Sagnac Interferometry Based on Ultraslow Polaritons in Cold Atomic Vapors

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(Received 23 December 2003; published 21 June 2004)

The advantages of light and matter-wave Sagnac interferometers—large area on one hand and high rotational sensitivity per unit area on the other—can be combined utilizing ultraslow light in cold atomic gases. While a group-velocity reduction alone does not affect the Sagnac phase shift, the associated momentum transfer from light to atoms generates a coherent matter-wave component which gives rise to a substantially enhanced rotational signal. It is shown that matter-wave sensitivity in a large-area interferometer can be achieved if an optically dense vapor at subrecoil temperatures is used. Already a noticeable enhancement of the Sagnac phase shift is possible, however, with far fewer cooling requirements.

DOI: 10.1103/PhysRevLett.92.253201 PACS numbers: 39.20.+q, 03.75.Dg, 06.30.Gv, 42.50.Gy

The relative phase accumulated by two counterpropagating waves in a rotating ring interferometer, first observed by Sagnac [1], is a well-established tool to detect intrinsically the rotation of a system. Two types of Sagnac gyroscopes have been developed which have very different characteristics depending on the type of wave phenomena they are based on [2]. Optical devices achieve high rotational sensitivity because of the large interferometer area [3]. On the other hand, the rotational sensitivity per unit area of matter-wave gyroscopes [4,5] exceeds that of optical ones by the ratio $mc^2/\hbar\omega \sim 10^{11}$. They suffer, however, from the smallness of the achievable loop areas and only recently short-time rotational sensitivities have been demonstrated for matter-wave interferometers which are comparable to state-of-the-art laser gyroscopes [6,7]. We here show that it should be possible to combine the large rotational sensitivity per unit area of matter-waves and the large-area typical for optical gyroscopes utilizing the coherence and momentum transfer associated with ultraslow light in cold atomic vapors with electromagnetically induced transparency (EIT) [8–11]. As the slowdown of light in EIT media is based on the rotation of dark-state polaritons from electromagnetic to atomic excitations [12], light waves can coherently be transformed into matter waves, if the excitation transfer is accompanied by momentum transfer and the atoms are allowed to move freely. Since the transfer is coherent and reversible it serves as a basis for a hybrid light-matter-wave Sagnac interferometer.

The rotational phase shift in an optical Sagnac interferometer is given by

$$\Delta\phi_{\text{light}} = \frac{4\pi}{\lambda c}\mathbf{\Omega}\cdot\mathbf{A}, \tag{1}$$

where $\mathbf{\Omega}$ and $\mathbf{A}$ are the vectors of the angular velocity and the loop area. Discussing Fresnel dragging in EIT media Leonhardt and Piwnicki suggested in [13] that the reduction of the group velocity in a solid attached to the rotating body should lead to the replacement of the vacuum speed of light c in (1) by the group velocity v_{gr} and thus to an enormous increase of rotational sensitivity. However, as will be shown here, the Sagnac phase shift in a solid medium is always given by the vacuum expression irrespective of the group velocity. In fact in the history of Sagnac interferometry there had been a longer discussion about the effect of refractive materials in the beam path of passive optical devices. It was finally recognized that materials that change the phase velocity of light have no effect on the Sagnac phase shift (see, e.g., [2]). However, if the coherence transfer from light to medium, associated with any group-velocity reduction, is accompanied by a *momentum transfer to freely moving particles* a traveling matter-wave component is created which can lead to a substantial enhancement of the rotational sensitivity.

Let us consider a circular light interferometer of radius R with an atomic vapor cell or trap in the beam path attached to a rotating body as shown in Fig. 1. The product of angular velocity Ω and radius R is assumed to be small compared to c such that nonrelativistic quantum mechanics applies [14]. Light propagation as well as center-of-mass motion of the atoms shall be confined to the periphery of the loop, e.g., by means of optical fibers

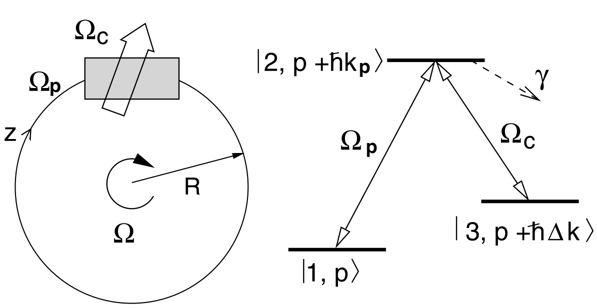

FIG. 1. (left) Setup of light Sagnac interferometer with vapor cell or trap attached to rotating body with angular velocity Ω. (right) Level scheme of atoms. p denotes momentum along the peripheral direction z. k_p is the wave number of the probe field propagating parallel to z. $\Delta k = k_p - k_c^{\parallel}$, where $k_c^{\parallel}$ is the component of the control-field wave vector in the z direction.

and atom potentials. Distances along the periphery will be denoted by the coordinate z. As indicated in Fig. 1 the atoms have three internal states $|1\rangle, |2\rangle, |3\rangle$ allowing for a Λ-type Raman coupling of the $|1\rangle - |2\rangle$ transition to the probe field described by the Rabi frequency $\Omega_p(z,t)$ and of the $|2\rangle - |3\rangle$ transition to the control field, characterized by the Rabi frequency $\Omega_c(z,t)$. For the present application Ω_c is assumed to be much larger than Ω_p.

Under two-photon resonance conditions, the control field generates electromagnetically induced transparency for the probe field associated with a reduction of the group velocity according to $v_{\rm gr} = c\cos^2\theta$ with $\tan^2\theta \equiv g^2 n/|\Omega_c|^2$ where n is the one-dimensional density of atoms and $g = d\sqrt{\omega_p/(2\hbar\epsilon_0 F)}$ is a coupling constant containing the probe frequency ω_p, the dipole moment d of the corresponding transition and the transversal cross section F of the probe beam. As has been demonstrated in several experiments [9–11] rather substantial group-velocity reductions can be achieved.

The atoms are described by one-dimensional matter fields $\Psi_1(z,t)$, $\Psi_2(z,t)$, and $\Psi_3(z,t)$ corresponding to the three internal states. Since the sources of the fields as well as the experimental apparatus are attached to the rotating frame, the dynamics of the probe light Ω_p and the matter-wave components Ψ_μ will be described in the rotating frame. For this the total Hamiltonian in a nonrotating frame H will be transformed according to $\exp\{iGt\}H\exp\{-iGt\}$, where $G = \Omega\hat{M}$ is the generator of a uniform rotation [15]. $\hat{M}$ is the total (orbital) angular momentum of matter fields and light in the direction of $\mathbf{\Omega}$. The Hamiltonian in the rotating frame leads to the following equations of motion:

$$\mathcal{D}\Psi_1 = \hbar\Omega_p^* e^{i(\omega_p t - k_p z)}\Psi_2, \tag{2}$$

$$\mathcal{D}\Psi_2 = \hbar(\omega_2 - i\gamma)\Psi_2 + \hbar\Omega_p e^{-i(\omega_p t - k_p z)}\Psi_1 + \hbar\Omega_c e^{-i(\omega_c t - k_c^{\parallel} z)}\Psi_3, \tag{3}$$

$$\mathcal{D}\Psi_3 = \hbar\omega_3\Psi_3 + \hbar\Omega_c^* e^{i(\omega_c t - k_c^{\parallel} z)}\Psi_2, \tag{4}$$

with

$$\mathcal{D} \equiv i\hbar\partial_t + \frac{\hbar^2}{2m}\partial_z^2 - i\hbar\Omega R\partial_z.$$

Here ω_p and ω_c are the probe and coupling frequencies in the rotating frame, which is the laboratory frame, and $k_p = \omega_p/c$ and $k_c^{\parallel}$ the corresponding wave-vector components in the z direction. γ describes losses by spontaneous emission from the excited state $|2\rangle$. We assume that initially, i.e., without applying the probe field all atoms are in state $|1\rangle$.

We proceed by introducing slowly varying matter-wave amplitudes $\Psi_1 = \Phi_1$, $\Psi_2 = \Phi_2 e^{-i(\omega_p t - k_p z)}$, and $\Psi_3 = \Phi_3 e^{-i((\omega_p-\omega_c)t - \Delta k z)}$, with $\Delta k = k_p - k_c^{\parallel}$ and assume single- and two-photon resonance of the carrier frequencies, i.e., $\omega_p = \omega_2 + \hbar k_p^2/2m$ and $\omega_p - \omega_c = \omega_3 + \hbar(\Delta k)^2/2m$. Neglecting second-order derivatives within a slowly varying envelope approximation we arrive at the set of equations

$$(\partial_t - \Omega R\partial_z)\Phi_1 = -i\Omega_p^*\Phi_2, \tag{5}$$

$$(\partial_t - (\Omega R - v_{\rm rec})\partial_z)\Phi_2 = +(ik_p\Omega R - \gamma)\Phi_2 - i\Omega_c\Phi_3 - i\Omega_p\Phi_1, \tag{6}$$

$$(\partial_t - (\Omega R - \eta v_{\rm rec})\partial_z)\Phi_3 = +i\eta k_p\Omega R\Phi_3 - i\Omega_c^*\Phi_2, \tag{7}$$

where we have introduced the recoil velocity $v_{\rm rec} \equiv \hbar k_p/m$. The dimensionless parameter $\eta \equiv \Delta k/k_p = 1 - k_c^{\parallel}/k_p$ describes the momentum transfer of the light fields to the atoms. In a degenerate Λ system with copropagating probe and control field there is no momentum transfer and thus $\eta = 0$.

Similarly we find the equation of motion of the probe field propagating parallel to the rotation in slowly varying envelope approximation

$$(\partial_t + c\partial_z - ik_p\Omega R)\Omega_p(z,t) = -ig^2\Phi_1^*\Phi_2. \tag{8}$$

We now make use of the assumption that the control field is strong compared to the probe field and treat the interaction with the fields in lowest order of perturbation in Ω_p. In this approximation the effect of the interaction on the coupling field Ω_c can be disregarded. Furthermore, we find for the matter-field $\Phi_1(z,t) = \sqrt{n} = \text{const}$, where n is the linear density of atoms. Assuming stationary conditions, the corresponding equations for Φ_2 and Φ_3 read as follows:

$$\begin{bmatrix} \Omega_c & -k_p\Omega R - i\gamma \\ -\eta k_p\Omega R & \Omega_c^* \end{bmatrix}\begin{bmatrix} \Phi_3 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} -\sqrt{n}\,\Omega_p \\ 0 \end{bmatrix} + \begin{bmatrix} i(v_{\rm rec} - \Omega R)\partial_z\Phi_2 \\ i(\eta v_{\rm rec} - \Omega R)\partial_z\Phi_3 \end{bmatrix}. \tag{9}$$

If the complex amplitudes of the matter-wave components change sufficiently slowly, we can analytically solve Eqs. (9) by treating the term on the second line as a perturbation. Substituting the corresponding result into the equation for the probe field (8) leads to the group velocity

$$v_{\rm gr} = c\cos^2\theta + \eta v_{\rm rec}\sin^2\theta. \tag{10}$$

One recognizes that the group velocity now contains an additional term due to the momentum transfer associated with the coherence transfer if $\eta \neq 0$ [16]. In the following we will assume that $\eta \geq 0$, although negative values of η are possible. If the mixing angle exceeds the critical value $\tan^2\theta_{\rm crit} \equiv c/v_{\rm rec} = mc^2/\hbar\omega_p$ the excitation propagates in the medium essentially as a matter wave with recoil velocity. In the case of $\eta < 0$ the minimum group velocity attainable is actually zero since the medium becomes opaque as soon as $v_{\rm gr}$ crosses the value zero.

Under stationary conditions and keeping only terms up to first order in the rotation velocity Ω, one finds the simple probe-field equation

$$\partial_z \ln\Omega_p(z) = i\frac{2\pi}{\lambda c}\Omega R\frac{\xi(z) + \eta(mc^2/\hbar\omega_p)}{\xi(z) + \eta}, \qquad (11)$$

where λ is the probe-field wavelength and we have introduced the parameter

$$\xi(z) \equiv \frac{\tan^2\theta_{\rm crit}}{\tan^2\theta(z)} = \frac{v_{\rm gr} - \eta v_{\rm rec}}{c - v_{\rm gr}}\frac{c}{v_{\rm rec}} \approx \frac{v_{\rm gr}(z)}{v_{\rm rec}} - \eta, \quad (12)$$

where in the last equation $v_{\rm gr} \ll c$ was assumed. (Note that $v_{\rm gr} \geq \eta v_{\rm rec}$.) Equation (11) describes a simple phase shift without absorption losses due to perfect EIT. Two counterpropagating probe fields will thus experience the Sagnac shift

$$\Delta\phi = \frac{2\pi\Omega R}{\lambda c}\int dz\,\frac{\xi(z)}{\xi(z) + \eta} + \frac{\Omega R}{\hbar/m}\int dz\,\frac{\eta}{\xi(z) + \eta}. \qquad (13)$$

The integration over z takes into account that the group velocity can be different in different parts of the interferometer loop (see Fig. 1). If $\xi \gg \eta mc^2/\hbar\omega_p$, i.e., for $v_{\rm gr} \to c$, Eq. (13) reproduces the Sagnac phase shift of an optical gyroscope, Eq. (1). On the other hand, if $\eta \neq 0$ and $\xi \ll \eta$ (13) approaches the matter-field phase shift. Furthermore one recognizes that in the absence of momentum transfer, i.e., for $\eta = 0$, the phase shift is for all values of ξ identical to that of a light interferometer. Thus the reduction of the group velocity alone does not affect the Sagnac phase. One should note that this is in contrast to Fresnel dragging, which is always present in a medium with a small group velocity. The difference to Fresnel dragging arises because in the Sagnac interferometer there is no motion of the medium relative to the source of the waves and consequently no simple Doppler shift. In order to distinguish rotational phase shifts from those resulting from linear accelerations it is necessary to use a symmetric interferometer setup. Since the matter wave is generated by a coherent transfer from light we can combine the rotational sensitivity per unit area with a large interferometer area utilizing well-established guiding techniques for light. This is the main result of the present Letter. The enhancement of the Sagnac phase shift due to an atomic vapor with EIT put into the beam path of a given optical interferometer is shown in Fig. 2. Here a circumference of the ring interferometer of 1 m as well as a medium length of 1 mm was assumed.

In order to estimate the potential enhancement of rotational sensitivity we now have to discuss the limitations on the minimum value of ξ, which arise mainly from two-photon Doppler shifts and velocity-changing collisions. As seen above, an enhancement of the rotational sensitivity requires $\eta > 0$. In this case there is a two-photon Doppler-shift caused by thermal motion of atoms in the rotating frame, which leads to a finite absorption of the medium. Since the absorption grows with decreasing group velocity, this represents an essential limitation to the hybrid interferometer. To discuss the effects of a thermal velocity distribution we decompose the three matter fields into velocity classes according to

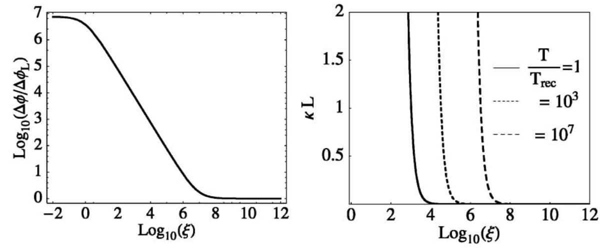

FIG. 2. (left) Sagnac phase shift of the EIT hybrid interferometer relative to the phase shift of an optical gyroscope of the same area, a ratio of medium length to circumference of 10^{-3} and $\eta = 1$; for $\xi \gg \eta mc^2/\hbar\omega_p$ we are in the light and for $\xi \ll \eta$ we are in the matter-wave regime. A recoil velocity $v_{\rm rec} = \hbar\omega_p/mc$ of 4 cm/s was assumed. (right) Absorption coefficient for an atomic sample in an elongated trap ($L = 1$ mm) and with opacity $\alpha = 10^2$.

$$\Phi_\mu(z,t) = \sum_v \Phi_\mu^v(z,t)e^{i(qz-\nu t)}, \qquad (14)$$

where $\hbar q = mv$ is the initial momentum of the atoms and $\hbar\nu = mv^2/2 - mv\Omega R$. We then can proceed as above and obtain the same equations of motion for the slowly varying velocity components Φ_μ^v as in Eqs. (9) except for the replacement $\Omega R \longrightarrow \Omega R - v$.

Keeping again only linear contributions of the angular velocity Ω to the phase, we find the following propagation equation for the probe field:

$$\begin{aligned}\partial_z \ln\Omega_p(z) = {} & ik_p\frac{\Omega R}{c}\frac{\xi(z) + \eta(mc^2/\hbar\omega_p)}{\xi(z) + \eta} \\ & - 2ik_p\left(\frac{\gamma\eta k_p c}{g^2 n}\frac{\overline{v^2}}{v_{\rm rec}^2}\frac{1}{\xi(z)(\xi(z) + \eta)}\right)^2 \\ & - k_p\frac{\gamma\eta k_p c}{g^2 n}\frac{\overline{v^2}}{v_{\rm rec}^2}\frac{1}{\xi(z)(\xi(z) + \eta)}, \end{aligned} \qquad (15)$$

which has the simple solution

$$\Omega_p(z) = \Omega_p(0)e^{i\phi(z)}e^{-\kappa z}. \qquad (16)$$

In Eq. (15) $\overline{v^2}$ denotes the mean square of the velocity distribution in the rotating frame with $\overline{v} = 0$.

The first term in Eq. (15) gives the Sagnac phase shift as derived previously, the second term is a constant phase contribution which, however, cancels in the difference phase of the two counterpropagating waves and is thus without consequence. The important new contribution due to the atomic velocity distribution is the third term in (15) which gives rise to absorption which grows with decreasing ξ. Thus ξ, the group velocity, cannot be made arbitrarily small since the growing losses will lead to a decreasing signal-to-noise ratio. Whether or not the matter-wave limit $\xi \ll \eta$ can be reached will depend on the velocity spread or temperature of the atoms. In deriving Eq. (15) we have made furthermore the following simplifying assumptions $\eta k_p^2 \overline{v}^2 \ll |\Omega_c|^2$, $|\Omega_c|^4/\gamma^2$. These are, however, well justified as long as the total amplitude decrease due to absorption is less than e^{-1} and $\xi \geq \alpha^{-1}$, where $\alpha \equiv g^2 nz/\gamma c$ is the opacity of the medium in the absence of EIT, which is typically in the range between 1 and 10^2. Figure 2 (right) shows the absorption coefficient

$$\kappa L = \eta \frac{(k_p L)^2}{\alpha} \frac{T}{T_{\rm rec}} \frac{1}{\xi(\xi + \eta)} \tag{17}$$

for three different temperatures T of the atomic sample relative to the recoil temperature $T_{\rm rec}$ for $L = 1$ mm, opacity $\alpha = 100$, and a recoil velocity typical for alkali of 4 cm/s. One recognizes that by approaching the matter-wave regime the absorption experiences a steep increase at a certain value of ξ. Thus in order to retain a reasonable signal-to-noise ratio, i.e., for absorption coefficients of $\kappa L \leq 1$, there is a minimum value of ξ (respectively, a minimum value of $v_{\rm gr}$). Figure 2 shows that it is only possible to take advantage of the maximum possible signal enhancement of $\mathrm{mc}^2/\hbar\omega = c/v_{\rm rec}$, if the atomic sample is cooled down to the recoil limit or even below. Nevertheless, it is still possible to gain about 2 orders of magnitude in signal strength as compared to the pure light regime by moderate cooling ($T \approx 10^3\, T_{\rm rec}$).

Another important limitation to the minimum value of ξ is set by the decoherence of motional degrees of freedom of the matter-wave component caused by collisions. Although superpositions of internal states can survive very many collisions, in particular, if they are hyperfine components of the electronic ground state, velocity kicks quickly destroy the coherence between motional states. To account for the latter in a quantitative way a kinetic theory needs to be developed. Such a theory goes, however, beyond the scope of the present Letter and will be the subject of future work. We here can only give an upper bound for the corresponding limit on ξ (or equivalently the group velocity $v_{\rm gr}$) by requiring that the pulse delay time τ_d is shorter than the average time $\tau_{\rm coll}$ between two successive velocity-changing collisions:

$$\left(\frac{v_{\rm gr}}{v_{\rm rec}}\right) \geq \frac{L}{v_{\rm rec}\tau_{\rm coll}} \sim L\varrho\sigma\sqrt{\frac{T}{T_{\rm rec}}}, \tag{18}$$

where σ is the collisional cross section and ϱ the (three-dimensional) density. E.g., in a trap of length $L = 1$ mm, $\varrho = 10^{12}\ \mathrm{cm}^{-3}$, assuming $\sigma = 10^{-12} \dots 10^{-10}\ \mathrm{cm}^2$ yields $v_{\rm gr}^{\rm min}/v_{\rm rec} \approx 0.1 \dots 10\sqrt{T/T_{\rm rec}}$. When a *coherent* sample of atoms such as a BEC is used, the limitation due to collisions is essentially absent since in the s-wave scattering limit the condensate of atoms in state $|1\rangle$ will only lead to a mean-field phase shift of the propagating polariton. Only when a sizable thermal cloud is present or when corrections beyond mean-field are important decoherence due to collisions cannot be neglected.

In the present Letter we have shown that it is possible to combine the advantages of light and mater-wave gyroscopes, to measure the Sagnac phase shift making use of the coherence and momentum transfer associated with ultraslow light propagation in cooled atomic vapors.

M. F. would like to acknowledge discussions with L. J. Wang, R. Chiao, and U. Leonhardt during the KITP workshop on "Fast and slow light and meta-materials" at the UC Santa Barbara as well as with I. Bialynicki-Birula. F. Z. acknowledges the support by the DFG through GRK 792 "Nichtlineare Optik und Ultrakurzzeitphysik."

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