Semileptonic Hyperon Decays and Cabibbo-Kobayashi-Maskawa Unitarity

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Using a technique that is not subject to first-order SU(3) symmetry breaking effects, we determine the V_{us} element of the Cabibbo-Kobayashi-Maskawa matrix from data on semileptonic hyperon decays. We obtain $V_{us} = 0.2250(27)$, where the quoted uncertainty is purely experimental. This value is of similar experimental precision to the one derived from K_{l3}, but it is higher and thus in better agreement with the unitarity requirement, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$. An overall fit, including the axial contributions and neglecting SU(3) breaking corrections, yields $F + D = 1.2670 \pm 0.0035$ and $F - D = -0.341 \pm 0.016$ with $\chi^2 = 2.96/3$ degrees of freedom.

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The determination of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2] is one of the main ingredients for evaluating the solidity of the standard model of elementary particles. This is a vast subject which has seen important progress with the determination [3,4] of ϵ'/ϵ and the observation [5,6] of CP violation in B decays.

While a lot of attention has recently been justly devoted to the higher mass sector of the CKM matrix, it is the low mass sector, in particular, V_{ud} and V_{us}, where the highest precision can be attained. The most sensitive test of the unitarity of the CKM matrix is provided by the relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta$. Clearly, the unitarity condition is $\Delta = 0$. The $|V_{ub}|^2$ contribution [7] is negligible (10^{-5}) at the current level of precision. The value $V_{ud} = 0.9740 \pm 0.0005$ is obtained from superallowed pure Fermi nuclear decays [8]. In combination with $V_{us} = 0.2196 \pm 0.0023$, derived from K_{e3} decay [9,10], this yields $\Delta = 0.0032 \pm 0.0014$. At face value, this represents a 2.3 standard deviation departure from unitarity [8].

In this Letter we reconsider the contribution that the hyperon beta decays can give to the determination of V_{us}. The conventional analysis of hyperon beta decay in terms of the parameters F, D and V_{us} is marred by the expectation of first-order SU(3) breaking effects in the axial-vector contribution. The situation is only made worse if one introduces adjustable SU(3) breaking parameters, as this increases the number of degrees of freedom (DOF) and degrades the precision. If on the contrary, as we do here, one focuses the analysis on the vector form factors, treating the rates and g_1/f_1 [11] as the basic experimental data, one has direct access to the f_1 form factor for each decay, and this in turn allows for a redundant determination of V_{us}. The consistency of the values of V_{us} determined from the different decays is a first confirmation of the overall consistency of the model. A more detailed discussion may be found in the Annual Reviews of Nuclear and Particle Sciences [12].

In 1964, Ademollo and Gatto proved [13] that there is no first-order correction to the vector form factor, $\Delta^1 f_1(0) = 0$. This is an important result: since experiments can measure $V_{us} f_1(0)$, knowing the value of $f_1(0)$ in $\Delta S = 1$ decays is essential for determining V_{us}.

The Ademollo-Gatto theorem suggests an analytic approach to the available data that first examines the vector form factor f_1 because it is not subject to first-order SU(3) symmetry breaking effects. An elegant way to do this is to use the *measured* value of g_1/f_1 along with the predicted values of f_1 and f_2 to extract a V_{us} value from the decay rate for each decay. If the theory is correct, these should coincide within errors and could be combined to obtain a best value of V_{us}. This consistency of the V_{us} values obtained from different decays then indicates the success of the Cabibbo model. A similar approach appears to have been taken in Ref. [14].

Four hyperon beta decays have sufficient data to perform this analysis: $\Lambda \to p\, e^- \bar{\nu}$, $\Sigma^- \to n\, e^- \bar{\nu}$, $\Xi^- \to \Lambda\, e^- \bar{\nu}$, and $\Xi^0 \to \Sigma^+ e^- \bar{\nu}$ [9]. Table I shows the results for them. In this analysis, both model-independent and model-dependent radiative corrections [11] are applied, and the q^2 variation of f_1 and g_1 is included. Also SU(3) values of $g_2 = 0$ and f_2 are used along with the numerical rate expressions tabulated in Ref. [11]. We have not, however, included SU(3) breaking corrections to the f_1 form factor, which will be discussed in the next section. The

TABLE I. Results from V_{us} analysis using measured g_1/f_1 values.

Decay Process	Rate (μsec^{-1})	g_1/f_1	V_{us}
$\Lambda \rightarrow pe^-\overline{\nu}$	3.161(58)	0.718(15)	0.2224 ± 0.0034
$\Sigma^- \rightarrow ne^-\overline{\nu}$	6.88(24)	−0.340(17)	0.2282 ± 0.0049
$\Xi^- \rightarrow \Lambda e^-\overline{\nu}$	3.44(19)	0.25(5)	0.2367 ± 0.0099
$\Xi^0 \rightarrow \Sigma^+ e^-\overline{\nu}$	0.876(71)	1.32(+.22/ − .18)	0.209 ± 0.027
Combined			0.2250 ± 0.0027

stated V_{us} errors are purely experimental, coming from experimental uncertainties in the hyperon lifetimes, branching ratios, and form factor ratios.

The four values are clearly consistent ($\chi^2 = 2.26/3$ DOF) with the combined value of $V_{us} = 0.2250 \pm 0.0027$. This value is nearly as precise as that obtained from kaon decay ($V_{us} = 0.2196 \pm 0.0023$) and, as observed in previous analyses [15–17], is somewhat larger. In combination with $V_{ud} = 0.9740 \pm 0.0005$ obtained from superallowed pure Fermi nuclear decays [8], the larger V_{us} value from hyperon decays beautifully satisfies the unitarity constraint $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$.

We will limit our discussion to the effects that are most relevant for the determination of V_{us}. Turning our attention first to SU(3) breaking corrections to the f_1 form factor, we find in the literature computations that use some version of the quark model, as in [18,19], or some version of chiral perturbation theory, as in [15,20,21].

The quark-model computations find that the f_1 form factors for the different $\Delta S = 1$ decays are reduced by a factor, the same for all decays, given as 0.987 in [18] and 0.975 in [19], a decrease, respectively, of 1.3% or 2.5%. This is a very reasonable result, the decrease arising from the mismatch of the wave functions of baryons containing different numbers of the heavier s quarks.

Evaluations of f_1 in chiral perturbation theory range from small negative corrections in [20] to larger positive corrections in [15,21]. Positive corrections in f_1 for *all* hyperon beta decays cannot be excluded but are certainly not expected in view of an argument [22] according to which one expects a negative correction to f_1, at least in the $\Sigma^- \rightarrow n\, e^-\bar{\nu}$ case. This result follows from the observation that the intermediate states that contribute to the positive second-order terms in the Ademollo and Gatto sum rule have, in this case, quantum numbers $S = -2$, $I = 3/2$; no resonant baryonic state is known with these quantum numbers. If we accept the hypothesis that the contribution of resonant hadronic states dominate, we can conclude that the correction to f_1 in Σ^- beta decay should be negative. We note that this argument also applies to K_{l3} decays, and that the corrections to these decays, computed with chiral perturbation theory, are, as expected, negative.

A modern revisitation of the quark-model computations will be feasible in the near future with the technologies of lattice QCD, and we would expect that a small negative correction would be obtained in *quenched* lattice QCD, an approximation that consists in neglecting components in the wave function of the baryons with extra quark-antiquark pairs. This is known to be an excellent approximation in low-energy hadron phenomenology [23].

Multiquark effects can be included in lattice QCD by forsaking the quenched approximation for a *full* simulation. Alternatively, one could resort to chiral perturbation theory to capture the major part of the multiquark contributions which will be dominated by virtual π, K, η states. Early results of a similar strategy applied to the K_{e3} decays [24] indicate that in that case a 1% determination of the $f_+(0)$ form factor is within reach, and we expect that a similar precision can be obtained in the case of hyperon decays. In the present situation, we consider it best not to include any SU(3) breaking corrections in our evaluation, nor to include an evaluation of a theoretical error. Our expectation that the corrections to $f_+(0)$ will be small and negative can only be substantiated by further work.

We next turn our attention to the possible effect of ignoring the g_2 form factor. In the absence of second class currents [25], the form factor g_2 can be seen to vanish in the SU(3) symmetry limit. The argument is very straightforward: the neutral currents $A^3_\alpha = \bar{q}\lambda^3\gamma_\alpha\gamma_5 q$ and $A^8_\alpha = \bar{q}\lambda^8\gamma_\alpha\gamma_5 q$ that belong to the same octet as the weak axial-vector current are even under charge conjugation, so that their matrix elements cannot contain a weak-electricity term, which is C odd. The vanishing of the weak electricity in the proton and neutron matrix elements of A^3_α, A^8_α implies the vanishing of the D and F coefficients for $g_2(0)$, so that, in the SU(3) limit, the $g_2(0)$ form factor vanishes for any current in the octet.

In hyperon decays, a nonvanishing $g_2(0)$ form factor can arise from the breaking of $SU(3)$ symmetry. Theoretical estimates [26] indicate a value for $g_2(0)/g_1(0)$ in the −0.2 to −0.5 range.

In determining the axial-vector form factor g_1 from the Dalitz Plot—or, equivalently, the electron-neutrino correlation—one is actually measuring $\tilde{g}_1$, a linear combination of g_1 and g_2 ($\tilde{g}_1 \approx g_1 - \delta g_2$ up to first order in $\delta = \Delta M/M$). This has already been noticed in past experiments and is well summarized in Gaillard and

Sauvage [17], Table 8. Therefore, in deriving $V_{us}^2 f_1^2$ (hence V_{us}) from the beta decay rate, there is in fact a small sensitivity to g_2. To first order, the rate is proportional to $V_{us}^2[f_1^2 + 3g_1^2 - 4\delta\, g_1 g_2] \approx V_{us}^2[f_1^2 + 3\tilde{g}_1^2 + 2\delta\, \tilde{g}_1 g_2]$. In fact, this is a second-order correction to the value of V_{us}, potentially of the same order of magnitude as the corrections to f_1.

Experiments that measure correlations with polarization—in addition to the electron-neutrino correlation—are sensitive to g_2. While the data are not yet sufficiently precise to yield good quantitative information, one may nevertheless look for trends. In polarized $\Sigma^- \to p\, e^- \bar{\nu}$ [27], negative values of g_2/f_1 are clearly disfavored (a positive value is preferred by 1.5σ). Since the same experiment unambiguously established that g_1/f_1 is negative, one concludes that allowing for nonvanishing g_2 would increase the derived value of $V_{us}^2 f_1^2$. In polarized $\Lambda \to p\, e^- \bar{\nu}$, the data favor [28] negative values of g_2/f_1 (by about 2σ). In this decay, g_1/f_1 is positive so that, again, allowing for the presence of nonvanishing g_2 would increase the derived value of $V_{us}^2 f_1^2$. In either case, we may conclude that making the conventional assumption of neglecting the g_2 form factor tends to *underestimate* the derived value of V_{us}. A more quantitative conclusion must await more precise experiments. We consider it to be of the highest priority to determine the g_2 form factor (or a stringent limit on its value) in at least one of the hyperon decays, ideally in Λ semileptonic decay, which at the moment seems to offer the single most precise determination of V_{us}.

The excellent agreement with the unitarity condition of our determination of V_{us}, which neglects SU(3) breaking effects, seems to indicate that such effects were overestimated in the past, probably as a consequence of the uncertainties of the early experimental results. We also find [12] that the g_1 form factor of the different decays, which is subject to first-order corrections, is well fitted by the F, D parameters [1], with $F + D = 1.2670 \pm 0.0035$ and $F - D = -0.341 \pm 0.016$ with $\chi^2 = 2.96/3$ DOF.

The value of V_{us} obtained from hyperon decays is of comparable experimental precision with that obtained from K_{l3} decays and is in better agreement with the value of θ_C obtained from nuclear beta decay. While a discrepancy between V_{us} and V_{ud} could be seen as a portent of exciting new physics, a discrepancy between the two different determinations of V_{us} can be taken only as an indication that more work remains to be done both on the theoretical and the experimental side.

On the theoretical side, renewed efforts are needed for the determination of SU(3) breaking effects in hyperon beta decays as well as in K_{l3} decays. While it is quite possible to improve the present situation on the quark-model front, the best hopes lie in lattice QCD simulations, perhaps combined with chiral perturbation theory for the evaluation of large-distance multiquark contributions.

We have given some indication that the trouble could arise from the K_{l3} determination of V_{us}, and we would like to encourage further experimental work in this field [29]. We are, however, convinced of the importance of renewed experimental work on hyperon decays, of the kind now in progress at the CERN Super Proton Synchrotron. The interest of this work goes beyond the determination of V_{us}, as it involves the intricate and elegant relationships that the model predicts.

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Note added.—The KTeV Collaboration has just announced a determination of $|V_{us}|$ based on neutral kaon semileptonic decay rates [30]. The result $|V_{us}| = 0.2252(\pm 0.005_{\text{KTeV}} \pm 0.009_{\text{ext}})$ is in beautiful agreement with our hyperon-derived value.

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