

Non-Fermi-Liquid Behavior in Itinerant Antiferromagnets

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We consider a two-dimensional itinerant antiferromagnet near a quantum-critical point. We show that, contrary to conventional wisdom, fermionic excitations in the ordered state are not the usual Fermi-liquid quasiparticles. Instead, down to very low frequencies, the fermionic self-energy varies as $\omega^{2/3}$. This non-Fermi-liquid behavior originates in the coupling of fermions to the longitudinal spin susceptibility $\chi_{\parallel}(q, \Omega)$ in which the order-induced “gap” in the spectrum at $q = 0$ dissolves into the Landau damping term at $v_F q > \Omega$. The transverse spin fluctuations obey the $z = 1$ scaling characteristic of spin waves, but remain overdamped in a finite range near the critical point.

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The Fermi-liquid (FL) theory is a powerful tool to study properties of interacting electrons. It states that, upon switching on the electron-electron interaction, elementary excitations near the Fermi surface remain fermionic, with well-defined energy and momentum. As a result, Fermi liquids have a constant uniform spin susceptibility and a linear in temperature, T , electronic specific heat.

In the past two decades, many new compounds were found which exhibit thermodynamic and transport properties dramatically different from those of a Fermi liquid [1]. Understanding the origin of this non-Fermi-liquid (NFL) behavior is one of the most important challenges in condensed matter physics.

Few avenues leading to the NFL behavior in dimension $D > 1$ have been proposed in the theory of interacting electron systems. One of the most widely studied scenarios ties the destruction of the Fermi liquid to a quantum-critical point (QCP) [2]. At a QCP, the fluctuations of the order parameter (OP) are gapless, and the effective electron-electron interaction mediated by this bosonic mode is long ranged. Scattering of electrons by gapless bosons is singular and destroys a Fermi liquid when the local static bosonic susceptibility $\chi_l = \int d^{D-1}q \chi(q, \omega = 0)$ diverges [3].

It is generally expected that, away from the QCP, the Fermi-liquid behavior is preserved. On the disordered side, the FL is protected by the gap in the OP fluctuations due to a finite correlation length. On the ordered side, where the OP acquires a mean value, the amplitude (longitudinal) bosonic excitations are gapped, while the gapless Goldstone excitations (phase or transverse modes) are harmless for fermions due to the Adler’s principle [4] that states that the fermion-boson vertex vanishes at the momentum transfer equal to the ordering wave vector \mathbf{Q} .

The main conclusion of this Letter is that, contrary to this general belief, a novel non-Fermi-liquid electron behavior emerges on the ordered side of a QCP

when $\mathbf{Q} \neq 0$. This NFL behavior originates from the interaction between electrons and longitudinal bosonic excitations. The key observation is that this mode becomes gapless and overdamped in the range $v_F q > \Omega$ due to Landau damping from the electrons, and gives rise to a strong electron-electron interaction even at the lowest energies. The NFL behavior persists down to a frequency ω_{\min} that vanishes if the fermionic bandwidth is infinite.

Below, we consider a SDW transition between a paramagnet and an itinerant antiferromagnet (AFM) with $\mathbf{Q} = (\pi, \pi)$ and the dynamical exponent $z = 2$ in $D = 2$. Such a transition is both one of the most studied and the most relevant experimentally, as it occurs in many heavy fermion materials and is believed to be responsible for their unusual properties.

We first present the results and discuss the physics and then report the details of the calculations. We measure the AFM order by the gap in the fermionic spectrum, m . Opening of this gap renormalizes the spin susceptibilities from their form on the paramagnetic side, $\chi_{\perp}^{-1}(\mathbf{q}, \Omega) = \chi_{\parallel}^{-1}(\mathbf{q}, \Omega) = (\mathbf{q} - \mathbf{Q})^2 - i\gamma|\Omega|$ [3]. Hereafter, \mathbf{q} denotes the deviation from (π, π) , except when noted. Conventional wisdom holds that, in the AFM state, at energies smaller than $2m$, the transverse spin excitations are the Goldstone spin waves, while the longitudinal excitations are gapped at all bosonic \mathbf{q} and Ω . We find, however, that this behavior holds only when $v_F q$ is smaller than the frequency, $2m > \Omega > v_F q$. In this range, $\chi_{\perp}^{-1} \approx q^2 - a\gamma\Omega^2/m$, has a spin-wave form [a is a number $O(1)$], while $\chi_{\parallel}^{-1} \approx q^2 + a\gamma\sqrt{m^2 - \Omega^2}/4$ has a gap, $2m$, for the propagating spin excitations.

In the opposite limit, $2m > v_F q > \Omega$, we find

$$\begin{aligned}\chi_{\parallel}^{-1} &\approx q^2 - 2i\gamma m|\Omega|/v_F q, \\ \chi_{\perp}^{-1} &\approx q^2 - i\gamma|\Omega|v_F q/m.\end{aligned}\quad (1)$$

The transverse spin excitations become overdamped, though they still show $z = 1$ scaling. More importantly, the constant $2m$ gap in the longitudinal spin excitations

dissolves into the Landau damping term, and the longitudinal excitations become *gapless* with the dynamical exponent $z = 3$. This implies that, at a given $v_F q \ll m$, there are two regions of Ω ($\Omega < v_F q$ and $\Omega > 2m - v_F q$), where $\chi_{\parallel}''(q, \Omega)$ is nonzero [see Fig. 3 (below)].

The gapless overdamped form of the longitudinal susceptibility strongly affects fermionic self-energy as the interaction vertex for χ_{\parallel} is not reduced by Adler's symmetry principle. Critical theories with $z = 3$ have been studied extensively in context of gauge theories and QC behavior in itinerant ferromagnets [5]. In two dimensions, the electron self-energy due to interaction with such a mode varies as $(i\omega)^{2/3}$. The analogy between our case and a quantum ferromagnet stems from the fact that in our case large ($\approx \mathbf{Q}$) momentum transfers on the paramagnetic side become small momentum transfers in the AFM state, due to the doubling of the unit cell. Note that $(i\omega)^{2/3}$ self-energy comes from $v_F q \propto \Omega^{1/3} \gg \Omega$ where Eq. (1) is valid. A similar power law was also found in electronic nematic phases [6].

For the infinite fermionic bandwidth, W , the $\omega^{2/3}$ behavior extends from $\omega \sim m$ down to the lowest energies. When W is finite, the longitudinal spin fluctuations have a gap of the order of m^2/W even for $\Omega < v_F q$. This scale sets the lower cutoff for $\omega^{2/3}$ behavior; the Fermi-liquid behavior is restored at lower energies. Still, in itinerant AFM, $m \ll W$ (see below), i.e., $m^2/W \ll m$; hence, Fermi-liquid behavior in the AFM ordered phase is violated in a wide frequency range.

The evolution of the spin response with $\Omega/v_F q$ is related to the fact that the mass term in $\chi_{\parallel}(q, \Omega)$ is proportional to the particle-hole polarization bubble $\tilde{\Pi}(q, \Omega) = \Pi(q, \Omega) - \Pi(q, 0)$ (see below). In a nested antiferromagnet, the AFM order opens a gap over the entire Fermi surface. In this case, $\tilde{\Pi}(\mathbf{q}, \Omega)$ is a constant independent on the ratio $\Omega/v_F q$. However, in an itinerant AFM without nesting, only parts of the Fermi surface close to the ‘‘hot spots’’ become gapped due to long-range spin ordering. Elsewhere on the Fermi surface a continuous particle-hole spectrum still exists. As a result, spin excitations can decay into particle-hole pairs. On the paramagnetic side, this process involves large momenta near \mathbf{Q} ; hence, the damping is linear in frequency, $\text{Im} \tilde{\Pi}(\mathbf{q}, \Omega) = \gamma|\Omega|$. In the ordered state, the lattice period is doubled, and the fermionic momenta \mathbf{k} and $\mathbf{k} + \mathbf{Q}$ become equivalent. As a result, a bosonic mode at $\mathbf{Q}_1 \sim \mathbf{Q}$ scatters a fermion from a point \mathbf{k}_F into a Fermi surface point $\mathbf{k}_F + \mathbf{q}$ with small $\mathbf{q} = \mathbf{Q}_1 - \mathbf{Q}$ (see Fig. 1). The polarization bubble then becomes $\tilde{\Pi}(\mathbf{q}, \Omega) \propto \gamma m \Omega / \sqrt{\Omega^2 - (v_F q)^2}$. It gives rise to a mass term for $\Omega \gg v_F q$, but reduces to $i\gamma m \Omega / v_F q$ in the limit $\Omega \ll v_F q$.

We now provide the details of the calculation. To investigate the properties of an itinerant antiferromagnet near a QCP we employ the spin fermion model, which has been widely used to study the properties of paramag-

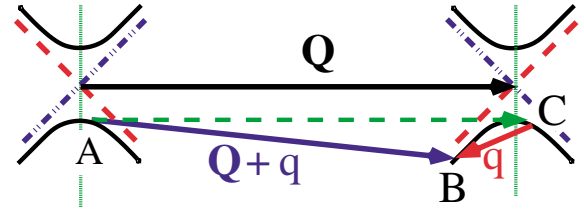


FIG. 1 (color online). Doubling of the unit cell in an AFM transition. Vertical dotted lines are the boundaries of the magnetic Brillouin zone. The Fermi surface at the ‘‘hot spots’’ at the paramagnetic side of the transition is the dashed red line; the dash-dotted blue line is its translation by \mathbf{Q} . Solid black lines denote the Fermi surface in the AFM state.

netic metals close to a magnetic instability [3]. Here we use it on the ordered side of the transition. The model Hamiltonian consists of three parts: the electronic band, $H_f^{(0)} = \sum_{\mathbf{k}, \alpha} \epsilon(\mathbf{k}) c_{\mathbf{k}, \alpha}^\dagger c_{\mathbf{k}, \alpha}$, the collective fermionic spin excitations, $H_s^{(0)} = \sum_{\mathbf{q}} \chi_0^{-1}(\mathbf{q}) \mathbf{S}_{\mathbf{q}} \mathbf{S}_{-\mathbf{q}}$, and the interaction of electrons with the spin fluctuations,

$$H_{sf}^{(0)} = g \sum_{\mathbf{k}, \mathbf{q}, \alpha, \beta} c_{\mathbf{k}, \alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} \mathbf{S}_{\mathbf{q}} c_{\mathbf{k}+\mathbf{q}, \beta} + \text{H.c.} \quad (2)$$

Here $\epsilon(\mathbf{k})$ is the bare quasiparticle dispersion, $\chi_0(\mathbf{q})$ is the bare static spin susceptibility, and g is a coupling constant that we take to be independent of momentum. The rationale for this model is that the antiferromagnetic interactions emerge from the energies up to the fermion bandwidth. Therefore to analyze the behavior of low-energy quasiparticles it is sufficient to separate the energy scales and treat the collective spin degrees of freedom as a separate bosonic field. This procedure is internally consistent if the coupling $g \ll W$.

Properties of the spin subsystem are determined by the bare susceptibility, which we take to have the Ornstein-Zernike form, $\chi_0(\mathbf{q}) = \chi_0 / [\xi_0^{-2} + (\mathbf{q} - \mathbf{Q})^2]$, as in earlier work [3]. In the paramagnetic phase, the magnetic correlation length, ξ_0 , is real and controls the separation from the AFM instability. The full dynamical susceptibility is $\chi(\mathbf{q}, \omega) = \chi_0 / [\xi_0^{-2} + (\mathbf{q} - \mathbf{Q})^2 + \tilde{\Pi}(\mathbf{q}, \omega)]$, where $\tilde{\Pi}(\mathbf{q}, \omega)$ is the full polarization bubble that has to be evaluated within the low-energy theory.

On the antiferromagnetic side of the transition ξ_0 becomes imaginary ($\xi_0^{-2} < 0$), so that the susceptibility diverges near the AFM wave vector, \mathbf{Q} , leading to a staggered magnetic moment, $\langle S_{\mathbf{Q}}^z \rangle$. The divergence is compensated by $\tilde{\Pi}(\mathbf{q}, 0)$ which acquires a finite value in the ordered phase. Furthermore, for a finite $\langle S_{\mathbf{Q}}^z \rangle$, the longitudinal (zz) and the transverse polarization bubbles differ, leading to the anisotropic susceptibility. The staggered moment, $\langle S_{\mathbf{Q}}^z \rangle$, is related to $\xi_0^{-2} < 0$, via the Goldstone requirement that the fully renormalized $\chi_{\perp}^{-1}(q \rightarrow 0, 0) = 0$.

Following the standard procedure, we introduce $m = g \langle S_{\mathbf{Q}}^z \rangle$, include the ‘‘condensed’’ part of the spin fluctuations, $\sum_{\mathbf{k}, \alpha} \alpha m c_{\mathbf{k}+\mathbf{Q}, \alpha}^\dagger c_{\mathbf{k}, \alpha}$, into the fermion part of the

Hamiltonian, and diagonalize it by Bogoliubov transformation to find $H_f = \sum_{n,\mathbf{k},\alpha} E_n(\mathbf{k}) a_{n,\mathbf{k},\alpha}^\dagger a_{n,\mathbf{k},\alpha}$. The dispersion of the fermions in the two new bands is given by $E_{\pm 1}(\mathbf{k}) = \epsilon_{\mathbf{k}} \pm \sqrt{\delta_{\mathbf{k}}^2 + m^2}$ with $\epsilon_{\mathbf{k}} = [\epsilon(\mathbf{k}) + \epsilon(\mathbf{k} + \mathbf{Q})]/2$ and $\delta_{\mathbf{k}} = [\epsilon(\mathbf{k}) - \epsilon(\mathbf{k} + \mathbf{Q})]/2$. The new and the original fermion operators are related by a unitary transformation $a_{+1,\mathbf{k},\alpha}^\dagger = u_{\mathbf{k}} c_{\mathbf{k},\alpha}^\dagger - \alpha v_{\mathbf{k}} c_{\mathbf{k}+\mathbf{Q},\alpha}^\dagger$, and $a_{-1,\mathbf{k},\alpha}^\dagger = \alpha v_{\mathbf{k}} c_{\mathbf{k},\alpha}^\dagger + u_{\mathbf{k}} c_{\mathbf{k}+\mathbf{Q},\alpha}^\dagger$, with $u_{\mathbf{k}}^2(v_{\mathbf{k}}^2) = [1 + (-)\delta_{\mathbf{k}}/\sqrt{\delta_{\mathbf{k}}^2 + m^2}]/2$. The interaction of fermions with the uncondensed part of the spin excitations is

$$H_{\text{sf}} = g \sum \Gamma_{n_1, n_2}^{\alpha, \beta}(\mathbf{k}, \mathbf{k} + \mathbf{q}) a_{n_1, \mathbf{k}, \alpha}^\dagger a_{n_2, \mathbf{k} + \mathbf{q}, \beta} \boldsymbol{\sigma}_{\alpha\beta} \mathbf{S}_{\mathbf{q}-\mathbf{Q}} + \text{H.c.} \quad (3)$$

$$\Gamma_{n_1, n_2}^{\alpha, \beta}(\mathbf{k}, \mathbf{k} + \mathbf{q}) = [u_{\mathbf{k}} u_{\mathbf{k}+\mathbf{q}} - \alpha \beta v_{\mathbf{k}} v_{\mathbf{k}+\mathbf{q}}][1 - \delta_{n_1, n_2}] - n_1 \delta_{n_1, n_2} [\alpha v_{\mathbf{k}} u_{\mathbf{k}+\mathbf{q}} + \beta u_{\mathbf{k}} v_{\mathbf{k}+\mathbf{q}}]. \quad (4)$$

The vertex for the Goldstone transverse mode, $\Gamma_{n,n}^{\alpha,\alpha}(\mathbf{k}, \mathbf{k} + \mathbf{q}) \propto |\mathbf{q}|$ at small \mathbf{q} , as required by Adler's principle. The spin polarization bubble $\Pi(q, \Omega)$ is

$$\Pi_{ij}(\mathbf{q}, \Omega) = \chi_0 g^2 T \sum_{\substack{n_1, n_2 \\ \mathbf{k}, \omega}} \sigma_{\alpha\beta}^{(i)} \sigma_{\beta\alpha}^{(j)} \left| \Gamma_{n_1, n_2}^{\alpha, \beta} \left(\mathbf{k} + \frac{\mathbf{q}}{2}, \mathbf{k} - \frac{\mathbf{q}}{2} \right) \right|^2 \times G_{n_1} \left(\mathbf{k} - \frac{\mathbf{q}}{2}, \omega_n - \frac{\Omega}{2} \right) G_{n_2} \left(\mathbf{k} + \frac{\mathbf{q}}{2}, \omega_n + \frac{\Omega}{2} \right), \quad (5)$$

where $G_i(\mathbf{k}, \omega_n)$ is the Green's function of the fermion in band i at the Matsubara frequency, $\omega_n = \pi T(2n + 1)$. The tensor Π_{ij} is diagonal, and its zz (xx and yy) component is the longitudinal (transverse) polarization.

The change in the static polarization upon entering the AFM state, $\Pi(q \rightarrow 0, 0) - \Pi_{m=0}(q \rightarrow 0, 0)$, extends over all energies up to the fermionic bandwidth. This is expected since this difference has to compensate negative ξ_0^{-2} that also comes from high energy fermion physics. The relationship between m and ξ_0^{-2} then depends on the regularization chosen for the high energies. We will simply measure the distance from the QCP on the AFM side in terms of m , and use the Goldstone condition $\xi_0^{-2} + \Pi_{\perp}(q \rightarrow 0, 0) = 0$. In contrast, $\tilde{\Pi}_{ij}(q, \Omega)$ are independent of the cutoff procedure and are fully accounted for in the low-energy theory. It is essential that $\Pi_{\parallel}(q \rightarrow 0, 0) - \Pi_{\perp}(q \rightarrow 0, 0) = O(1/W)$ and, hence, to this order, $\xi_0^{-2} + \Pi_{\parallel}(q, \Omega) = \tilde{\Pi}_{\parallel}(q, \Omega)$.

Evaluation of $\tilde{\Pi}_{\parallel}$ and $\tilde{\Pi}_{\perp}$ is straightforward. We consider a two-dimensional Fermi surface with tetragonal symmetry, so that there are four pairs of hot spots which contribute additively to $\Pi(q, \Omega)$. Without loss of generality, we choose the Fermi velocities at the hot spots \mathbf{k}_F and $\mathbf{k}_F + \mathbf{Q}$ along the x and the y directions, respectively. It is convenient to introduce $k_{\pm} = (k_x \pm k_y)/2$, so that $\epsilon_{\mathbf{k}} = v_F k_+$ and $\delta_{\mathbf{k}} = v_F k_-$. These directions are inequivalent

as the Fermi surface exists for arbitrary \mathbf{k}_- , but only for $|\mathbf{k}_+| > m/v_F$ (see Fig. 1). Evaluating the integrals in Eq. (5), we find for a given pair of hot spots ($\Omega > 0$)

$$\tilde{\Pi}_{\perp}(\mathbf{q}, \Omega) = \frac{\gamma \Omega \text{sgn}(v_F q_+ - \Omega)}{8} \times \left[\frac{4m^2}{(v_F q_+ - \Omega)^2 - v_F^2 q_-^2} - 1 \right]^{-1/2}, \quad (6)$$

$$\tilde{\Pi}_{\parallel}(\mathbf{q}, \Omega) = \frac{\gamma \Omega \text{sgn}(\Omega - v_F q_+)}{8} \times \left[\frac{4m^2}{(v_F q_+ - \Omega)^2 - v_F^2 q_-^2} - 1 \right]^{1/2}, \quad (7)$$

where $\gamma = g^2 \chi_0 / (2\pi v_F^2)$, and $\Omega = \Omega + i\delta_{\Omega}$. The full $\Pi(q, \Omega)$ are the sums over the pairs of hot spots, i.e., the sum of contributions from the wave vectors $(\pm q_+, \pm q_-)$ and $(\pm q_-, \pm q_+)$.

The behavior of the susceptibilities is particularly simple if we set $q_+ = 0$ in Eqs. (6) and (7). When $\Omega, v_F q_- \gg m$, $\tilde{\Pi}_{\perp}(q, \Omega) = \tilde{\Pi}_{\parallel}(q, \Omega) \sim i\gamma|\Omega|$, as at the QCP with $z = 2$. For $m \gg \Omega \gg v_F q_-$, $\tilde{\Pi}_{\perp} \propto \Omega^2/m$; i.e., $\chi_{\perp}^{-1} \propto q_-^2 - (\gamma/m)\Omega^2$. In the same regime $\tilde{\Pi}_{\parallel} \approx 2\gamma m$, and therefore longitudinal excitations have a gap. In the opposite limit $m \gg v_F q \gg \Omega$, we find $\tilde{\Pi}_{\perp} \propto i|\Omega|(v_F q)/m$ and $\tilde{\Pi}_{\parallel} \propto im\Omega/v_F q$. Consequently, the two spin susceptibilities have the forms given in Eq. (1); i.e., spin excitations are overdamped and *gapless*, with $z = 1$ ($z = 3$) for the transverse (longitudinal) channel.

Since an undamped pole in χ_{\perp} occurs only when $\Omega \gg v_F q$, the propagating spin waves exist only above a threshold value of staggered magnetization, $m_c \sim \gamma v_F^2 \sim g$ [i.e., $\langle S_Q^z \rangle = O(1)$]. For $m < m_c$, the spin wave would be at $\Omega < v_F q$, but it is damped there. At $m = m_c$, the spin-wave pole first splits from the upper edge of the low-frequency continuum and a propagating spin-wave emerges, initially with the residue $Z \propto m - m_{\text{min}}$. This behavior is illustrated in Fig. 2. For χ_{\parallel} , there is no distinction between different values of m —in all cases, for $v_F q \ll m$, there are two separate regions where dissipation is present: One is at $\Omega < v_F q_-$, and the other at $\Omega > 2m - v_F q_-$, as shown in Fig. 3.

For a general \mathbf{q} , when both q_+ and q_- are comparable, the details of the momentum and frequency dependence of the susceptibility are somewhat lengthy, but follow straightforwardly from Eqs. (6) and (7). The Landau damping is now effective for $\Omega < v_F(|q_+| + |q_-|) \ll 2m$. At small frequencies $\chi_{\parallel}^{-1} \propto q_-^2 - i\gamma m \Omega / (v_F \sqrt{|q_+^2 - q_-^2|})$. The threshold for the spin-wave creation along the direction $q_+ = \alpha q_-$ is $m_c = \gamma v_F^2 (\alpha + 1)^{3/2} (\sqrt{\alpha} + 1) / (1 + \alpha^2)$, and the spin-wave pole first appears along the diagonals of the Brillouin zone, at $m \geq \gamma v_F^2$.

The regime $\Omega \ll v_F |q_+ - q_-| \ll m$ is particularly important for the electron self-energy, $\Sigma(\mathbf{k}, \omega)$. To the lowest order in the interaction,

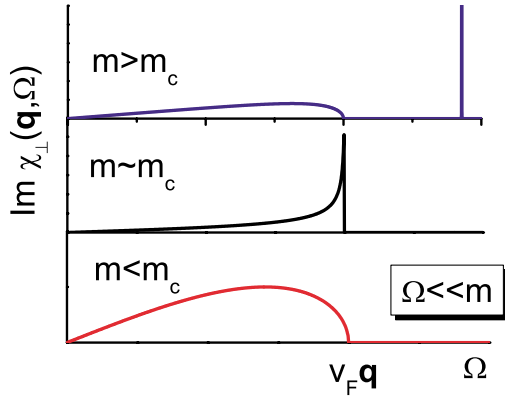


FIG. 2 (color online). Transverse spin susceptibility at $\Omega, v_F q \ll m$. The propagating spin wave is manifested as a delta-function peak at $\Omega(q)$ in the top panel.

$$\Sigma_{n,\alpha}(\mathbf{k}, \omega) = T \sum_{\substack{q, \Omega \\ \beta n_1}} \sigma_{\alpha\beta}^{(i)} \sigma_{\beta\alpha}^{(j)} \Gamma_{nn_1}^{\alpha\beta}(\mathbf{k}, \mathbf{q}) \Gamma_{n_1 n}^{\beta\alpha}(\mathbf{q}, \mathbf{k}) \times \chi_{ij}(\mathbf{k}-\mathbf{q}, \omega - \Omega) G_{n_1}(\mathbf{q}, \Omega). \quad (8)$$

We find that there are two nontrivial contributions to the self-energy. The first comes from the large momentum transfers $v_F q \geq 2m$ and gives $\Sigma_1^{(1)} \approx (ig^2\sqrt{2}/v_F)(\sqrt{4m^2 - i\gamma a\omega - 2m}) \propto \omega/m$ at small energies. At QCP this term gave rise to the $\sqrt{i\omega}$ dependence of the self-energy at hot spots [3]. Finite staggered magnetization, m , renders it harmless for the fermions.

The second contribution appears for $m \neq 0$ from the interaction between the electrons and the Landau damped longitudinal spin excitations. This contribution is dominated by $v_F q \propto \Omega^{1/3}$ for which the criterium $v_F q > \Omega$ is satisfied. In contrast to the transverse Goldstone mode, the interaction vertex for χ_{\parallel} is not reduced

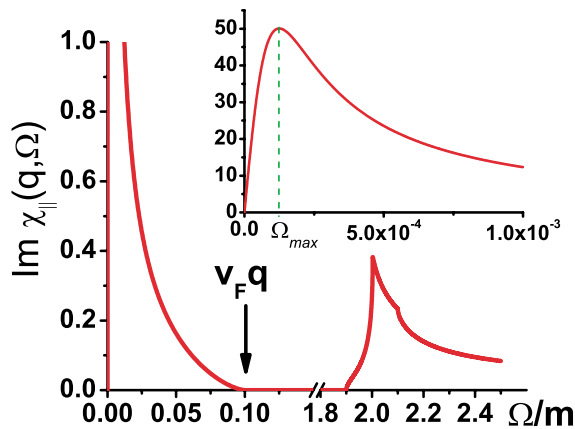


FIG. 3 (color online). Dissipative part of the longitudinal susceptibility. Below energies $\sim 2m$, the onset of damping occurs at $\Omega = v_F q$. Notice the break in the horizontal scale. Inset: Low energy behavior of $\text{Im } \chi_{\parallel}(\mathbf{q}, \Omega)$. The maximum is at $\Omega_{\text{max}} \propto q^3$. Notice that in the Landau-damped region $\text{Im } \chi_{\parallel}(\mathbf{q}, \Omega)$ is much larger than that in the continuum.

by Adler's symmetry principle. As a result, the corresponding contribution to the self-energy is $\Sigma_1^2 \approx (\pi/4\sqrt{3})(g^2/v_F)(i\omega)^{2/3}/(m\gamma)^{1/3}$, as in $z = 3$ critical theories [5]. The fractional ω dependence with exponent $2/3$ implies that both $\text{Re } \Sigma_1$ and $\text{Im } \Sigma_1$ are of the same order, giving rise to a non-Fermi-liquid behavior. Furthermore, as the Landau damping is due to small momentum transfer, the NFL form of the self-energy persists everywhere on the Fermi surface, not only at the former hot spots. This is not surprising since, in the presence of the long-range AFM order, the entire Fermi surface becomes "hot" as the points \mathbf{k}_F and $\mathbf{k}_F + \mathbf{Q}$ become equivalent. Finally, in complete analogy to $z = 3$ gauge and ferromagnetic critical theories [5], vertex corrections and the momentum-dependent part of the self-energy are small for $m \leq g \ll W$. Hence, the result $\Sigma \propto (-i\omega)^{2/3}$ remains valid to all orders in the interaction.

We believe that this scenario of Fermi-liquid breakdown on the ordered side of QCP is quite general. The requirement that the gap in the optical mode of the order parameter dissolves into the Landau damping due to small momentum scattering in the enlarged unit cell is satisfied not only for AFM, but also for charge-density wave and other transitions with a finite \mathbf{Q} .

The most obvious experimentally observable consequence of this scenario is the existence of the two regions of Ω where $\text{Im } \chi_{\parallel}(q, \Omega) \neq 0$ (see Fig. 3). This can be probed by inelastic neutron scattering experiments. The fractional exponent in the self-energy also leads to the non-Fermi-liquid, quantum-critical behavior for the temperature dependence of the electronic specific heat and resistivity [7], which is accessible experimentally.

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- [1] For a review, see P. Coleman *et al.*, *J. Phys. Condens. Matter* **13**, R723 (2001).
- [2] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, England, 1999).
- [3] Ar. Abanov, A.V. Chubukov, and J. Schmalian, *Adv. Phys.* **52**, 119 (2003).
- [4] S. L. Adler, *Phys. Rev.* **137**, B1022 (1965).
- [5] J. Polchinski, *Nucl. Phys.* **B422**, 617 (1994); B. L. Altshuler, L. Ioffe, and A. J. Millis, *Phys. Rev. B* **52**, 5563 (1995); R. Haslinger *et al.*, *Phys. Rev. Lett.* **90**, 077002 (2003); A. Chubukov, C. Pepin, and J. Rech, *cond-mat/0311420*.
- [6] V. Oganesyan, S. A. Kivelson, and E. Fradkin, *Phys. Rev. B* **64**, 195109 (2001).
- [7] A. J. Millis, *Phys. Rev. B* **48**, 7183 (1993).