Superfluidity of Trapped Dipolar Fermi Gases

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We derive the phase diagram for ultracold trapped dipolar Fermi gases. Below the critical value of the dipole-dipole interaction energy, the BCS transition into a superfluid phase ceases to exist. The critical dipole strength is obtained as a function of the trap aspect ratio. Alternatively, for a given dipole strength there is a critical value of the trap anisotropy for the BCS state to appear. The order parameter exhibits a novel nonmonotonic behavior at the criticality.

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One of the most challenging goals of modern atomic, molecular, and optical physics is to observe the superfluid (BCS) transition [1] in trapped Fermi gases. The possibility of such a transition for gases with attractive short-range interactions has been predicted in Ref. [2] and has been the subject of very intensive experimental investigations since then (for the latest experimental results, see [3]). In typical experiments evaporative cooling is used to cool fermions. However, since the Pauli principle forbids the s-wave scattering for fermions in the same internal state, Fermi-Fermi [4] or Fermi-Bose [5] mixtures have to be used to assure collisional thermalization of the gas. Such a combination of evaporation and sympathetic cooling allows us to reach temperatures $T \simeq 0.1T_F$, where T_F is the Fermi temperature at which the gas exhibits quantum degeneracy. Unfortunately, critical temperatures for the BCS transition, T_c, are predicted to be much smaller than T_F. Nowadays, the standard way to overcome this difficulty is to increase T_c by employing a Feshbach resonance and by modifying the atomic scattering length a_s to large negative values. Such "resonance superfluidity" should lead to superfluid transition at $T_c \approx 0.1T_F$ [6]. Another way to achieve the BCS regime is to use the cooling scheme that can overcome the Pauli blocking, such as appropriately designed laser cooling [7]. Yet another promising route is to place the Fermi gas in an optical lattice and enter the "high T_c" regime [8].

The temperature of the BCS transition in a two-component Fermi gas depends dramatically on the difference of concentrations of the two components, which presents another experimental obstacle [9]. This problem, however, is not relevant for a polarized Fermi gas with long-range interactions, such as dipole-dipole ones. For this reason there has been recently a considerable interest in the BCS transition in dipolar Fermi gases. The possibility of the Cooper pairing has been predicted in Ref. [10]. The critical temperature (including many-body corrections) and the order parameter for a homogeneous gas have been obtained in Ref. [11]. It is worth mentioning that possible realizations of dipolar gases include ultracold heteronuclear molecular gases [12], atomic gases in a strong dc electric field [13], atomic gases with laser-induced dipoles [14], or with magnetic dipoles [15]. For dipolar moments d of the order of 1 D and densities n of 10^{12} cm^{-3}, T_c should be in the range of 100 nK, i.e., experimentally feasible.

Dipole-dipole interaction is not only of long range but also anisotropic, i.e., partially attractive and partially repulsive. Thus, the nature of the interaction for trapped gases may be controlled by the geometry of the trap. For a dipolar Bose gas in a cylindrical trap with the axial (radial) frequency $\omega_z(\omega_\rho)$, there exists a critical aspect ratio $\lambda = (\omega_z/\omega_\rho)^{1/2}$, above which the Bose-condensed gas collapses if the atom number is too large [16], and below which the condensate exhibits the roton-maxon instability [17]. The trap geometry is also expected to control the physics of trapped dipolar Fermi gases. So far, however, only partial results are known [18]: analytic corrections to T_c in "loose" traps, solution of the case of an infinite "slab" with $\omega_\rho = 0$, and ω_z finite. In the latter case there exists a critical frequency above which the superfluid phase does not exist.

In this Letter we solve the fundamental problem of the effects of trap geometry on the BCS transition in trapped dipolar Fermi gases. We calculate the phase diagram in the plane $\Gamma - \lambda^{-1}$, where $\Gamma = 36nd^2/\pi\mu$ is the strength of the dipole-dipole interaction relative to the chemical potential μ. Below the critical value of the interaction, $\Gamma < \Gamma_c$, the BCS transition does not take place. Similarly, for a given dipole interaction strength there is a critical value of λ^{-1}, above which the BCS state appears. We determine the dependence of Γ_c on λ^{-1} and calculate the order parameter at the criticality.

A dipolar Fermi gas in a cylindrically symmetric trap is described by the Hamiltonian

$$\hat{H} = \int_{\mathbf{r}} \hat{\psi}^\dagger(\mathbf{r}) \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{trap}}(\mathbf{r}) - \mu \right] \hat{\psi}(\mathbf{r}) + \frac{1}{2}\int_{\mathbf{r},\mathbf{r}'} \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r}')V_{\text{dip}}(\mathbf{r}-\mathbf{r}')\hat{\psi}(\mathbf{r}')\hat{\psi}(\mathbf{r}), \quad (1)$$

where m is the mass of atoms, $V_{\text{trap}}(\mathbf{r}) = m[\omega_\rho^2(x^2+y^2) + \omega_z^2 z^2]$ is the trapping potential, μ is the chemical potential, and $V_{\text{dip}}(\mathbf{r}) = (d^2/r^3)(1 - 3z^2/r^2)$. The dipoles are

assumed to be polarized along the z direction, and $\hat{\psi}^\dagger(\mathbf{r})$ and $\hat{\psi}(\mathbf{r})$ are the atomic creation and annihilation operators that fulfill the canonical anticommutation relations. Our goal is to apply the BCS theory (see, e.g., [1]) to the system described by Eq. (1) and calculate the critical temperature and the superfluid order parameter $\Delta(\mathbf{r}, \mathbf{r}') = V_{\mathrm{dip}}(\mathbf{r} - \mathbf{r}')\langle\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r}')\rangle$.

As shown in Ref. [11], the BCS pairing is dominated by the p-wave scattering with zero projection of the angular momentum on the z axis, $l_z = 0$, in which the interaction is attractive. Contributions of higher angular momentum, although present due to the long-range character of the dipole-dipole interaction, are numerically small (see also Ref. [10]). In the p-wave channels with $l_z = \pm 1$ the interaction is repulsive and, therefore, leads only to renormalizations of a Fermi-liquid type and is neglected here. Therefore, for the considered pairing problem we can model the dipole-dipole interaction by the following (off-shell) scattering amplitude:

$$\Gamma_1(\mathbf{p}, \mathbf{p}', E) = p_z p_z' \tilde{\gamma}_1(E), \tag{2}$$

where $\mathbf{p}$ is the incoming momentum, $\mathbf{p}'$ is the outgoing one, and $\tilde{\gamma}_1(E)$ is some function of the energy E. The amplitude Γ_1 describes anisotropic scattering only in the p-wave channel with the projection of the angular momentum $l_z = 0$. The function $\tilde{\gamma}_1(E)$ obeys the equation

$$\tilde{\gamma}_1(E) - \tilde{\gamma}_1(E') = \int^{\Lambda} \frac{d\mathbf{p}}{(2\pi)^3} \tilde{\gamma}_1(E) \left\{ \frac{p_z^2}{p^2 - E + i0} - \frac{p_z^2}{p^2 - E' + i0} \right\} \tilde{\gamma}_1(E'), \tag{3}$$

that follows from the Lipmann-Schwinger equation for the off-shell scattering amplitude [19]; $\tilde{\gamma}_1(E)$ is assumed to be negative in order to guarantee the BCS pairing. The cutoff parameter Λ ensures the convergence of the integral and, in fact, can be expressed in terms of the observable scattering data corresponding to the on-shell scattering amplitude with $p = p'$ and $E = p^2/m$. It follows from Eq. (3) that $\tilde{\gamma}_1(E)$ is inversely proportional to E, $\tilde{\gamma}_1(E) = \gamma_1(2mE)^{-1}$, with some coefficient γ_1. Therefore, the on-shell amplitude is energy independent, as it should be for low-energy scattering on the dipole-dipole potential (see Ref. [20]).

The coefficient γ_1 determines the value of the critical temperature T_c of the BCS transition in a spatially homogeneous gas and can be expressed through the dipole moment d using the results of Ref. [11]. In a homogeneous gas, the order parameter has the form $\Delta(\mathbf{p}) = p_z \Delta_0$ with some constant Δ_0, and the linearized gap equation reads

$$\frac{1}{\tilde{\gamma}_1(\mu)} = -\int \frac{d\mathbf{p}}{(2\pi)^3} \frac{p_z^2}{2\xi_p} \left[\tanh\frac{\xi_p}{2T_c} - 1 \right], \tag{4}$$

where $\xi_p = p^2/2m - \mu$; the bare interaction is renormalized in terms of the scattering amplitude with $\tilde{\gamma}_1(\mu) = \gamma_1/p_F^2$ at the Fermi energy $\varepsilon_F = \mu = p_F^2/2m$ along the lines of Ref. [21] (p_F is the Fermi momentum). After integrating over p we obtain the equation for T_c:

$$1 = \frac{1}{3}|\gamma_1|\nu_F \left[\ln\frac{2\mu}{T_c} - \frac{8}{3} - \ln\frac{\pi}{4} + C \right], \tag{5}$$

where $\nu_F = m p_F/2\pi^2$ is the density of states at the Fermi energy and $C = 0.5772$ is the Euler constant. Comparing the solution of Eq. (5) with the result of Ref. [11] for T_c, we find $\gamma_1 = -24d^2/\pi$.

In the ordinary space, the scattering amplitude Γ_1 is

$$\Gamma_1(\mathbf{r}, \mathbf{r}', E) = \partial_z \delta(\mathbf{r}) \partial_{z'} \delta(\mathbf{r}') \tilde{\gamma}_1(E), \tag{6}$$

where $\mathbf{r}$ and $\mathbf{r}'$ are the relative distances between the two incoming and outgoing particles, respectively. Therefore, the order parameter in the trapped gas, $\Delta(\mathbf{r}_1, \mathbf{r}_2) \sim \langle\psi(\mathbf{r}_1)\psi(\mathbf{r}_2)\rangle$, has the form $\Delta(\mathbf{r}_1, \mathbf{r}_2) = \partial_z \delta(\mathbf{r}) \Delta_0(\mathbf{R})$, where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, and the corresponding equation for the critical temperature is

$$\frac{\Delta_0(\mathbf{R})}{\tilde{\gamma}_1(\mu)} = -\int_{\mathbf{R}'} \left[\sum_{\mathbf{n}_1, \mathbf{n}_2} M_{\mathbf{n}_1\mathbf{n}_2}(\mathbf{R}) M_{\mathbf{n}_1\mathbf{n}_2}(\mathbf{R}') \frac{\tanh(\xi_1/2T) + \tanh(\xi_2/2T)}{2(\xi_1 + \xi_2)} - \int \frac{d\mathbf{p}}{(2\pi)^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{p_z^2}{2\xi_p + q^2/4m} \exp[i\mathbf{q}(\mathbf{R} - \mathbf{R}')] \right] \Delta_0(\mathbf{R}'). \tag{7}$$

Here $\xi_i = \xi(\mathbf{n}_i)$, $\mathbf{n} = (n_x, n_z, n_z)$ are the harmonic oscillator quantum numbers, $\xi(\mathbf{n}) = \hbar[\omega_z(n_z + 1/2) + \omega_\rho(n_x + n_y + 1)] - \mu$, and $M_{\mathbf{n}_1\mathbf{n}_2}(\mathbf{R}) \equiv M^{(z)}_{n_{1z}n_{2z}}(z) \times M^{(\rho)}_{n_{1x}n_{2x}}(x) M^{(\rho)}_{n_{1y}n_{2y}}(y)$ with $M^{(z)}_{n_1n_2}(z) = \frac{1}{2}[\varphi_{n_1}(z)\partial_z\varphi_{n_2}(z) - \varphi_{n_2}(z)\partial_z\varphi_{n_1}(z)]$, $M^{(\rho)}_{n_1n_2}(x) = \varphi_{n_1}(x)\varphi_{n_2}(x)$, and similar expression for $M^{(\rho)}_{n_1n_2}(y)$; φ_n denote the harmonic oscillator wave functions.

The gap equation (7) is still hardly tractable numerically. We thus employ additional approximations. We assume a large number of particles such that the chemical potential μ is much larger than the trap frequencies, $\mu \gg \omega_z, \omega_\rho$. Therefore, while calculating the functions $M_{\mathbf{n}_1\mathbf{n}_2}(\mathbf{R})$, we can use the WKB approximation for the wave functions φ_n of the states with energies near the Fermi energy, which are the most important for the BCS pairing. Another simplification is due to the fact that the BCS order parameter $\Delta_0(\mathbf{R})$ varies slowly on an interparticle distance scale $n^{-1/3} \sim \hbar/p_F$, where $p_F = \sqrt{2m\mu}$ is now the Fermi momentum in the center of the trap in

the Thomas-Fermi approximation. As a result, the pairing correlations are pronounced only between states that are close in energy, and one can neglect $q^2/4m$ in the denominator of the second term in Eq. (7), as well as rapidly oscillating terms in the functions $M_{\mathbf{n}_1\mathbf{n}_2}(\mathbf{R})$. These functions become then proportional to the Chebyshev polynomials $U_{n-1}(z/l_{zN})$, $T_n(x/l_{\rho N})$, and $T_n(y/l_{\rho N})$, where $l_{iN} = \sqrt{2N\hbar/n\omega_i} = l_{0i}\sqrt{2N}$.

The critical aspect ratio λ_c corresponds to vanishing critical temperature. We therefore calculate the gap equation in the limit $T \ll \omega_i$, using the ansatz $\Delta_0(\mathbf{R}) \to \Delta_0(\mathbf{r}) = \Delta_0(zR_{\mathrm{TF}}^{(z)}, xR_{\mathrm{TF}}^{(\rho)}, yR_{\mathrm{TF}}^{(\rho)})$, where x, y, z are now dimensionless $|x|, |y|, |z| \le 1$, and $R_{\mathrm{TF}}^{(i)}$ denote the radius of the gas cloud in the i direction, calculated in the Thomas-Fermi approximation, $R_{\mathrm{TF}}^{(i)} = p_F/m\omega_i$. Tedious analytic calculation leads then to

$$\frac{3}{\Gamma}(1-r^2)\Delta_0(\mathbf{r}) = (1-r^2)^{3/2}\left\{\ln\frac{2\mu(\mathbf{r})}{\omega_z} - \frac{2}{3}(4-\ln 4)\right\} \times \Delta_0(\mathbf{r}) - \frac{3\pi^2}{2}\int_{\mathbf{r}}' K(\mathbf{r},\mathbf{r}')\Delta_0(\mathbf{r}'), \quad (8)$$

where $\Gamma = |\gamma_1|\nu_F$, $\mu(\mathbf{r}) = \mu - V_{\mathrm{trap}}(\mathbf{r})$, and

$$K(\mathbf{r},\mathbf{r}') = \sum_{n_z>0;n_x,n_y\ge 0} \delta_{n_x}\delta_{n_y} \ln\left[n_z + \frac{\omega_\rho}{\omega_z}(n_x+n_y)\right] \int_{M_z}^1 d\zeta \int_{M_x}^1 d\alpha \int_{M_y}^1 d\beta\, \delta(1-\zeta-\alpha-\beta)\frac{4}{\pi^2}\frac{\sqrt{(\zeta-z^2)(\zeta-z'^2)}}{\zeta} \times U_{n_z-1}\left(\frac{z}{\sqrt{\zeta}}\right)U_{n_z-1}\left(\frac{z'}{\sqrt{\zeta}}\right)\frac{4}{\pi^2}\frac{1}{\sqrt{(\alpha-x^2)(\alpha-x'^2)}}T_{n_x}\left(\frac{x}{\sqrt{\alpha}}\right)T_{n_x}\left(\frac{x'}{\sqrt{\alpha}}\right)\frac{4}{\pi^2}\frac{1}{\sqrt{(\beta-y^2)(\beta-y'^2)}} \times T_{n_y}\left(\frac{y}{\sqrt{\beta}}\right)T_{n_y}\left(\frac{y'}{\sqrt{\beta}}\right), \quad (9)$$

with $\delta_n = 2$ for $n > 0$, $\delta_0 = 1$, and $M_s = \max(s^2, s'^2)$ for $s = x, y, z$. The above equation can be viewed as the equation for an extremum of the quadratic form

$$F[\Delta_0] = \frac{1}{2}\int_{\mathbf{r},\mathbf{r}'} \Delta_0(\mathbf{r})[L(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') - K(\mathbf{r},\mathbf{r}')]\Delta_0(\mathbf{r}'),$$

where $L(\mathbf{r}) = 3(1-r^2)/\Gamma - (1-r^2)^{3/2}\{\ln[2\mu(\mathbf{r})/\omega_z] - 2(4-\ln 4)/3\}$. The extremum can be found numerically using the ansatz

$$\Delta_0(\mathbf{r}) = (1-r^2)^{3/2}\sum_{m_z,m_\rho} c_{m_z m_\rho} U_{m_z}(z^2)T_{m_\rho}(x^2+y^2),$$

with $m_z, m_\rho \ge 0$. The form F becomes now a quadratic form of the unknown coefficients $c_{m_z m_\rho}$, and the extremum corresponds to the eigenvector of the matrix $M_{m_z,m_\rho,n_z,n_\rho}$ of the quadratic form with the smallest eigenvalue, which tends to zero at criticality, where the interaction Γ and the trap frequencies ω_i obey a certain constraint. The parameter Γ can be written as $\Gamma = 36n(0)d^2/\pi\mu$, where $n(0) = (2m\mu)^{3/2}/6\pi^2\hbar^3$ is the gas density in the center of the trap. Hence, for a given dipole moment d, Γ depends only on the chemical potential μ. On the other hand, μ and the total number of particles N in the trap are related via $3N = \mu^3/\omega_z\omega_\rho^2$. Therefore, fixing of Γ and N determines the product of the trap frequencies $\omega_z\omega_\rho^2$, so that the only free parameter left is the trap aspect ratio λ. We may thus determine its critical value λ_c from the above mentioned constraint. The problem is therefore reduced to finding the set of coefficients $c_{m_z m_\rho}$ and the aspect ratio λ such that for a given Γ and N the lowest eigenvalue of the matrix $M_{m_z,m_\rho,n_z,n_\rho}$ of the quadratic form F is zero.

The calculation of the matrix elements $M_{m_z,m_\rho,n_z,n_\rho}$ is naturally divided into two parts [see Eq. (9)]. The local part with the kernel $L(\mathbf{r})$ can be easily computed using, for instance, the Monte Carlo integration routines, such as the VEGAS algorithm from the GNU Scientific Library [22]. The calculation of the nonlocal part with the kernel $K(\mathbf{r},\mathbf{r}')$ is more time-consuming, but one can considerably speed it up by using a two-dimensional spline interpolation method to interpolate the integrand for the last integrations over α and β.

The results are presented in two figures. Figure 1 shows the desired relation between Γ and the inverse aspect ratio λ^{-1}. The three curves correspond to three different numbers of particles. As could be expected, for the larger number of particles, the critical aspect ratio λ_c is smaller because the interaction is stronger. For a pancake trap, $\lambda^{-1} < 1$, the interaction is predominantly repulsive, and higher values of Γ for fixed λ are required to achieve the BCS transition. On the other hand, for a cigar trap, $\lambda^{-1} > 1$, the interaction is predominantly attractive and the BCS transition occurs at smaller values of Γ. The existence of

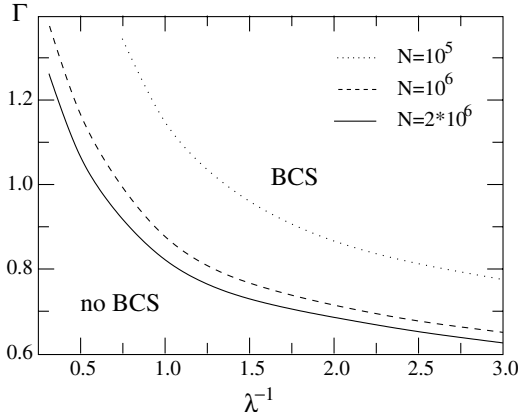

FIG. 1. The critical lines Γ_c versus the inverse aspect ratio λ^{-1} for different numbers of particles. The BCS pairing takes place above the depicted curves.

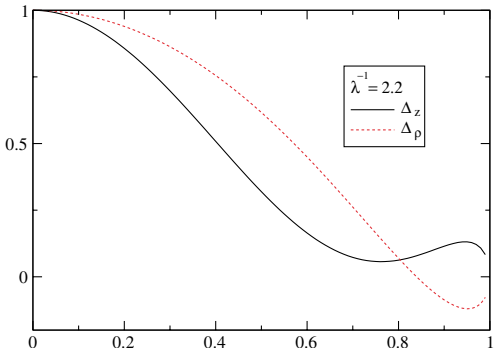

FIG. 2 (color online). The order parameter for the aspect ratio $\lambda = 0.45$ (cigar shape trap). The solid line shows $\Delta_0(z, \rho = 0)$, and the dashed line corresponds to $\Delta_0(z = 0, \rho)$.

the critical interaction strength (for a given value of λ) is a result of the discreteness of the spectrum in the trap and of the specific structure of the order parameter ($\sim p_z$). The latter allows pairing only between particles in the states, where quantum numbers n_z differ by an odd number (intershell pairing). Therefore, the pairing correlations have to be strong enough to overcome the corresponding energy difference.

Figure 2 shows the order parameter $\Delta_0(\mathbf{r})$ for the cigar trap with $\lambda^{-1} = 2.2$. Amazingly, the order parameter is a nonmonotonic function of the distance from the trap center, in contrast to the case of the BCS order parameter in a two-component Fermi gas with short-range interactions [23]. This effect persists, although being less pronounced, for the case of a pancake geometry. In the axial direction, the order parameter $\Delta(z, \rho = 0)$ develops a minimum at $\rho < 1$, whereas in the radial direction $\Delta(z = 0, \rho)$ becomes negative in the outer part of the cloud. This completely new behavior, originating from the anisotropy of the interpaticle interaction, can have profound consequences for the form and spectrum of the elementary excitations. We expect an appearance of excitations with wave functions concentrated in the inner region of the atomic cloud, where $\Delta \simeq 0$. This problem, however, goes beyond the scope of this Letter.

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