Positronium in Intense Laser Fields

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The dynamics and radiation of positronium are investigated in intense laser fields. Our two-body quantum mechanical treatment displays the tunneling, free-evolution, and recollision dynamics of electron and positron in both the oscillating laser electric and laser magnetic field components. In spite of significant momentum transfer of the numerous incoming laser photons, recollisions of both particles are shown to occur automatically after tunneling ionization, along with substantial x-ray and gamma-ray emission during recombination and annihilation processes.

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The highly nonlinear interaction of gaseous atoms with superintense laser pulses has been demonstrated to give rise to the emission of coherent high-harmonic radiation up to the x-ray regime, with numerous applications in high-resolution spectroscopy and diagnostics (see, e.g., [1]). Since the kinetic energy and, consequently, the frequency of the emitted radiation of a particle increase with rising laser intensity, considerable interest has been directed towards understanding the complex relativistic quantum dynamics of atoms and ions in ultraintense laser pulses [2]. Once electrons reach velocities non-negligible to that of light, however, the magnetically induced Lorentz force, or, in other words, the momentum transfer, of the numerous incoming photons induces a separation of electrons and ionic core in the laser propagation direction, resulting in a strong reduction of high-harmonic yields [2–4]. While highly charged ions and crystals were studied towards a reduction of this effect [5,6], there is still a clear lack for an efficient system, where radiation pressure does not induce substantial ionization in the laser propagation direction and thus reduces coherent high-frequency generation. On a different front, there is also a quest for physics beyond atomic and classical plasma physics in ultraintense lasers, such as nuclear reactions [7] and QED effects [8].

In this Letter, the dynamics and high-harmonic generation of positronium is investigated in intense laser fields. The two-body system is shown to display unique properties: While tunnel ionization of electron and positron may occur almost oppositely in the laser polarization direction, both particles sense the identical drift in the laser propagation direction due to their equal magnitudes of mass and charge (see Fig. 1). Periodic electron-positron recollisions are shown to occur automatically in spite of the influence of the Lorentz force. In addition to substantial coherent x-ray generation during electron-positron recombinations, we predict gamma radiation in the less likely event of laser-enhanced annihilations of both particles.

Positronium consists of an electron and a positron and is known to be unstable. While ortho-positronium annihilates into three photons with a lifetime of 1.4×10^{-7} s, para-positronium does so with two photons and a lifetime of 1.25×10^{-10} s [9]. The presence of a strong laser field may induce substantial reductions [10] or enhancements [11,12] of the annihilation process into gamma photons. However, even for the shorter lifetime of para-positronium it is sufficient for the interaction with many cycles of a femtosecond laser pulse.

The laser field will be described by the vector potential $\mathbf{A} = (A, 0, 0)$ with laser polarization direction $\mathbf{e}_x$ and propagation direction $\mathbf{e}_z$. Being interested in the tunneling regime with moderately intense laser field strengths,

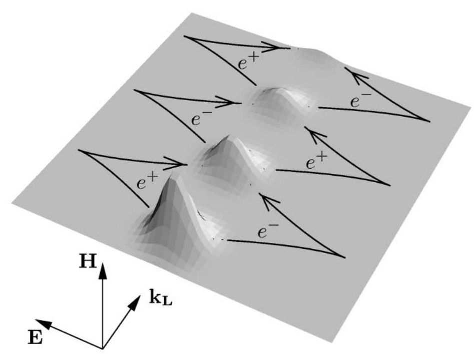

FIG. 1. Schematic diagram displaying positronium dynamics in an intense laser field. The bound system depicted by the density of its wave function may ionize in the laser field. Once free, both electron e^- and positron e^+ could be described as classical particles. Their trajectories are shown by the solid lines. The electric field $\mathbf{E}$ accelerates both particles in opposite directions, while the Lorentz force due to the magnetic field $\mathbf{H}$ leads to an identical drift in the propagation direction $\mathbf{k}_\mathbf{L}$. Without an initial center-of-mass motion, these trajectories are symmetric and thus both particles overlap periodically, giving rise to possible recombinations and thus high-frequency light emission.

we restrict ourselves to Schrödinger dynamics in two dimensions, although beyond the dipole approximation. The quantum dynamics of positronium in the laser field, taking fully into account the Lorentz force due to the laser magnetic field component, is thus governed by the following Hamiltonian (in atomic units as throughout this Letter):

$$H = \frac{[-i\nabla_{\mathbf{x}_-} + \frac{\mathbf{A}(t-z_-/c)}{c}]^2}{2} + \frac{[-i\nabla_{\mathbf{x}_+} - \frac{\mathbf{A}(t-z_+/c)}{c}]^2}{2} - \frac{1}{|\mathbf{x}_- - \mathbf{x}_+|}, \qquad (1)$$

where $\mathbf{x}_i$ and $-i\nabla_{\mathbf{x}_i}$ ($i \in \{-, +\}$) are the operators of the coordinates and momenta of the electron and positron, respectively. Further, we introduce relative $\mathbf{r} = (x, y, z) = \mathbf{x}_- - \mathbf{x}_+$ and center of mass $\mathbf{R} = (X, Y, Z) = (\mathbf{x}_- + \mathbf{x}_+)/2$ coordinates. In accordance with the Ehrenfest theorem, the center of mass transversal canonical momenta $-i\partial_X$, $-i\partial_Y$ as well as $i(\partial_t + c\partial_Z)$ turn out to be conserved quantities, i.e., commute with H. The eigenvalues are defined to be P_x, P_y, g, respectively, and $\mathcal{E}$ is a separation constant that can be understood as the energy of the system before the interaction. Therefore, we consider the ansatz

$$\Phi(\mathbf{r}, \mathbf{R}, \tau) = \exp\left[i\left(P_x X + P_y Y - \frac{g}{c} Z - \mathcal{E}\tau\right)\right]\phi(\mathbf{r}, \tau), \qquad (2)$$

singling out eigenfunctions of the conserved quantities in the wave function and introducing the running time $\tau = t - (Z/c)$. Employing a $1/c$ expansion of the vector potential as a function of the coordinates of the relative motion of the particles, we obtain the following equation for ϕ:

$$i(1 - B_z)\partial_\tau \phi(\mathbf{r}, \tau) = \left\{\left[\frac{1}{i}\nabla - \frac{\mathbf{A}(\tau)}{c}\right]^2 + \frac{\mathbf{P}^2}{4} + \frac{P_x \dot{A}(\tau) z}{2c^2} - \frac{1}{r} - \mathcal{E} - \frac{1}{4c^2}\partial_\tau^2\right\}\phi(\mathbf{r}, \tau). \qquad (3)$$

Here the center of mass velocity is introduced via $\mathbf{V} = \mathbf{P}/2$ with $\mathbf{B} = \mathbf{V}/c$ and $\mathbf{P} = (P_x, P_y, (\mathcal{E} - g)/c)$. The applied expansion takes into account the leading multipole operators describing the magnetically induced relative motion of electron and positron in the laser propagation direction. Since P_x is a conserved quantity, this operator describes an oscillation of the relative coordinate in the propagation direction, while for atoms the analogous term would lead to a drift. For consistency, the higher-order term for the center-of-mass motion $-(1/4c^2)\partial_\tau^2$ in Eq. (3) will be neglected in the following.

We proceed by carrying out a transformation to the length gauge, $\exp[-i\chi(\mathbf{r}, \tau)]\phi = \Psi$ with $\chi(\mathbf{r}, \tau) = \mathbf{A}(\tau) \cdot \mathbf{r}/c$. Further, we employ the reasonable assumptions proposed earlier by Lewenstein and co-workers [13] for atomic high-harmonic generation (HHG) in the dipole approximation: (a) The contribution of all bound states except the ground state is neglected; (b) the depletion of the ground state is neglected; (c) in the continuum the electron is treated as a free particle in the laser field. Then the time dependent wave function can be written as

$$|\Psi(\tau)\rangle = |0\rangle + \int_{-\infty}^{\infty} d^3\tilde{p}\, b(\tilde{\mathbf{p}}, \tau)|\tilde{\mathbf{p}}\rangle. \qquad (4)$$

Here $|0\rangle$ is the ground state and $b(\tilde{\mathbf{p}}, \tau)$ denotes the amplitude of the corresponding continuum state $|\tilde{\mathbf{p}}\rangle$. Integration of Eq. (3) with the ansatz (4) yields for the ionization amplitude b:

$$b(\tilde{\mathbf{p}}, \tau) = \frac{-i}{1 - B_z}\int_{-\infty}^{\tau} \langle \mathbf{p} - \mathbf{A}(\tau')/c - \mathbf{\Lambda}(\tau')|H_I(\tau')|0\rangle \times \exp[-iS(\mathbf{p}, \tau, \tau')]d\tau', \qquad (5)$$

where the quasiclassical action is $S(\mathbf{p}, \tau, \tau') = \int_{\tau'}^{\tau} d\tau''\{[\mathbf{p} - \mathbf{A}(\tau'')/c - \mathbf{\Lambda}(\tau'')]^2 + I_p\}/(1 - B_z)$. Here the new variable $\mathbf{p} = \tilde{\mathbf{p}} + \mathbf{A}(\tau)/c + \mathbf{\Lambda}(\tau)$ is introduced with $\mathbf{\Lambda}(\tau) = \Lambda(\tau)\mathbf{e}_z = B_x A(\tau)/[c(1 - B_z)]\mathbf{e}_z$ and $H_I(\tau) = -E(\tau)x(1 - B_z) - B_x E(\tau)z$, $\mathbf{E}(\tau) = -\frac{1}{c}\partial_\tau \mathbf{A}$ is the electric field, and I_p is the positronium ionization potential.

We calculate the dipole moment $\mathbf{x}_d(\tau) = \langle\Psi(\tau)|\mathbf{r}|\Psi(\tau)\rangle$ of which the Fourier transform $\mathbf{x}_d([\omega - \mathbf{k}\cdot\mathbf{V}]/[1 - B_z])$ yields the Doppler shifted harmonic spectrum, where ω and $\mathbf{k}$ denote the emitted radiation frequency and wave vector, respectively. In the tunneling regime the integrals can be obtained using the saddle point method, $\nabla_\mathbf{p} S(\mathbf{p}, \tau, \tau') = 0$. Sustaining all momenta up to first order in $1/c$ yields: $p_x(\tau, \tau') = \int_{\tau'}^{\tau} d\tau'' A(\tau'')/c(\tau - \tau')$, $p_y = 0$, $p_z(\tau, \tau') = B_x p_x(\tau, \tau')$. The matrix elements can be evaluated analytically employing the relation [14] $\mathbf{d}(\mathbf{p}) = (d_x, d_y, d_z) = \langle\mathbf{p}|\mathbf{r}|0\rangle = 4i(I_p)^{5/4}\mathbf{p}/[\pi(\mathbf{p}^2 + I_p)^3]$ (to be approximated for the numerical calculations later as in Ref. [6]). Taking into account also the pole in the τ' integration finally results in

$$x_d(\tau) = \frac{1}{\sqrt{i}}\sum_{\tau_0}\left(\frac{\pi}{\tau - \tau_0}\right)^{3/2}\frac{I_p^{3/4}}{2|E(\tau_0)|} \times d_x^*\left[p_x(\tau, \tau_0) - \frac{A(\tau)}{c}\right] \times \exp\left\{-\frac{2}{3}\frac{I_p^{3/2}}{|E(\tau_0)|}(1 + B_z)\right\} \times \exp\{-iS[\mathbf{p}(\tau, \tau_0), \tau, \tau_0]\} + \text{c.c.} \qquad (6)$$

The birth times τ_0 are determined, for given τ, by the condition $p_x(\tau, \tau_0) = A(\tau_0)/c$. The leading terms in $1/c$ have been sustained only in the phases because they play a far less substantial role in the prefactor being identical to the dipole case. For the maximal harmonic frequency $\omega_{\max}$ we obtain from Eq. (6) $\omega_{\max}(1 - \mathbf{k}\cdot\mathbf{B}/|\mathbf{k}|) = n_{\max}\omega_L(1 - B_z) = 3.17U_p + I_p$ for maximal harmonic number $n_{\max}$, $U_p = c^2\xi^2/4$, $\xi = A_0/c^2$, and amplitude A_0 of the laser vector potential.

The dipole moment in Eq. (6) displays essential differences in comparison with atomic HHG, which shows especially in the real exponential factor in Eq. (6): $C := \exp[-2I_p^{3/2}(1 + B_z)/(3|E(\tau_0)|)]$. We stress that in the atomic HHG case [4] there is an additional contribution via exchanging I_p in C by $I_p^a + c^2\xi^4/4$ due to the drift in the laser propagation direction which can strongly reduce the amplitude and thus the efficiency of HHG. Here I_p^a denotes the atomic ionization potential. The expression C, without the factor $1 + B_z$, coincides with the corresponding result within the dipole approximation for atoms. This means that for positronium, the magnetically induced drift in the laser propagation direction has no diminishing impact on HHG.

Figure 2 displays the hard x-ray harmonic spectrum of positronium via the Fourier transform of its dipole moment via a fivefold ($d\mathbf{p}, d\tau, d\tau'$) saddle point integration [6] versus Ne^{7+} ions via Eq. (5) in Ref. [4]. Ne^{7+} with $I_p^a = 240.31$ eV in the chosen laser field appears to be the presently most favorable choice for obtaining 50 keV x rays at approximately the same cutoff as for positronium [5,6]. For Ne^{7+} the influence of the magnetic field component yields a very small recollision probability so that the harmonic yield is ca. 15 orders of magnitude smaller than for positronium. Figure 2(b) shows hardly any structure due to the lack of interference between quantum trajectories [6]. We add further that neutral titanium with a similar ionization potential as for positronium interacting with identical low-frequency fields as in Fig. 2(a) would give rise to a harmonic spectrum with approximately half the cutoff frequency and a yield even lower than that for the ion spectrum in Fig. 2(b).

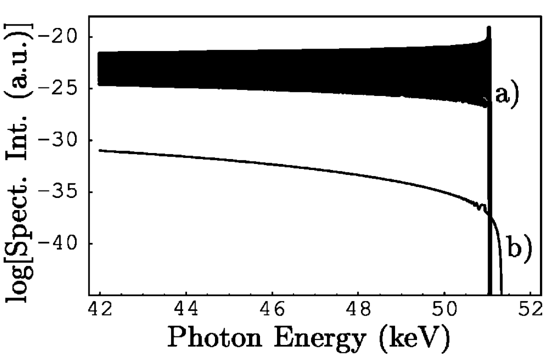

FIG. 2. Differential high-harmonic spectral intensities $(dI_n/d\Omega)\sin^{-2}\theta$ at odd harmonic numbers n as a function of the photon energy of the respective harmonic with θ being the harmonic emission angle with respect to the polarization direction. (a) The spectrum of a single positronium with $\sim 4.1 \times 10^6$ harmonics via a coherent field of intensity $I = 8.63 \times 10^{12}$ W/cm^2 at a wavelength of $\lambda = 100$ μm close to that provided by the Centre Laser Infra-rouge Orsay free-electron laser [15], $\xi = 0.25$ and $\mathbf{B} = 0$. (b) The spectrum of a Ne^{7+} ion in a yttrium aluminum garnet laser field with $\lambda = 1054$ nm at $I = 1.5 \times 10^{17}$ W/cm^2 ($\xi = 0.35$). At those parameters both systems yield approximately the same cutoff frequency, although with a yield in the spectral intensity of the cutoff harmonics differing by about 15 orders of magnitude.

Assuming the high ion density of 10^{19} cm^{-3}, positronium densities 3×10^7 cm^{-3} such as demonstrated in a Penning trap [16], and a focal volume of $V_f \simeq (10\lambda)^3$ with wavelength λ, we lose only 6 out of the 15 orders of magnitude of the positronium versus ion yield in Fig. 2. The spreading of both electron and positron wave packets induces the less significant loss of a factor of $\sim \frac{1}{2}$ compared to the case with a resting nucleus. Thus, hard x-ray harmonic yields from positronium should be substantially larger than from gaseous ions in comparable setups.

Finally, we need to discuss the process of positronium annihilation in strong laser fields [Figs. 3(a) and 3(b)] because it presents (i) a limitation for the available time for HHG via our mechanism and (ii) an additional source of high-frequency radiation. Thus, in what follows we estimate both the lifetime of positronium in strong laser fields and the generation of γ radiation, which turns out to be of narrow bandwidth and to clearly prevail over the spontaneous background.

Without external laser field, ortho-positronium, e.g., annihilates spontaneously into three photons, with energies varying between zero and $\omega_c = c^2$ [Fig. 3(c)] [9]. Some essential channels involve two γ and one low-frequency photon. Those channels may be enhanced significantly by stimulating either the γ or the low-frequency emission with the corresponding intense coherent fields. Stimulated annihilation via coherent fields with high frequencies of order $\omega_c = c^2$ has been predicted already [17], while low-frequency photons have been investigated only for stimulating channels with single γ photons in extremely intense laser fields [Fig. 3(a)] [12]. In the feasible moderately intense low-frequency light of interest here [Fig. 3(b)], the laser radiation stimulates multiple emission of low-frequency photons and this way the corresponding annihilation channel with two γ photons.

We proceed with an estimation of the positronium lifetime via calculating the Feynman diagram in Fig. 3(b) (plus the same diagram with photon exchange). The intense laser field is included to all orders because

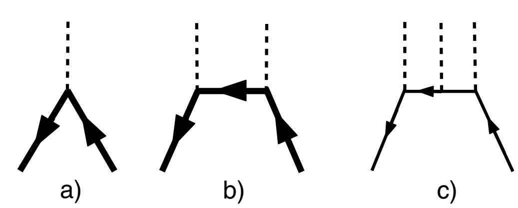

FIG. 3. Feynman diagrams displaying γ-photon emission via electron-positron annihilation by (a) multiphoton annihilation in superstrong laser fields at $\xi \gtrsim 100$ with emission of $n \gtrsim 10^6$ laser photons along with one γ quantum; (b) annihilation in a laser field at $\xi \ll 100$ with emission of laser photons along with two γ quanta; (c) annihilation of ortho-positronium with emission of three photons without laser field. Bold lines correspond to the electron (positron) Volkov states, i.e., involving a laser field; dashed lines correspond to the emitted photons.

electrons and positrons are dressed as Volkov states. The influence of the laser field on the propagator is assumed negligible as the laser frequency ω_L and the number s of involved photons is such that $s\omega_L \ll \omega_c$. The differential probability of laser-induced annihilation with stimulated emission of s low-frequency photons and two γ quanta with energies $\omega_{1,2} \approx \omega_c$ and the first γ quantum within the solid angle $d\Omega$ then reads: $dW_s^{\mathrm{ind}} \approx (r_0^2/2)[\omega_1^2/(\omega_2\omega_c)]\rho c F_s d\Omega$. Here $F_s = |1 - \xi^2/2 - s\omega_L/\omega_c|J_{s/2}^2(\zeta)$ for even s (para-positronium) and $F_s = (\xi^2/2)[J_{(s-1)/2}^2(\zeta) + J_{(s+1)/2}^2(\zeta)]$ for odd s (ortho-positronium), $\zeta := (\xi^2/4)(\omega_c/\omega_L)$, Bessel functions $J_n(\zeta)$, classical electron radius r_0, and $\rho = a_B^{-3}$ with Bohr radius a_B. In a weak laser field with $\xi \ll 1$ the above mentioned formula for dW_s^{ind} matches the known result in Ref. [11]. The total probability of induced annihilation for ortho-positronium becomes of the order $W_o^{\mathrm{ind}} = \sum_s W_s^{\mathrm{ind}} \approx r_0^2\rho c\xi^2$ and for para-positronium $W_p^{\mathrm{ind}} \approx r_0^2\rho c$. In comparison with the corresponding spontaneous annihilation probabilities $W_o^{\mathrm{sp}} \approx \alpha r_0^2\rho c$ and $W_p^{\mathrm{sp}} \approx \pi r_0^2\rho c$ with $\alpha = 1/c$ [9], we obtain a decrease of the ortho-positronium lifetime by the factor ξ^2/α. In laser fields with $\xi \lesssim 1$ of interest here, we then estimate a ortho-positronium lifetime of 10^{-9} s (10^{-10} s for para-positronium), being well sufficient for HHG.

Furthermore, laser-induced annihilation via Fig. 3(b) will be a source of high-frequency radiation. Since stimulated low-frequency and γ radiation mutually depend on each other for the annihilation channel in Fig. 3(b), we intuitively predict that intense laser fields also stimulate γ photons. The spectrum is further expected to be narrow and in excess of the corresponding spontaneous radiation, because with annihilation being accompanied by stimulated emission of laser photons, energy and momentum conservation allow an emission of γ quanta only with certain frequencies in each direction. The spectral bandwidth $\Delta\omega = \max\{\Delta\omega_D, \Delta s\omega_L\}$ of the induced γ radiation due to annihilation is determined either by the laser Doppler linewidth $\Delta\omega_D \approx 10^{-4}$ at room temperature or by the range of the harmonic order Δs of involved stimulated photons. This means a bandwidth much narrower than that of the spontaneous process: $\Delta\omega \ll \omega_c$. The ratio of stimulated versus spontaneous γ-radiation probabilities $\eta := (dW^{\mathrm{ind}}/d\Omega)/(\Delta\omega dW^{\mathrm{sp}}/d\Omega d\omega)$ is largest for $\zeta \gg 1$. Then $s \approx \zeta$ within the range $\Delta s \approx \zeta^{1/3}$, which yields for ortho-positronium $\eta \approx (\xi^2/\alpha)(\omega_c/\Delta\omega) = (\xi^{4/3}/\alpha)(\omega_c/\omega_L)^{2/3}$. This means $\eta \approx 10^5$ in the tunneling regime with $\xi = 10^{-1}$ (for para-positronium $\eta \approx 1$). Thus, laser-induced γ radiation via positronium annihilation in the tunneling regime of interest here can exceed the spontaneous background in its narrow bandwidth.

Regarding the γ-ray yield via laser-induced annihilation, a brilliance at photon energy 0.5 MeV of $B \simeq 10^8$ photons/[s mm^2 mrad^2(0.1% bandwidth)] can be estimated for an ortho-positronium density of 10^7 cm^{-3} and an interaction length of 1 cm. This appears detectable though moderate as compared to large-scale γ-ray sources in the same spectral region [18].

In conclusion, positronium was shown to provide various promising dynamical features in strong laser fields such as energetic recollisions of its constituents with substantial coherent x-ray generation and γ-ray emission.

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