanism ($\overline{\Xi}_j$). Note that $1/T_1 \propto 1/\lambda_{\rm SO}^2$.

$$\langle\dot{\mathbf{S}}\rangle = \boldsymbol{\omega}\times\langle\mathbf{S}\rangle - \Gamma\langle\mathbf{S}\rangle + \mathbf{Y}, \tag{7}$$

where $\boldsymbol{\omega} = \omega\boldsymbol{l}$, with $\omega = g\mu_B B/\hbar$ and $\boldsymbol{l} = \mathbf{B}/B$. For a generic $\delta\mathbf{B}(t)$ [with $\langle\delta\mathbf{B}(t)\rangle = 0$], in the Born-Markov approximation [19,20], we find $\Gamma_{ij} = \Gamma^r_{ij} + \Gamma^d_{ij}$, with

$$\Gamma^r_{ij} = \delta_{ij}(\delta_{pq} - l_p l_q)J^+_{pq}(\omega) - (\delta_{ip} - l_i l_p)J^+_{pj}(\omega) - \delta_{ij}\varepsilon_{kpq}l_k I^-_{pq}(\omega) + \varepsilon_{ipq}l_p I^-_{qj}(\omega), \tag{8}$$

$$\Gamma^d_{ij} = \delta_{ij}l_p l_q J^+_{pq}(0) - l_i l_p J^+_{pj}(0), \tag{9}$$

where $J^\pm_{ij}(w) = \mathrm{Re}[J_{ij}(w) \pm J_{ij}(-w)]$ and $I^\pm_{ij}(w) = \mathrm{Im}[J_{ij}(w) \pm J_{ij}(-w)]$ are given by the spectral function

$$J_{ij}(w) = \frac{g^2\mu_B^2}{2\hbar^2}\int_0^{+\infty}\langle\delta B_i(0)\delta B_j(t)\rangle e^{-iwt}\,dt. \tag{10}$$

The inhomogeneous part in Eq. (7) is given by

$$2\Upsilon_i = l_j J^-_{ij}(\omega) - l_i J^-_{jj}(\omega) + \varepsilon_{ipq}I^+_{pq}(\omega) + \varepsilon_{iqk}l_k l_p[I^+_{pq}(\omega) - I^+_{pq}(0)], \tag{11}$$

where ε_{ijk} is the antisymmetric tensor. Equation (7) describes spin decay in a number of problems, such as electron scattering off impurities in bulk systems, nuclear spin scattering [19], etc. In our notation, the spin decay comes from the symmetric part of Γ, whereas the antisymmetric part leads to a correction to $\boldsymbol{\omega}$ in Eq. (7). The tensor Γ^r describes spin decay due to processes of energy relaxation such as emission or absorption of a phonon. Thus, the T_1 time is entirely determined by Γ^r [see Eq. (16)]. The tensor Γ^d can be nonzero due only to elastic scattering of spin, i.e., due to dephasing. Γ^d contributes to the decoherence time T_2, and so does Γ^r. In many cases, however, the latter contribution is negligible, and Γ^d entirely dominates the spin decoherence [19]. This is in strong contrast to what we find here for an electron localized in a QD. To illustrate this, we first consider an example when Γ^d dominates the spin decoherence and then return to our case. A textbook example is $\langle \delta B_i(0)\delta B_j(t)\rangle = \bar{b}^2\delta_{ij}\exp(-|t|/\tau_c)$. Choosing $\boldsymbol{l} = (0,0,1)$, we obtain from Eqs. (8)–(10) the nonzero elements: $\Gamma^r_{xx} = \Gamma^r_{yy} = \Gamma^r_{zz}/2 = \gamma_n^2\bar{b}^2\tau_c/(1+\omega^2\tau_c^2)$, and $\Gamma^d_{xx} = \Gamma^d_{yy} = \gamma_n^2\bar{b}^2\tau_c$, where $\gamma_n = g\mu_B/\hbar$. The longitudinal component $\langle S_z\rangle$ decays over the time $T_1 = \Gamma_{zz}^{-1} = 1/\Gamma^r_{zz}$. The transverse components decay over the time $T_2 = 1/(\Gamma^r_{xx} + \Gamma^d_{xx})$. At $\omega \gg 1/\tau_c$, the contribution of Γ^r_{xx} to T_2 is negligible, and hence, $T_2 \ll T_1$. The latter relation has widely been quoted in the literature on quantum computing. In stark contrast to this example, we show now below that there are no *intrinsic* dephasing mechanisms for our case, which would justify this relation for the electron spin in GaAs QDs at $T \ll \hbar\omega_0$.

We start with calculating the spin decay due to the mechanism (6). Here, Γ^d_{ij} is identically zero, due to the transverse nature of the fluctuating field $\delta\mathbf{B}$. This can be inferred from Eqs. (9) and (10), noticing that each term in (9) contains $\boldsymbol{l}\cdot\delta\mathbf{B} = 0$. In order to calculate Γ^r_{ij}, we first find the main axes of the tensor $J_{ij}(w)$ [see Eq. (10)]. $J_{ij}(w)$ is diagonal in the frame (X,Y,Z) (see inset of Fig. 1), which is obtained from (x',y',z) by a rotation with Euler angles φ', θ, and χ. Here, the angles $\varphi' = \varphi - \pi/4$ and θ give $\mathbf{B}$ in the frame (x',y',z), and χ depends on the details of $U(\mathbf{r})$. It can be determined from $\langle \delta B_X \delta B_Y(t)\rangle = 0$. For $U(\mathbf{r}) = U(r)$, we find [21]

$$\tan 2\chi = \frac{2(\lambda_+^2 - \lambda_-^2)l_{x'}l_{y'}l_z}{\lambda_+^2(l_{y'}^2 - l_z^2 l_{x'}^2) + \lambda_-^2(l_{x'}^2 - l_z^2 l_{y'}^2)}. \tag{12}$$

We now consider $U(r) = m^*\omega_0^2 r^2/2$ and evaluate $\boldsymbol{\Omega}(t)$ of Eq. (6) for the ground state $\psi(\mathbf{r}) = \exp(-r^2/2\lambda^2)/\lambda\sqrt{\pi}$, where $\lambda^{-2} = \hbar^{-1}\sqrt{(m^*\omega_0)^2 + (eB_z/2c)^2}$. Using [22]

$$y = \frac{-i}{\hbar m^*\omega_0^2}\hat{L}_d\left(p_y - \frac{eB_z}{\hbar c}x\right), \tag{13}$$

we find $\Omega_{x'}$ from $U_{\rm ph}(\mathbf{r})$ by substituting

$$\exp(i\mathbf{q}_\parallel\mathbf{r}) \to \frac{-iq_{y'}}{m^*\omega_0^2\lambda_-}\exp(-q_\parallel^2\lambda^2/4). \tag{14}$$

$\Omega_{y'}$ is obtained from $U_{\rm ph}(\mathbf{r})$, using (14) with the prefactor $q_{y'}/\lambda_- \to q_{x'}/\lambda_+$. Finally, we obtain

$$\begin{aligned}\mathrm{Re}\,J_{XX}(w) = \frac{\omega^2 w^3(N_w+1)}{(2\Lambda_+ m^*\omega_0^2)^2}\sum_j \frac{\hbar}{\pi\rho_c s_j^5}\int_0^{\pi/2} d\vartheta \sin^3\vartheta \\ \times e^{-(w\lambda\sin\vartheta)^2/2s_j^2}\left|F\left(\frac{|w|}{s_j}\cos\vartheta\right)\right|^2 \\ \times\left(e^2\overline{\beta}_{j\vartheta}^2 + \frac{w^2}{s_j^2}\Xi_j^2\right),\end{aligned} \tag{15}$$

where $N_w = (e^{\hbar w/T} - 1)^{-1}$, and s_j is the sound velocity for branch j. For GaAs, we use $s_1 = 4.73\times 10^5$ cm/s and $s_2 = s_3 = 3.35\times 10^5$ cm/s. Furthermore, $\Xi_j = \delta_{j,1}\Xi_0$ with $\Xi_0 = 6.7$ eV, and $\overline{\beta}_{1,\vartheta} = 3\sqrt{2}\pi h_{14}\kappa^{-1}\sin^2\vartheta\cos\vartheta$, $\overline{\beta}_{2,\vartheta} = \sqrt{2}\pi h_{14}\kappa^{-1}\sin 2\vartheta$, $\overline{\beta}_{3,\vartheta} = \sqrt{2}\pi h_{14}\kappa^{-1}(3\cos^2\vartheta - 1)\sin\vartheta$, with $h_{14} = -0.16$ C/m^2 and $\kappa = 13.1$. The effective SO length Λ_+ in Eq. (15) is given by

$$\frac{2}{\Lambda_\pm^2} = \frac{1-l_{x'}^2}{\lambda_-^2} + \frac{1-l_{y'}^2}{\lambda_+^2} \pm \sqrt{\left(\frac{1-l_{x'}^2}{\lambda_-^2} + \frac{1-l_{y'}^2}{\lambda_+^2}\right)^2 - \frac{4l_z^2}{\lambda_+^2\lambda_-^2}}.$$

$\mathrm{Re}\,J_{YY}(w)$ is obtained from Eq. (15) by substituting $\Lambda_+ \to \Lambda_-$, and $J_{ZZ}(w) = 0$. $\mathrm{Im}\,J_{XX}(w)$ and $\mathrm{Im}\,J_{YY}(w)$ are irrelevant for our discussion; see further. From Eq. (8), we obtain $\Gamma^r_{XX} = J^+_{YY}(\omega)$, $\Gamma^r_{YY} = J^+_{XX}(\omega)$, $\Gamma^r_{ZZ} = J^+_{XX}(\omega) + J^+_{YY}(\omega)$, $\Gamma^r_{XY} = -I^-_{YY}(\omega)$, and $\Gamma^r_{YX} = I^-_{XX}(\omega)$. Since $\Gamma_{ij}/\omega \lesssim \omega\lambda^2/\omega_0\Lambda_\pm^2 \ll 1$, we can solve the secular equation iteratively [23]. In the general case,

$$\frac{1}{T_1} := l_p l_q \Gamma_{pq} = \Gamma_{ZZ} = \Gamma^r_{ZZ}, \tag{16}$$

$$\frac{1}{T_2} := \frac{1}{2}(\delta_{pq} - l_p l_q)\Gamma_{pq} = \frac{1}{2}(\Gamma_{XX} + \Gamma_{YY}). \tag{17}$$

Then, the solution of Eq. (7) reads $\langle S_X(t)\rangle = S_\perp e^{-t/T_2}\sin(\omega t + \phi)$, $\langle S_Y(t)\rangle = S_\perp e^{-t/T_2}\cos(\omega t + \phi)$, and $\langle S_Z(t)\rangle = S_T + (S_Z^0 - S_T)e^{-t/T_1}$, with the thermodynamic value of spin being $\mathbf{S}_T = \boldsymbol{l}(\boldsymbol{l}\cdot\mathbf{Y})T_1 = -(\boldsymbol{l}/2)\tanh(\hbar\omega/2k_BT)$, and the initial value $\langle \mathbf{S}(0)\rangle = (S_\perp\sin\phi, S_\perp\cos\phi, S_Z^0)$. For our special situation with purely transverse fluctuations ($\Gamma^d = 0$), we obtain

$$\frac{1}{T_1} = \frac{2}{T_2} = J^+_{XX}(\omega) + J^+_{YY}(\omega). \tag{18}$$

We plot $1/T_1$ as a function of B for $\theta = \pi/2$ and $\alpha = 0$ on Fig. 1 (solid curve). In agreement with experiment [3], we find that $1/T_1$ has a plateau in a wide range of B fields, due to a crossover from the piezoelectric-transverse (dashed curve) to the deformation potential (dot-dashed curve) mechanism of electron-phonon interaction. For arbitrary φ, θ, and α, we have $1/T_1 = f/T_1(\theta = \pi/2, \alpha = 0)$ with

$$f = \frac{1}{\beta^2}[(\alpha^2 + \beta^2)(1 + \cos^2\theta) + 2\alpha\beta\sin^2\theta\sin 2\varphi].$$

Note that $\sqrt{f}$ describes an ellipsoid in the frame (x',y',z), i.e., $f = x'^2 + y'^2 + z^2$, with dimensionless x', y', z obeying $(x'/a)^2 + (y'/b)^2 + (z/c)^2 = 1$, where $a = 1 + \alpha/\beta$, $b = 1 - \alpha/\beta$, and $c = \sqrt{a^2 + b^2}$. Thus, if $\alpha = \beta$, then $b = 0$, i.e., $1/T_1$ *vanishes* if $\mathbf{B} \parallel y'$. The same is true for $\alpha = -\beta$ and $\mathbf{B} \parallel x'$. The case $\alpha = \pm\beta$ was considered previously for extended electron states in a two-dimensional electron gas (2DEG) [24]. Note that the Hamiltonian (1) conserves the spin component $\sigma_{y'(x')}$ for $\alpha = \beta$ ($\alpha = -\beta$) and $\mathbf{B} \parallel y'$ (x'). This spin conservation results in T_1 being infinite to all orders in the SO

Hamiltonian (3). At the same time, $1/T_2$ reduces to the next order contribution of (3). However, as we show below, a single-phonon process is inefficient in inducing dephasing, and therefore, $1/T_2$ can be nonzero only in the next order in electron-phonon interaction. Next, we note that a long-lived spin state also occurs in a different GaAs structure, namely, for a 2DEG grown in the (110) direction. Then, the normal to the 2DEG plane component of spin is conserved [25], provided $\alpha = 0$.

We discuss now other SO mechanisms. In Eq. (3), we omitted the so-called k^3 terms of the Dresselhaus SO coupling [25]; i.e., $H_{SO} \propto \hbar^{-2}\beta d^2(\sigma_x\{p_x, p_y^2\} - \sigma_y\{p_y, p_x^2\})$. They are parametrically small ($d^2/\lambda^2 \ll 1$) in the 2D limit, compared to the retained ones. However, their contribution to the spin decay can be important, if $gm^*/m_0 \lesssim (d/\lambda)^2 \cos\theta$ and $\lambda^3 \lesssim d^2\lambda_{SO}$, since the orbital effect of B contributes here in the first place. Otherwise, the orbital effect is given by the second order contribution of (3), i.e., by $H_{SO}^{(2)} = -H_{SO}\hat{L}_d^{-1}H_{SO}$, and can be important, if $gm^*/m_0 \lesssim (\lambda/\lambda_{SO})\cos\theta$. For in-plane B fields, however, these mechanisms are negligible.

Additional spin decay mechanisms arise from the *direct* spin-phonon interaction [9]. The strain field produced by phonons couples to the electron spin via the SO interaction, resulting in the term $\Delta H' = (V_0/4)\varepsilon_{ijk}\sigma_i\{u_{ij}, p_k\}$, where p_i is the bulk kinetic momentum, u_{ij} is the phonon strain tensor, and $V_0 = 8 \times 10^7$ cm/s for GaAs. A similar mechanism occurs in a B field, due to g-factor fluctuations caused by lattice distortion. This yields $\Delta H'' = \tilde{g}\mu_B\sum_{i\neq j}u_{ij}\sigma_i B_j$, where $\tilde{g} \approx 10$ for GaAs. The contribution of these mechanisms to the spin-flip rates in QDs has been estimated in Ref. [9]. Except for the $\alpha = \pm\beta$ cases discussed above, the direct mechanisms are usually negligible in QDs. Here, we find that such spin-phonon couplings do not violate the equality $T_2 = 2T_1$. For this, we note that $\Gamma_{ij}^d = 0$ for a generic $\delta B_i = \sum_{\mathbf{q}} M_i(\mathbf{q})(b_{-\mathbf{q}}^\dagger + b_{\mathbf{q}})$ in Eq. (5), if $q|M_i(\mathbf{q})|^2 \to 0$ at $q \to 0$. Obviously, this condition is satisfied for the direct spin-phonon mechanisms, since $u_{ij} = 0$ at $q = 0$. The same follows for the Hamiltonian (1) with the phonon potential (5) and an arbitrary H_{SO}; the physical explanation is that the potential of long-wave phonons is constant over the dot size and, thus, commutes with H_{SO}. Finally, we note that, at temperatures $T \sim \hbar\omega_0$, there can be dephasing mechanisms [26], which can result in $T_2 \ll T_1$.

In conclusion, we have shown that the decoherence time T_2 of an electron spin in a GaAs QD is as large as the relaxation time T_1 for the spin decay based on SO mechanisms.

We acknowledge support from the Swiss NSF, NCCR Basel, EU RTN "Spintronics," U.S. DARPA, ARO, and ONR.

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