Alternative String Theory in Twistor Space for $N = 4$ Super-Yang-Mills Theory

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In this Letter, an alternative string theory in twistor space is proposed for describing perturbative $N = 4$ super-Yang-Mills theory. Like the recent proposal of Witten, this string theory uses twistor worldsheet variables and has manifest spacetime superconformal invariance. However, in this proposal, tree-level super-Yang-Mills amplitudes come from open string tree amplitudes as opposed to coming from D-instanton contributions.

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In a recent paper [1], Witten has shown that the simple form of maximal helicity violating (MHV) amplitudes of Yang-Mills theory has a natural generalization to the nonmaximal helicity violating amplitudes. MHV amplitudes involve processes with two negative-helicity gluons and an arbitrary number of positive-helicity gluons, whereas non-MHV amplitudes can involve arbitrary numbers of both positive and negative-helicity gluons. Since the simple structure of MHV amplitudes has been useful for phenomenologists interested in describing gluon scattering [2], the recent generalization of these formulas to non-MHV amplitudes could have important implications.

In addition to generalizing the structure of MHV amplitudes to non-MHV amplitudes, Witten also constructed a topological B model from twistor worldsheet variables and argued that this string theory could be used to compute the MHV and non-MHV scattering amplitudes. Although his topological B model is quite elegant, it has the unusual feature that D instantons, which are normally associated with nonperturbative scattering amplitudes in string theory, are needed for computing the perturbative Yang-Mills amplitudes. For details on this model and the twistor approach to super-Yang-Mills, see the review and references in [1]. In this paper, an alternative string theory will be proposed which has the property that perturbative amplitudes in the string theory reproduce the perturbative Yang-Mills amplitudes.

The formula for D-instanton contributions of degree d to n-gluon tree-level amplitudes is [1,3]

$$B(\lambda_r, \bar{\lambda}_r) = \int d^{2d+2}a\, d^{2d+2}b\, d^{4d+4}\gamma \prod_{r=1}^{n} \int d\sigma_r [\mathrm{vol}(GL(2))^{-1}] \prod_{r=1}^{n-1} (\sigma_r - \sigma_{r+1})^{-1} (\sigma_n - \sigma_1)^{-1} \prod_{r=1}^{n} \left[\frac{\lambda_r^2}{\lambda_r^1} - \frac{\lambda^2(\sigma_r)}{\lambda^1(\sigma_r)} \right] \times \exp\left[i \bar{\lambda}_r^{\dot{\alpha}} \lambda_r^1 \frac{\mu \dot{\alpha}(\sigma_r)}{\lambda^1(\sigma_r)} \right] \mathrm{Tr}\left[\phi_1\left(\frac{\psi^A(\sigma_1)}{\lambda^1(\sigma_1)}\right) \phi_2\left(\frac{\psi^A(\sigma_2)}{\lambda^1(\sigma_2)}\right) \dots \phi_n\left(\frac{\psi^A(\sigma_n)}{\lambda^1(\sigma_n)}\right)\right] \tag{1}$$

where $P_r^{\alpha}\dot{\alpha} = \lambda_r \alpha \bar{\lambda}_r \dot{\alpha}$ is the momentum of the rth state,

$$\lambda^{\alpha}(\sigma) = \sum_{k=0}^{d} a_k^{\alpha} \sigma^k, \qquad \mu^{\dot{\alpha}}(\sigma) = \sum_{k=0}^{d} b_k^{\dot{\alpha}} \sigma^k,$$

$$\psi^A(\sigma) = \sum_{k=0}^{d} \gamma_k^A \sigma^k,$$

$\phi_r(\psi^A)$ is the $N = 4$ superfield whose lowest component is the positive-helicity gluon and whose top component is the negative-helicity gluon, and the $[\mathrm{vol}(GL(2))^{-1}]$ factor can be used to remove one of the a integrals and three of the σ integrals.

For maximal helicity violating amplitudes (i.e., $n-2$ positive-helicity gluons and 2 negative-helicity gluons), the above formula when $d = 1$ has been shown to give the correct n-point amplitude. For nonmaximal helicity violating amplitudes, it has been suggested that this formula may also give the correct n-point amplitude where one has $n - d - 1$ positive-helicity gluons and $d + 1$ negative-helicity gluons. Although there is a possibility that the formula of (1) needs to be modified for nonmaximal amplitudes by contributions from instantons of lower degree, it has been recently verified that no such modifications are necessary when $d = 2$ and $n = 5$ [3]. It will be assumed below that the formula of (1) correctly reproduces the super-Yang-Mills tree amplitudes for any d and n.

In this Letter, a new string theory in twistor space is proposed which reproduces the formula of (1) using ordinary open string tree amplitudes as opposed to D-instanton contributions. This string theory shares many aspects in common with the orginal idea of Nair in [4]. The worldsheet matter variables in this string theory consist of a left and right-moving set of super-twistor variables,

$$Z_L^I = (\lambda_L^{\alpha}, \mu_L^{\dot{\alpha}}, \psi_L^A), \qquad Z_R^I = (\lambda_R^{\alpha}, \mu_R^{\dot{\alpha}}, \psi_R^A), \tag{2}$$

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for $\alpha, \dot{\alpha} = 1$ to 2 and $A = 1$ to 4, a left and right-moving set of conjugate supertwistor variables,

$$Y_{LI} = (\overline{\mu}_{L\alpha}, \overline{\lambda}_{L\dot{\alpha}}, \overline{\psi}_{LA}), \qquad Y_{RI} = (\overline{\mu}_{R\alpha}, \overline{\lambda}_{R\dot{\alpha}}, \overline{\psi}_{RA}), \quad (3)$$

and a left- and right-moving current algebra,

$$j_L^C, \qquad j_R^C, \quad (4)$$

where C is Lie-algebra valued and j_L^C and j_R^C satisfy the usual operator-product-expansions of a current algebra, i.e.,

$$j_L^C(y) j_L^D(z) \to \frac{f^{CDE} j_L^E(z)}{y - z} + \frac{k g^{CD}}{(y - z)^2}, \qquad j_R^C(\overline{y}) j_R^D(\overline{z}) \to \frac{f^{CDE} j_R^E(\overline{z})}{\overline{y} - \overline{z}} + \frac{k g^{CD}}{(\overline{y} - \overline{z})^2}. \quad (5)$$

The current algebra can be constructed from free fermions, a Wess-Zumino-Witten model, or any other model.

The worldsheet action is

$$S = \int d^2 z (Y_{LI} \nabla_R Z_L^I + Y_{RI} \nabla_L Z_R^I) + S_G, \quad (6)$$

where S_G is the worldsheet action for the current algebra and (∇_R, ∇_L) contains a worldsheet GL(1) connection for which Z_L^I and Z_R^I have charge $+1$, and Y_{LI} and Y_{RI} have charge -1.

Quantizing this worldsheet action gives rise to left and right-moving Virasoro ghosts, (b_L, c_L) and (b_R, c_R), as well as left- and right-moving GL(1) ghosts, (u_L, v_L) and (u_R, v_R). The untwisted left-moving stress tensor is

$$T_0 = Y_{LI} \partial_L Z_L^I + T_G + b_L \partial_L c_L + \partial_L (b_L c_L) + u_L \partial_L v_L, \quad (7)$$

where T_G is the left-moving stress tensor for the current algebra, and the left-moving GL(1) current is

$$J = Y_{LI} Z_L^I. \quad (8)$$

To have vanishing conformal anomaly, the current algebra must be chosen such that the central charge contribution from T_G is 28. Note that there is no GL(1) anomaly because of cancellation between bosons and fermions in J.

The open string theory is defined using the conditions

$$Z_L^I = Z_R^I, \qquad Y_{LI} = Y_{RI}, \qquad j_L^C = j_R^C, \qquad c_L = c_R, \qquad b_L = b_R, \qquad v_L = v_R, \qquad u_L = u_R \quad (9)$$

on the open string boundary. Unlike a usual open string theory where Lie-algebra indices come from Chan-Paton factors, the Lie-algebra indices in this open string theory come from a current algebra.

The physical integrated and unintegrated open string vertex operator for the super-Yang-Mills states is

$$V = \int dx\, j^C(z) \Phi_C[Z(z)], \qquad U = c(z) j^C(z) \Phi_C[Z(z)]. \quad (10)$$

There are no massive states in this string theory since there are no operators such as e^{ikx} which can carry negative conformal weight. Note that in usual string theory, massive states are described by operators of zero conformal weight where the positive conformal weight of the ∂x factors is canceled by the negative conformal weight of the e^{ikx} factor. The superfields $\Phi_C(Z)$ are similar to those defined in [1], namely, for a super-Yang-Mills state with momentum $P_r^{\alpha\dot{\alpha}} = \lambda_r^\alpha \bar{\lambda}_r^{\dot{\alpha}}$,

$$\Phi_C[Z(z_r)] = \delta\left[\frac{\lambda_r^2}{\lambda_r^1} - \frac{\lambda^2(z_r)}{\lambda^1(z_r)}\right] \exp\left[i \bar{\lambda}_r^{\dot{\alpha}} \lambda_r^1 \frac{\mu^{\dot{\alpha}}(z_r)}{\lambda^1(z_r)}\right] \times \phi_C\left[\frac{\psi^A(z_r)}{\lambda^1(z_r)}\right], \quad (11)$$

where $\phi_C(\psi^A)$ is the same $N = 4$ superfield as in (1). Note that $\Phi_C(Z)$ is GL(1) neutral and has zero conformal weight.

Tree-level open string scattering amplitudes are computed in the usual manner from the disk correlation function

$$A = \langle U_1(z_1) U_2(z_2) U_3(z_3) \int dz_4\, V_4(z_4) \ldots \int dz_n\, V_n(z_n) \rangle, \quad (12)$$

where different twistings of the stress tensor are used to compute different helicity violating amplitudes. For amplitudes involving $(n - d - 1)$ positive-helicity gluons and $d + 1$ negative-helicity gluons, the twisted stress tensor is defined as

$$T_d = T_0 + \frac{d}{2} \partial J, \quad (13)$$

where T_0 and J are defined in (7) and (8). Note that T_d has no conformal anomaly since J has no GL(1) anomaly.

So after twisting, Z^I has conformal weight $-\frac{d}{2}$ and Y_I has conformal weight $(d + 2)/2$. This means that the disk correlation function of (12) involves an integration over the $4d + 4$ bosonic and $4d + 4$ fermionic zero modes of Z^I, except for the one bosonic zero mode which can be removed using the worldsheet GL(1) gauge invariance. Performing the correlation function for the current algebra gives the contribution

$$\mathrm{Tr}[\phi_1 \ldots \phi_n] \prod_{r=1}^{n-1} (z_r - z_{r+1})^{-1} (z_n - z_1)^{-1}, \quad (14)$$

and the (b, c) correlation function gives the factor $(z_1 - z_2)(z_2 - z_3)(z_3 - z_1)$.

So one obtains the formula

$$A = \int d^{2d+2}a\, d^{2d+2}b\, d^{4d+4}\gamma \int dz_1 \ldots \int dz_n [\mathrm{vol}(GL(2))]^{-1} \prod_{r=1}^{n-1} (z_r - z_{r+1})^{-1} (z_n - z_1)^{-1} \prod_{r=1}^{n} \delta\left(\frac{\lambda_r^2}{\lambda_r^1} - \frac{\lambda^2(z_r)}{\lambda^1(z_r)}\right)$$
$$\times \exp\left[i\bar{\lambda}_r^{\dot{\alpha}} \lambda_r^1 \frac{\mu \dot{\alpha}(z_r)}{\lambda^1(z_r)} \right] \mathrm{Tr}\left[\phi_1\left(\frac{\psi^A(z_1)}{\lambda^1(z_1)}\right) \phi_2\left(\frac{\psi^A(z_2)}{\lambda^1(z_2)}\right) \ldots \phi_n\left(\frac{\psi^A(z_n)}{\lambda^1(z_n)}\right)\right], \tag{15}$$

where

$$\lambda\alpha(z) = \sum_{k=0}^{d} a_k \alpha z^k, \quad \mu^{\dot{\alpha}}(z) = \sum_{k=0}^{d} b_k^{\dot{\alpha}} z^k,$$
$$\psi^A(z) = \sum_{k=0}^{d} \gamma_k^A z^k,$$

$(a_k^\alpha, b_k^{\dot{\alpha}}, \gamma_k^A)$ are the zero modes of Z^I on a disk, and the SL(2) part of GL(2) can be used to fix three of the z_r integrals and reproduce the (b, c) correlation function. This formula clearly agrees with the formula of (1) for the D-instanton amplitude where the σ variable from the $D1$-string worldvolume has been replaced with the z variable from the open string boundary.

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