Coulomb Scattering in a 2D Interacting Electron Gas and Production of EPR Pairs

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We propose a setup to generate nonlocal spin Einstein-Podolsky-Rosen pairs via pair collisions in a 2D interacting electron gas, based on constructive two-particle interference in the spin-singlet channel at the $\pi/2$ scattering angle. We calculate the scattering amplitude via the Bethe-Salpeter equation in the ladder approximation and small r_s limit and find that the Fermi sea leads to a substantial renormalization of the bare scattering process. From the scattering length, we estimate the current of spin-entangled electrons and show that it is within experimental reach.

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In recent years, the spin degree of freedom of electrons has become of central interest in semiconductor research [1]. This is particularly so for spin-based quantum information processing, where the basic resources are Einstein-Podolsky-Rosen (EPR) pairs such as nonlocal spin singlets formed by two electrons that are spatially separated [2]. A number of recent publications have described ways to produce such spin-correlated two-electron states [3–5], as well as orbital entanglement [6]. Here we propose a new scheme based on electron flow in an open system that avoids experimental complications, such as coupling different materials or transport channels [3,5,6] or implementing small quantum dots with well-controlled Coulomb blockade effects [4]. Our concept is based on a two-particle interference mechanism that is well known from elementary scattering theory [7]: the cross section for two electrons in vacuum, given in terms of the scattering amplitude f by $\lambda_{S/T}(\theta) = |f(\theta) \pm f(\pi - \theta)|^2$, favors singlet (+) over triplet (−) states in the outgoing channel around the scattering angle $\theta = \pi/2$. Thus, in principle, two-particle scattering processes can be used to generate EPR pairs. However, in the context of solid state systems, the question immediately arises whether this scattering effect remains operational, and moreover observable, in the presence of a Fermi sea consisting of many interacting electrons, such as a typical two-dimensional electron gas (2DEG) formed in GaAs heterostructures. In this Letter, we will show that within Fermi liquid theory the answer is affirmative.

We focus on 2DEGs since these systems are promising candidates for the observation of such effects. Indeed, recent experiments [8] have demonstrated that in a 2DEG the flow of electrons as well as scattering off impurities can be controlled and monitored via atomic force microscopy (AFM) technology. Thus, we believe that a setup as shown in Fig. 1(a) is experimentally realizable and should allow the observation of the angular and density dependence of the scattering cross section. Once the EPR pairs are created, their singlet character can be tested [9] by a noise measurement in a beam-splitter configuration [2] or by tests of Bell inequalities [10]. We note that the experimental observation of this type of entanglement would provide support for the applicability of Fermi liquid theory to 2D systems, which has been questioned by Anderson [11]. Indeed, one can hardly imagine singlet pairs of particles that are separated by mesoscopic distances without well-defined fermionic quasiparticles.

For electrons incident from unpolarized sources, we expect the ratio of singlets $|S\rangle$ to triplets $|T_\mu\rangle$ ($\mu = 0, \pm$) to be 1:3, this mixed state being described by the density matrix $1/4|S\rangle\langle S| + 1/4\sum_\mu |T_\mu\rangle\langle T_\mu|$. Our goal is to calculate the ratio of scattered triplets to scattered

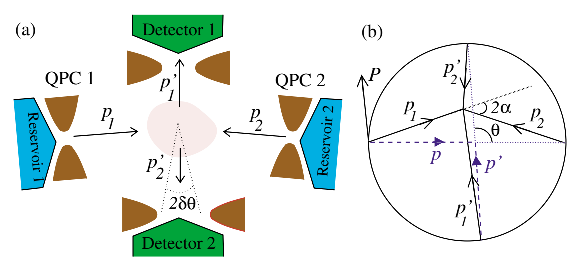

FIG. 1 (color online). (a) Proposed setup: two quantum point contacts filter electrons from two reservoirs with initial momenta $\mathbf{p}_1 \simeq -\mathbf{p}_2$. The two detectors (with an aperture angle $2\delta\theta$) are placed such that only electrons that collide (shaded area) at a scattering angle around $\pi/2$ are registered. Because of interference, the scattering amplitude vanishes at $\pi/2$ for the spin-triplet states, allowing only the spin-entangled singlets to be collected: one electron of the singlet state in detector 1 and its partner in detector 2. The scattering cross section and the electron flux could be measured via an AFM tip [8]. (b) Scattering parameters: $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_1' + \mathbf{p}_2'$ is the total momentum, $\mathbf{p} = (\mathbf{p}_1 - \mathbf{p}_2)/2$ and $\mathbf{p}' = (\mathbf{p}_1' - \mathbf{p}_2')/2$ are the relative momenta, and $\theta = \angle(\mathbf{p}, \mathbf{p}')$ is the scattering angle between them. The initial ($\mathbf{p}_1, \mathbf{p}_2$) and final ($\mathbf{p}_1', \mathbf{p}_2'$) momenta are connected by a circle of radius $p = p'$ due to energy and momentum conservation.

singlets for an aperture angle $2\delta\theta$ of the detectors around $\theta = \pi/2$. For small $\delta\theta$ this ratio is $R \simeq \delta\theta^2 |f'/f|^2_{\theta=\pi/2}$, where f is the many-body scattering amplitude and $f' = df/d\theta$. Solving the Bethe-Salpeter equation in the small r_s limit, we shall find that $|f'/f|^2 \sim 1$ at $\theta = \pi/2$. This means that for small $\delta\theta$ the scattering process dominantly produces singlets in the direction of detectors 1 and 2; see Fig. 1(a). As an experimental check, this ratio can be increased by reducing the amount of singlets in the incoming channels, which in turn can be achieved, e.g., by using spin polarized electron sources or devices that act as spin filters, such as a quantum dot [12] or a quantum point contact [13].

Setup.—We begin with a description of the setup, shown in Fig. 1(a). Electrons escaping from thermal reservoirs with momenta $\mathbf{p}_1 \simeq -\mathbf{p}_2$ are filtered by two quantum point contacts (QPC) and injected into a 2DEG where they scatter off each other. To collect after the collision only the entangled singlets (EPR pairs), we place two detectors such that only collisions of electrons with final momenta $\mathbf{p}_1'$, $\mathbf{p}_2'$ and with scattering angle $\theta \in [\pi/2 - \delta\theta, \pi/2 + \delta\theta]$ are registered. Below we estimate the expected singlet flux (current).

As seen from Fig. 1(b), the energies of both incident electrons need to be known, in general, to determine the scattering angle $\theta \simeq \angle(\mathbf{p}', \mathbf{p})$. However, for the special case with opposite momenta $\mathbf{p}_2 \simeq -\mathbf{p}_1$, θ is easily determined by $\theta \simeq \angle(\mathbf{p}_1', \mathbf{p}_1)$. Moreover, the energies are then individually conserved, $p_1 \simeq p_2 \simeq p_1' \simeq p_2'$ $(p_i = |\mathbf{p}_i|)$, which ensures that the outgoing scattering states are unoccupied $(p_{1,2}' > k_F)$. Finally, to have well-defined quasiparticle states with long lifetimes [14], we assume the electrons to be injected with small excitation energies $\xi_i = \hbar^2 p_i^2/2m - E_F \ll E_F$, where $E_F = \hbar^2 k_F^2/2m$ is the Fermi energy and m the effective mass.

Scattering t matrix.—Let us evaluate the scattering t matrix for two electrons in the presence of the Fermi sea. The condition $\mathbf{p}_2 = -\mathbf{p}_1$ defines the Cooper channel, and thus we can follow the work by Kohn and Luttinger on interaction-induced superconductivity in a 3D Fermi liquid [14]. In contrast to Ref. [14], we consider here a 2D system where the screened Coulomb potential is nonanalytic. Moreover, we need the complete angular dependence of the scattering amplitude—rather than only the asymptotic of its Legendre or Fourier coefficients. The t matrix can be obtained from the (direct) vertex Γ governed by the Bethe-Salpeter equation [14,15]

$$\Gamma(\tilde{p}', \tilde{p}; \tilde{P}) = \Lambda(\tilde{p}', \tilde{p}; \tilde{P}) + \frac{i}{\hbar(2\pi)^3} \int d\tilde{k} \Lambda(\tilde{k}, \tilde{p}; \tilde{P}) G(\tilde{k}_1) G(\tilde{k}_2) \Gamma(\tilde{p}', \tilde{k}; \tilde{P}), \tag{1}$$

where G is the single-particle Green function and Λ is the irreducible vertex (we suppress the spin indices as spin is conserved). Here $\tilde{k}_{1,2} = \tilde{P}/2 \pm \tilde{k}$ and $\tilde{p}_i = (\mathbf{p}_i, \omega_i)$, with the frequencies ω_i. The full vertex for singlet/triplets contains the direct and exchange parts, i.e., $\Gamma(\tilde{p}', \tilde{p}; \tilde{P}) \pm \Gamma(-\tilde{p}', \tilde{p}; \tilde{P})$. From the vertex we obtain the t matrix: $t = \Gamma(\omega_1 + \omega_2 \to \xi_1 + \xi_2)$ [15], and from it the scattering length $\lambda = |f|^2$ via the 2D scattering amplitude $f = -tm/\hbar^2\sqrt{2\pi p}$ [16]. The scattering lengths for singlet/triplets are then given by $\lambda_{S/T}(\theta) = |f(\theta) \pm f(\theta - \pi)|^2$.

The bare Coulomb interaction in 2D is given by [17] $V_C(\mathbf{q}) = 2\pi e_0^2/q$, with $e_0^2 = e^2/4\pi\epsilon_0\epsilon_r$ and the dielectric constant ϵ_r. In a first stage, we use the RPA approximation to account for screening by the Fermi sea [15]; this yields a renormalized G and a screened interaction $V(\tilde{q}) = V_C(q)/[1 - V_C(q)\chi^0(\tilde{q})]$ given in terms of the bubble susceptibility diagram χ^0, where $\tilde{q} = (\mathbf{q}, \omega_q) = \tilde{p}' - \tilde{p}$ is the momentum transfer. We can consider [18] the usual static Thomas-Fermi screening $V(q) = 2\pi e_0^2/(q + k_s)$, with the screening momentum $k_s = 2me_0^2/\hbar^2$ and $r_s = k_s/k_F\sqrt{2}$. Note that RPA requires a high density, $r_s \ll 1$.

In addition to the single interaction line $V(q)$, the irreducible vertex Λ contains, in lowest order in V, two diagrams: the crossed diagram and the "wave function modification," shown in Figs. 2(c) and 2(d) of Ref. [14]. It is easy to see that these are smaller than V by a factor r_s [19]. Altogether, this justifies the ladder approximation, which consists in keeping only the single interaction line in $\Lambda \simeq V$. Note that the ladder approximation requires $\lambda k_F \ll 1$ [15]. This condition is consistent with RPA, e.g., in the Born approximation $t \simeq V_C \Rightarrow \lambda k_F \sim [V_C(k_F)m]^2/2\pi\hbar^4 \sim (k_s/k_F)^2 \sim r_s^2$. Finally, we can approximate G by the free propagator G_0 [14]. We start with the zero temperature $T = 0$ case and discuss finite T later.

Before solving Eq. (1) to all orders, we first consider its second-order iteration $\Gamma^{(2)}(\mathbf{p}', \mathbf{p}; \tilde{P}) = V(\mathbf{p}' - \mathbf{p}) + i/\hbar(2\pi)^3 \int d\tilde{k} V(\mathbf{k} - \mathbf{p}) G(\tilde{k}_1) G(\tilde{k}_2) V(\mathbf{p}' - \mathbf{k})$. The ω_k integration of the Green functions yields [15]

$$\frac{i}{2\pi\hbar} \int d\omega_k G_0(\mathbf{k}_1, \Omega/2 + \omega_k) G_0(\mathbf{k}_2, \Omega/2 - \omega_k) = \frac{N(k_1, k_2)}{\hbar\Omega - \xi_{k_1} - \xi_{k_2} + 2i\eta N(k_1, k_2)} =: D(k_1, k_2), \tag{2}$$

with $N(k_1, k_2) = 1 - n(k_1) - n(k_2)$, $n(k) = \Theta(-\xi_k)$, and the $\tilde{P}$ frequency is $\hbar\Omega \to \xi_1 + \xi_2$. We now take $\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2 \Rightarrow \mathbf{k} = \mathbf{k}_1 = -\mathbf{k}_2$ and $q = |\mathbf{p}' - \mathbf{p}| = 2k_F|\sin\theta/2|$. This yields a single discontinuity in the numerator when $\xi_k = 0$, which coincides with the zero of the denominator at $\xi_k = \xi$ for incident electrons with vanishing excitation energies $\xi = \xi_{1,2} \to 0$. Thus, the main contribution to the energy integration comes from virtual states at the Fermi surface, i.e., $\xi_k \simeq 0$. We set $k = k_F$ in V and integrate only on $D(k, k)$. Writing $\nu := (1/2\pi) \int_0^\infty dk k D(k, k)$, we obtain

$$\nu \simeq \frac{m}{2\pi\hbar^2} \log\frac{\xi}{E_F}. \tag{3}$$

For finite temperatures with $\xi \ll k_B T \ll E_F$, the occupation function $n(k) = (1 + e^{\xi_k/k_BT})^{-1}$ cuts the log divergence and ξ is replaced by k_BT in Eq. (3). This logarithmic divergence reveals a 2D Cooper singularity (see below) very much like in 3D [15]. Next, we examine the situation slightly away from the Cooper channel, i.e., for nonvanishing total momentum. Indeed, for experimental reasons it is preferable to have a small but finite angle $2\alpha = \angle(\mathbf{p}_1, -\mathbf{p}_2)$ between the incident particles to avoid flux misalignment with no collision at all. In this case, at $T = 0$ and for $p_1 = p_2$, we find that $N(k, \phi) \simeq \Theta(k - k_F - p\alpha|\sin\phi|) - \Theta(k_F - k - p\alpha|\sin\phi|)$ depends on the integration angle $\phi = \angle(\mathbf{k}, \mathbf{p})$, and, as a consequence, we find now $\nu(\phi) \simeq (m/2\pi\hbar^2)\log(2\alpha|\sin\phi|)$ in the limit $p \to k_F$. However, we recall that both outgoing scattering states must be unoccupied, which at $T = 0$ requires $p'_{1,2} > k_F$. For $\pi/2$ scattering, $p'_{1,2} = p_1(\cos\alpha \pm \sin\alpha) \Rightarrow p_1 \gtrsim k_F(1 + \alpha)$; see Fig. 1(b). Thus, $\alpha < \xi/E_F$ and the relevant cutoff in Eq. (3) remains ξ or k_BT.

We repeat this procedure in each order in V and rewrite Eq. (1) as $t(\theta) = v(\theta) + (\nu/2\pi)\int d\phi v(\phi)t(\theta - \phi)$, where $v(\phi) = 2\pi e^2/(2k_F|\sin\phi/2| + k_s)$ is the potential V at the Fermi surface. To solve this equation, we expand v and t into Fourier series: $v(\phi) = \sum_n v_n e^{in\phi}$, etc. We finally get, for the t matrix at the Fermi surface,

$$t(\theta) = \sum_n \frac{v_n}{1 - \nu v_n} e^{in\theta}, \tag{4}$$

with the Fourier coefficients

$$v_n = \frac{4e_0^2}{k_F \cos\gamma} \sum_{\mathrm{odd}\, m \geq 1} \frac{\cos(m\gamma)}{2n + m} \tag{5}$$

and $\gamma = \arcsin(r_s/\sqrt{2})$. Below, we use this result to evaluate the scattering length.

Scattering length.—To illustrate how the scattering process gets renormalized by the Fermi sea, we compare the above result (4) with the t matrix $|t_C(\theta)| = e_0\hbar[\tanh(\pi m e_0^2/k_F\hbar^2)\pi/mk_F\sin^2(\theta/2)]^{1/2}$ obtained for the bare Coulomb potential V_C in 2D [16]. We use typical parameters for a GaAs 2DEG: $\epsilon_r = 13.1$, $r_s = 0.86$, and a sheet density $n = 4 \times 10^{15}\ \mathrm{m}^{-2}$ [17], and we assume $\xi < k_BT = 10^{-2}E_F$ ($T = 2$ K). In Fig. 2(a) we plot the scattering length $\lambda(\theta) = |tm/\hbar^2\sqrt{2\pi k_F}|^2$ as function of the scattering angle θ (without antisymmetrization). The reduction in amplitude due to the Fermi sea is seen to be quite substantial (compared to the bare t_C), which can be traced back to the relatively large screening k_s ($r_s = 0.86$). We also have $t \to 0$ as k_BT, $\xi \to 0$.

The higher-order terms appearing in the iteration of the Bethe-Salpeter equation further reduce the scattering (compare t to the first order V). For large T or away from the Cooper channel ($\alpha \sim 1$), the logarithmic divergence ν disappears, and the contribution of higher-order terms becomes negligible; this yields the Born approximation $t \simeq V$. For very small r_s, we can drop $\nu v_n \ll 1$ in Eq. (4)

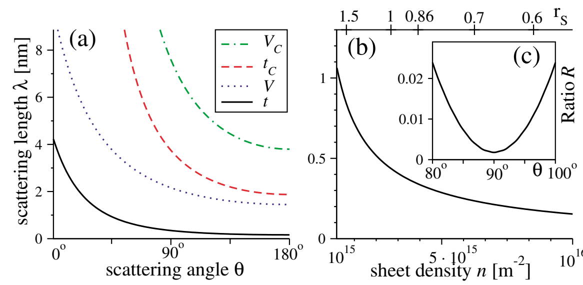

FIG. 2 (color online). Plots of scattering quantities obtained from the t matrix (4) for $r_s = 0.86$ (GaAs) and $k_BT/E_F = 10^{-2}$. (a) Angular dependence of the scattering length $\lambda(\theta)$. We compare λ to its Born approximation given by $t \simeq V$ and to the bare scattering of two particles (no Fermi sea) given by the exact result (t_C) or by the first order (V_C). (b) Dependence of $\lambda(\pi/2)$ on the sheet density n (see $r_s = me_0^2/\hbar^2\sqrt{\pi n}$ in top axis). (c) Ratio $R(\theta, \delta\theta)$ of triplets/singlets detected at a scattering angle θ for an aperture $\delta\theta = 5°$.

and find the bare potential, $t \simeq V_C$. In Fig. 2(b) we see the significant reduction of the scattering length λ as the density n is increased, which could be tested experimentally via a top gate.

Production of EPR pairs.—We now turn to the production of entangled electrons in the spin-singlet state. We consider detectors placed at θ, with a small aperture angle of $2\delta\theta$ (Fig. 1). We introduce the scattering lengths around θ for singlets and triplets, $\bar{\lambda}_{S/T}(\theta) = 2\int_{\theta-\delta\theta}^{\theta} d\theta'|f(\theta') \pm f(\pi - \theta')|^2$. A useful measure is the ratio between the number $N_{T/S}$ of detected triplets/singlet, $R(\theta, \delta\theta) = N_T/N_S = 3\bar{\lambda}_T/\bar{\lambda}_S$. Expanding for small $\delta\theta$ around $\theta = \pi/2$, we find in leading order $\bar{\lambda}_S \simeq 8|f(\pi/2)|^2\delta\theta$, and $\bar{\lambda}_T \simeq (8/3)|f'(\pi/2)|^2\delta\theta^3$, which yields

$$R(\pi/2, \delta\theta) \simeq \left|\frac{f'(\pi/2)}{f(\pi/2)}\right|^2 \delta\theta^2. \tag{6}$$

Using Eq. (4), we find $|f'/f| = 0.48$ at $\theta = \pi/2$ for GaAs, and note that this ratio remains of order unity for a wide parameter range $k_BT/E_F = 10^{-1}$–10^{-10}, and $r_s = 0.1$–1. Therefore this setup indeed allows the selection of singlets (EPR pairs) at detectors 1 and 2, provided the aperture angle is sufficiently small—even for electrons injected with some angular spread. For example, $R(90°, 5°) = 0.2\%$ or $R(85°, 5°) = 0.7\%$; see Fig. 2(c). The Born approximation $|V'/V| = 1/2(1 + r_s) \simeq 0.266$ is approached for $k_BT/E_F > 10^{-1}$.

To estimate the singlet current for a given input current I, we assume that the incident electrons occupy the lowest transverse mode in the QPC, giving plane waves of transverse width w (typically $w \simeq 100$ nm). The probability for the singlets to be scattered into the detectors is $P_S = (1/4)\bar{\lambda}_S/w = 0.06\%$, where $\bar{\lambda}_S = 0.24$ nm for $\delta\theta = 5°$. It is advantageous to inject simultaneously the two electrons from the reservoirs (e.g., by opening both QPCs simultaneously) [20]. Then the singlet current is given by $I_S = P_SI = 0.6$ pA for $I = 1$ nA. The total scattering length for unpolarized electrons is $\lambda_{\rm tot} = (1/4) \int_{-\pi/2}^{\pi/2} d\theta[\lambda_S(\theta) + 3\lambda_T(\theta)] = 3.4$ nm in GaAs (compared to 11 nm in Born approximation). This is consistent with the ladder approximation ($\lambda_{\rm tot} k_F = 0.53$) and yields the total scattering probability $P_{\rm tot} = \lambda_{\rm tot}/w = 3\%$.

It is crucial to count at detectors 1 and 2 only electrons that have scattered off each other at $\theta = \pi/2$, while avoiding the counting of uncorrelated electrons that are accidentally scattered into the detectors due to, e.g., impurities. This requirement could be fulfilled by coincidence measurements or with the help of an ac modulation applied to each reservoir with different frequencies ω_1 and ω_2. This would enable a frequency selection of the electrons that have interacted, as they are modulated by the two frequencies $\omega_1 \pm \omega_2$. We also note that the electrons diffracted at the exit of the QPCs could be refocused by a lensing effect [21] obtained via an appropriately shaped top gate.

Besides the observation of the scattering length and its density and angular dependence, our proposal could be further tested by adding a beam splitter to probe the singlet state via noise measurement [2,9], by performing tests of Bell's inequality [9,10] with spin filters [12,13], or by using *p-i-n* junctions [22] to transform singlets into entangled photon pairs. An alternative test requires spin filters, obtained, e.g., by tuning the QPCs into the spin-filtering regime [13] or by replacing them by spin-filtering quantum dots [12]. Then, with increasing spin polarization $\mathcal{P}$, the probability of incoming singlets $\rho_S = (1 - \mathcal{P}^2)/2$ is suppressed, and the singlet current at $\theta = \pi/2$ vanishes at full polarization $\mathcal{P} = 1$.

We now comment on the Kohn-Luttinger instability [14]. The crossed diagram—which in 2D can lead to an instability only for excitations with $q > 2k_F$ [23]—does not lead to a strong renormalization of the scattering vertex, as the associated temperature is infinitesimal, $k_B T/E_F \sim \exp(-10^3)$ [24]. This is larger than the value $\sim\exp(-10^5)$ found in 3D [14], despite the asymptotic decay for $v_n \sim n^{-2}$ being slower than in 3D, $v_l \sim e^{-l}$ (the decay is polynomial because the 2D potential is nonanalytic). The crossed diagram, given by the polarization propagator, has asymptotic $\sim n^{-3/2}$ instead of $\sim l^{-4}$ in 3D. Finally, we also checked that the repulsive electron-electron interaction is not appreciably affected by polar or acoustic phonons [24].

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