D-Term Inflation without Cosmic Strings

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We present a superstring-inspired version of *D*-term inflation that does not lead to cosmic string formation and appears to satisfy the current cosmic microwave background constraints. It differs from minimal *D*-term inflation by a second pair of charged superfields that makes the strings nontopological (semilocal). The strings are also Bogomol'nyi-Prasad-Sommerfield strings, so the scenario is expected to survive supergravity corrections. The second pair of charged superfields arises naturally in several brane and conifold scenarios, but its effect on cosmic string formation had not been noticed so far.

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The Wilkinson Microwave Anisotropy Probe (WMAP) [1] data have put in sharp focus the question of identifying the microscopic origin of the inflaton, the field or fields that fuel inflation. *D*-term inflation in supersymmetric (SUSY) gauge theories is a promising hybrid inflation scenario that seems to survive supergravity corrections [2,3] (see [4] for a recent discussion). Unfortunately, it suffers from a severe drawback: at the end of inflation cosmic strings form, giving a contribution to the primordial density perturbations comparable to that of inflation, in contradiction with the cosmic microwave background (CMB) data [5].

While this problem may be circumvented in various ways (for instance, by invoking a curvaton [6]), much of the beautiful simplicity and predictive power of the original minimal *D*-term inflation model is lost in these modifications. It would be desirable to have a model that has all the good features of the model of Refs. [2,3] but without the production of cosmic strings. The purpose of this Letter is to present such a model.

The idea is extremely simple. The minimal *D*-term model is a supersymmetric Abelian gauge model where the inflaton is a neutral scalar field that couples to two charged scalar fields with opposite U(1) charges. The model has a Fayet-Iliopoulos (FI) D term. At the end of inflation, the charged fields acquire nonzero vacuum expectation values (VEVs), the gauge field undergoes the Higgs mechanism, and Nielsen-Olesen (NO) strings form. To avoid string formation, we include a second identical pair of charged fields with the same couplings to the inflaton and to the gauge field. At the end of inflation the charged scalars acquire VEVs and cause a Higgs mechanism, as before, but now the vacuum manifold is simply connected and the strings are nontopological. They are semilocal (SL) strings [7,8] whose cosmological formation rates have been studied in Ref. [9] and whose number density falls to zero when the couplings are in the Bogomol'nyi limit—the region of interest here.

The addition of the second pair of charged fields is motivated by a possible interpretation as four-dimensional effective actions of type II superstrings compactified on Calabi-Yau (CY) manifolds. These are expected to have $N = 2$ SUSY in four dimensions. Each pair of charged $N = 1$ superfields forms an $N = 2$ hypermultiplet, and the two $N = 2$ hypermultiplets have opposite U(1) charges. The neutral $N = 1$ superfield (the inflaton) and the U(1) gauge multiplet combine into an $N = 2$ vector multiplet.

D-term inflation has been studied in heterotic string compactifications where the FI terms arise from an anomalous U(1) [10]. Instead, the model presented here seems to be related to type II string compactifications on CY spaces near singular points (see also [11]). The hypermultiplets are typically associated with string or brane states wrapping around cycles in the internal space whose size goes to zero at the singularity, making the states massless. The FI terms appear when the singularity is resolved by replacing the singular point with a small "sphere" (whose size determines the FI parameter). FI terms may also arise in brane-antibrane backgrounds [12] and, therefore, in brane inflation [13]. Thus, the analysis of these models gives a potential window into string theory [11,12,14].

Specific examples have been studied, e.g., in connection with topology change transitions in CY spaces [15,16], phases of SUSY Abelian theories [17], and confinement [18] in string theory, where the presence of the second, oppositely charged hypermultiplet was forced by the homology of the vanishing cycles or chains [19].

If we ignore gravity, the model presented here has $N = 2$ SUSY. Coupling to gravity breaks $N = 2$ to (local) $N = 1$ due to the FI terms (see [12,20,21] and references therein). A recent study [12] shows that the NO strings produced after inflation in the one-hypermultiplet model are Bogomol'nyi-Prasad-Sommerfield (BPS) strings (that is, they break half of the $N = 1$ *local* SUSY, such as in the

global case [22]). For SUSY to be (partially) unbroken, the transformation of the gravitino (and all other fermions) must be zero. The holonomy of the local SUSY parameter going around the string receives two contributions, one from the deficit angle in the metric transverse to the string, the other from an Aharonov-Bohm interaction with the U(1) magnetic field of the string. Cancellation of these two contributions allows for the existence of killing spinors and unbroken SUSY [23], provided the gravitino is charged under the U(1), which is therefore an R symmetry. We will show that in our model the (embedded) NO strings are also BPS but are not expected to form after inflation.

The model.—Minimal D-term inflation (see [3], for instance) requires two chiral superfields ϕ_+, ϕ_-, with opposite U(1) charges $\pm g$, a neutral chiral superfield S, and a U(1) gauge multiplet. We add another pair of charged chiral superfields $\tilde{\phi}_+, \tilde{\phi}_-$ [24]. In the absence of gravity, the model has $N = 2$ SUSY, so each pair of chiral superfields assembles into a nonchiral $N = 2$ hypermultiplet, and the neutral superfield and the gauge multiplet into an $N = 2$ vector multiplet. Each hypermultiplet transforms under the $SU(2)_R$ symmetry between the two supercharges. It is always possible to add an $N = 2$ FI $\vec{k} \cdot \vec{P}$ term (a *P term*), where $\vec{P}$ are the $SU(2)_R$ triplets of auxiliary fields belonging to the U(1) Abelian vector multiplet. The choice $\vec{k} \propto (0, 0, 1)$ leads to $N = 1$ D terms, while $\vec{k} \propto (1, 0, 0)$ leads to F terms. Here we take $\vec{k} = (0, 0, g\omega^2/2)$.

In global supersymmetry this choice can be made without loss of generality and shows that D-term and F-term models are equivalent, by $SU(2)_R$ rotations, and part of a larger class of so-called P-term models [4]. Coupling to $N = 1$ supergravity breaks this equivalence [4,20]. Loosely speaking, if the direction of supergravity is "aligned" with the FI terms, one obtains a D-term model. If it is misaligned, F-term or P-term inflation results. The higher dimensional origin of the FI terms leads us to suppose that the breaking of supergravity is triggered by the FI terms themselves and therefore aligned with them [21], so the four-dimensional model is a D-term model.

The full matter Lagrangian and supersymmetry transformations can be found in Refs. [24,25]. After eliminating auxiliary fields, the bosonic part reads:

$$\mathcal{L} = \tfrac{1}{2}|D_\mu \phi_+|^2 + \tfrac{1}{2}|D_\mu \phi_-|^2 + \tfrac{1}{2}|D_\mu \tilde{\phi}_+|^2 + \tfrac{1}{2}|D_\mu \tilde{\phi}_-|^2 + \tfrac{1}{2}|\partial_\mu S|^2 - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - V, \quad (1)$$

with the tree-level scalar potential V given by

$$V = \frac{g^2}{2}\Big\{\frac{1}{4}[|\phi_+|^2 + |\tilde{\phi}_+|^2 - |\phi_-|^2 - |\tilde{\phi}_-|^2 - \omega^2]^2 + |\phi_+\phi_- - \tilde{\phi}_-\tilde{\phi}_+|^2 + S^2[|\phi_+|^2 + |\phi_-|^2 + |\tilde{\phi}_+|^2 + |\tilde{\phi}_-|^2]\Big\}, \quad (2)$$

where A_μ is the U(1) gauge field, $D_\mu = \partial_\mu + i g_\pm A_\mu$, and $g_\pm = \pm g$ is the charge of the $\phi_\pm, \tilde{\phi}_\pm$ fields. From the $N = 2$ point of view, these belong to two hypermultiplets of opposite charge, $h_1 = (\phi_+, \phi_-^*)$ and $h_2 = (\tilde{\phi}_-, \tilde{\phi}_+^*)$. Note the *extra* accidental SU(2) symmetry between $(\phi_+, \tilde{\phi}_+)$ and also between $(\tilde{\phi}_-, \phi_-)$ due to the charge assignments. It will be important in what follows. This SU(2) is not broken when adding a FI term, whereas $SU(2)_R$ is.

The potential (3) has two different types of minima:

Supersymmetric minima: with $V = 0$, that is,

$$S = 0, \qquad |\phi_+\phi_- - \tilde{\phi}_+\tilde{\phi}_-| = 0, \qquad |\phi_+|^2 + |\tilde{\phi}_+|^2 - |\phi_-|^2 - |\tilde{\phi}_-|^2 = \omega^2. \quad (3)$$

Because of the FI term, gauge symmetry is broken and the hypersymmetric Higgs mechanism takes place; all fields acquiring mass have $m^2 = \omega^2 g^2$ since we are automatically in the Bogomol'nyi limit.

In the analogous model with only one hypermultiplet (only $\phi_\pm$), the vacuum manifold after inflation is simply connected and cosmic strings form via the Kibble mechanism. In our model, the vacuum manifold is not simply connected, but the condition of finite energy per unit length still correlates the phases of all scalars far from the string core and causes the quantization of magnetic flux. Moreover, the Bogomol'nyi equations force $\phi_- = \tilde{\phi}_- = 0$, so there is a vacuum selection effect [24–26].

There are BPS string solutions involving the remaining $\phi_+, \tilde{\phi}_+$ fields which are SL strings [7] [recall the SU(2) symmetry between them]. The stability properties and cosmological formation rates of SL strings are very different from those of NO strings (see below).

Nonsupersymmetric minima: given by

$$\phi_\pm = \tilde{\phi}_\pm = 0, \qquad S^2 > \frac{\omega^2}{2} = S_c^2. \quad (4)$$

These are local minima with potential $V = g^2\omega^4/8$, and S is a flat direction. Clearly, as the (false) vacuum energy is nonzero, all SUSYs are broken. SUSY breaking causes mass splitting within each hypermultiplet:

$$m_\pm^2 = S^2 g^2 \mp \frac{g^2}{2}\omega^2, \qquad m_\psi^2 = g^2 S^2, \quad (5)$$

where $m_\pm$ is the mass of field $\phi_\pm, \tilde{\phi}_\pm$, whereas m_ψ is the mass of their superpartner fermions.

Inflation.—Standard D-term inflation assumes a chaotic inflationary scenario [27], where S will have initial random values, and the regions where $S > S_c$ will inflate. During inflation, the system is in the false vacuum, supersymmetry is broken, and the potential gets corrections from the different masses of the bosons and fermions (5).

These corrections are known for one hypermultiplet [3,4,20,28]. Moreover, the cosmological predictions of P-term inflationary models were investigated in Refs. [4,20] and constrained by the WMAP data [1] in Ref. [29], showing that the predictions can be within the observed values, but problems arise with the cosmic strings formed afterwards.

The correction is easily generalized to C hypermultiplets ($C = 2$ here) of equal g^2 [30], and in the limit $S \gg S_c$ it can be approximated by

$$V_{\rm eff} = \frac{g^2}{8}\omega^4\left\{1 + \frac{Cg^2}{8\pi^2}\left[\ln\frac{S^2g^2}{\Lambda^2} + \frac{3}{2}\right]\right\}. \quad (6)$$

This potential is a hybrid inflation type potential [30]. We follow the standard procedures to get information about the cosmological predictions arising from this potential: The value of the inflaton when the cosmologically interesting scales leave the horizon S_N can be obtained from the number N of e-foldings:

$$N = \frac{1}{M_{\rm Pl}^2}\int_{S_{\rm end}}^{S_N} dS\frac{V}{V'}. \quad (7)$$

$S_{\rm end}$ represents the end of inflation. This will be well before gauge symmetry breaking takes place [31], because slow roll fails at $S \gtrsim S_c$. In any case, the precise value of $S_{\rm end}$ is not important in evaluating N, since the major contribution will come from S_N. In order to keep the theory under control, we need $S_N < M_{\rm Pl}$:

$$S_N \sim \sqrt{\frac{NCg^2}{2\pi^2}}M_{\rm Pl} \lesssim \tau M_{\rm Pl} \Rightarrow g \lesssim 0.1, \quad (8)$$

where we have used $N = 55$ [32], $C = 2$, and $\tau \sim 0.2$. From the Cosmic Background Explorer normalization, we can obtain that the energy scale S_c at which the gauge symmetry breaking will take place is similar to grand unified theory scale

$$S_c = \frac{\omega}{\sqrt{2}} \sim \sqrt{\frac{5.2\times10^{-4}}{2\pi}}\left(\frac{C}{N}\right)^{1/4}M_{\rm Pl} \sim 10^{16}\ {\rm GeV}. \quad (9)$$

The slow-roll parameters (ε, η), the spectral index (n), and the relative amplitude of tensor to scalar perturbations (R) are within the phenomenological values [33]

$$\varepsilon = \frac{g^2C}{16\pi^2N} \lesssim 2\times10^{-6}, \qquad \eta = \frac{-1}{2N} \sim -0.009,$$
$$n = 1 - \frac{1}{N} \sim 0.98, \qquad R = 16\varepsilon \lesssim 3\times10^{-5}. \quad (10)$$

These numbers are not very different from those in Refs. [4,20], as expected, since some parameters are independent of C, and in others the difference between $C = 1$ and $C = 2$ is very mild. Thus, the good agreements with the cosmological parameters obtained for the one-hypermultiplet case also apply here. We now turn to the differences, which arise after gauge symmetry breaking.

Extended objects after inflation.—Once inflation is over, and the field S approaches the value S_c, the fields $\phi_\pm$, $\tilde{\phi}_\pm$ will roll down to the supersymmetric vacua (3). In the one-hypermultiplet case, NO cosmic strings will form via the Kibble mechanism. These defects create perturbations to the metric of the same order of magnitude as inflation [34]. This is a problem since observational CMB data [1] constrain the contribution of cosmic strings to less then a few percent of the total. Vortons will also form in this case, and their cosmological constraints will be much more stringent [35].

By contrast, with two hypermultiplets the defects are SL strings, whose stability depends on the ratio of the scalar (m_s) and vector (m_v) masses. The parameter $\beta = m_s^2/m_v^2$ (analogous to the κ parameter that distinguishes type I from type II superconductors) separates stable strings ($\beta < 1$) from unstable strings ($\beta > 1$).

The cosmological formation and evolution of a network of such strings was analyzed in Ref. [9] with the conclusion that no strings will form if $\beta \geq 1$. Here we are in the Bogomol'nyi limit, $\beta = 1$, so neither vortons nor strings are expected after inflation (other than perhaps a few transient strings that will quickly disappear). In fact, the solution to the Bogomol'nyi equations is a *one-parameter family* of magnetic vortices, degenerate in energy but with different core structure, with string "widths" ranging from the NO string to wider and wider vortices looking more and more like CP^1 lumps [8]. They are all BPS states, partially breaking SUSY in their core [25], and therefore stable. Nevertheless, there is a zero mode linking these states which, once excited, will invariably drive the vortices to flux tubes of greater and greater radius [36].

The zero mode plays a very important role in preventing the cosmological formation of the strings [25,36], so we should immediately worry about whether supergravity effects would lift this degeneracy. In the bosonic case, the zero mode survives coupling to gravity [37], and the same is true here: the fattened BPS states all carry the same U(1) flux and deficit angle as the NO string, so the holonomy cancellation [12] that is needed for unbroken supersymmetry still holds. Thus, coupling to supergravity will preserve all of these BPS states, and their degeneracy, for the same reason that it preserves their topological cousins in the minimal model [12,21]. As a result, the flat space analysis of Refs. [9,25] goes through and we do not expect strings to form at the end of inflation.

Scalar gradients in this model will also contribute to the CMB anisotropy. There are two main sources. One is the ϕ_-, $\tilde{\phi}_-$ fields, which have been observed to give a very small contribution to the energy density in the one-hypermultiplet case [38]. The other is the contribution of the fattened vortices in the ϕ_+, $\tilde{\phi}_+$ fields, which are more akin to (stabilized) textures. The CMB constraints on textures are much weaker than those on strings [39] but a more detailed study is needed, in particular, since textures can contribute extra power on large angular scales.

These conclusions depend on supersymmetry not being completely broken until much later; otherwise, the results depend on how the breaking proceeds. If SUSY is broken due to soft mass terms, there will be several scenarios depending on the masses [38]: the strings may remain SL, they can turn into NO vortices, or strings will disappear. Also, a vorton problem may arise, depending on the

nature of the breaking and the detailed dynamics of the zero mode, if the latter survives [25,35].

For completeness, we give an estimate of the CMB signal deep in the stability region $\beta < 1$ assuming that the SU(2) symmetry is not broken. In this case, SL strings can form with a density comparable to NO strings, and scaling behavior is expected, but the reduced mass per unit length also weakens the CMB signal [40]. The net result should be a contribution to the CMB anisotropy, which is a few percent (2%–5%) of that of NO strings, and fairly insensitive to β.

Finally, the connection with superstrings leaves open a number of interesting questions. First, the FI parameters are expected to be spacetime dependent, and such cosmological models have not been investigated to our knowledge. Second, if several vector multiplets are present (as is the case in most brane and conifold models), inflation could be driven by multiple scalar fields, and the situation could be more complicated [5].

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