New Dark Energy Constraints from Supernovae, Microwave Background, and Galaxy Clustering

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Using supernova, cosmic microwave background, and galaxy clustering data, we make the most accurate measurements to date of the dark energy density ρ_X as a function of cosmic time, constraining it in a rather model-independent way, assuming a flat universe. We find that Einstein's simplest scenario, where $\rho_X(z)$ is constant, remains consistent with these new tight constraints and that a big crunch or big rip is more than 50 Gyr away for a broader class of models allowing such cataclysmic events. We discuss popular pitfalls and hidden priors.

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The nature of dark energy has emerged as one of the deepest mysteries in physics. When strong evidence for its existence first appeared from supernova observations in 1998 [1,2], the most pressing question was whether it was real or an observational artifact. Since then, the supernova evidence has both withstood the test of time and strengthened [3–5], and two other lines of evidence have independently led to the same conclusion: measurements of cosmological clustering with the cosmic microwave background (CMB) and large-scale structure (LSS) (e.g., [6,7]) and observation of CMB/LSS correlations due to the late integrated Sachs-Wolfe effect [8]. Now that its current density has been accurately measured [Wilkinson Microwave Anisotropy Probe+Sloan Digital Sky Survey gives $\rho_X(0) = (4.8 \pm 1.2) \times 10^{-27}$ kg/m^3 [7], corresponding to $(9.3 \pm 2.3) \times 10^{-124}$ in Planck units and $\Omega_\Lambda \approx 0.7$], the next pressing question is clearly whether its density ρ_X stays constant over time (like Einstein's cosmological constant) or varies. The latter is predicted by most models attempting to explain dark energy either as a dynamic substance, "quintessence" (e.g., [9]), or via some form of modified gravitational theory, perhaps related to extra dimensions or string physics (e.g., [10]). See Ref. [11] for reviews with more complete lists of references.

The recent discovery of 16 type Ia supernovae (SNe Ia) [5] with the Hubble space telescope during the Great Observatories Origins Deep Survey Advanced Camera for Surveys Treasury survey bears directly on this question. By discovering six out of the seven highest redshift SNe Ia known, all at $z > 1.25$, this search team [5] was able to pinpoint for the first time the transition epoch from matter domination to dark energy domination when the cosmic expansion began to accelerate. It is therefore timely to revisit this question regarding if and how the dark energy density varies with time. This is the goal of the present Letter. Given our profound lack of understanding of dark energy and the profusion of theoretical models in the recent literature, we focus on measuring the function $\rho_X(z)$ in as model independent a fashion as possible, emphasizing what we do and do not know given various assumptions about how $\rho_X(z)$ is parametrized, about data sets used, and about modeling thereof. We will see that the new data are powerful enough to make previous measurements of $\rho_X(z)$ (e.g., [12–14]) tighter and more robust and also to extend them back firmly into the epoch of cosmic deceleration.

Analysis technique.—We wish to measure the dimensionless *dark energy function*, $X(z) \equiv \rho_X(z)/\rho_X(0)$, the dark energy density in units of its present value. We do this as described in Ref. [13], fitting to SN Ia, CMB, and LSS information, obtaining the results shown in Fig. 1.

The measured distance-redshift relations of SNe Ia provide the foundation for probing the dark energy function $X(z)$. In a flat universe, the dimensionless luminosity distance $d_L(z)H_0/c = (1+z)\Gamma(z)$, where $\Gamma(z) = \int_0^z dz'/E(z')$ is the dimensionless comoving distance and

$$E(z) \equiv [\Omega_m(1+z)^3 + (1-\Omega_m)X(z)]^{1/2} \qquad (1)$$

is the cosmic expansion rate relative to its present value. We use the "gold" set of 157 SNe Ia published by Riess *et al.* in Ref. [5] and analyze it using flux-averaging statistics [13,15] to reduce bias due to weak gravitational lensing by intervening matter. We assume spatial flatness as motivated by inflation and discuss the importance of this and other assumptions below. We use CMB and LSS data to help break the degeneracy between the dark energy function $X(z)$ and Ω_m. For the CMB, we use only the measurement of the CMB shift parameter [16], $R \equiv \Omega_m^{1/2}\Gamma(z_{\rm CMB}) = 1.716 \pm 0.062$ from CMB (WMAP, CBI, ACBAR) [6,17], where $z_{\rm CMB} = 1089$. The only large-scale structure information we use is the linear growth rate $f(z_{\rm 2df}) = 0.51 \pm 0.11$ measured by the 2 degree field (2dF) galaxy redshift survey (2dFGRS) [3,18], where $z_{\rm 2df} = 0.15$ is the effective redshift of this survey and $f \equiv (d\ln D/d\ln a)$ is determined by solving the equation for the linear growth rate D, $D''(\tau) + 2E(z)D'(\tau) - \frac{3}{2}\Omega_m(1+z)^3 D = 0$, where primes denote $d/d(H_0 t)$.

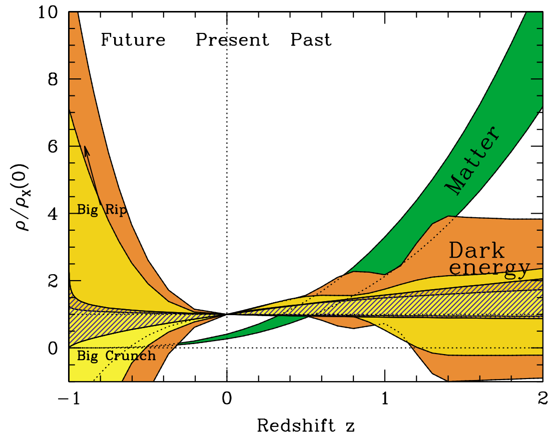

FIG. 1 (color online). 1σ constraints on the density of matter and dark energy from SN Ia (Riess sample, flux averaged with $\Delta z = 0.05$), CMB, and LSS data, all in units of the current dark energy density. From inside out, the four nested dark energy constraints are for models making increasingly strong assumptions, corresponding, respectively, to the four-parameter spline, the three-parameter spline, the two-parameter (f_∞, w_i) case, and the one-parameter constant w case (hatched area). The Universe starts accelerating when the total density slope $d\ln\rho/d\ln(1+z) > -2$, which roughly corresponds to when dark energy begins to dominate, i.e., to where the matter and dark energy bands cross. In the distant future, the Universe recollapses if the dark energy density ρ_X goes negative and ends in a "big rip" if it keeps growing $[d\ln\rho_X/d\ln(1+z) < 0]$.

Note that the CMB and LSS measurements we use (R and f) do *not* depend on the Hubble parameter H_0 and are quite insensitive to assumptions made about $X(z)$. The SN Ia measurements used are also independent of H_0, since we marginalize them over the intrinsic SN Ia luminosity calibration.

We run a Monte Carlo Markov chain (MCMC) based on the MCMC engine of Ref. [19] to obtain a few million samples of Ω_m and $X(z)$. The dark energy bands in Fig. 1 correspond to the central 68% of the X values at each z and the matter band does the same for $\rho_m(z)/\rho_X(0) = (1+z)^3\Omega_m/(1-\Omega_m)$.

Results.—Figure 1 shows our main results, the constraints on the dark energy function $X(z) = \rho_X(z)/\rho_X(0)$ for four different parametrizations, and illustrates that the assumptions one makes about the curve $X(z)$ have an important effect on the results. The most common way of measuring dark energy properties in the literature has been to parametrize the dark energy function X by merely one or two free parameters, constraining these by fitting to observed data. Table I includes the historically most popular parametrizations, expressed as functions of the dimensionless cosmic scale factor $a \equiv (1+z)^{-1}$. Parametrization A simply assumes that $X(a)$ is a power law, with the single equation-of-state parameter w determining its logarithmic slope. From the identity $\partial\ln\rho_X/\partial\ln a = -3(1+w_x)$, it follows that parametrization B corresponds to the popular parametrization $w_x(z) = w_0 + w_0'z$ [20], which has been widely used in the literature. It has the drawback of being rather unphysical for $w_0' > 0$, with the dark energy density $\rho_X(z)$ blowing up as $e^{3w_0'z}$ at high redshift. Parametrization C avoids this [21] and corresponds to $w_x = w_1 + w_a(1-a)$, but blows up exponentially in the future as $a \to \infty$ for $w_a > 0$. In contrast, our parametrization D remains well behaved at all times: both early on and in the distant future, the dark energy approaches either a constant equation of state w_i or a constant density, depending on the sign of $(1+w_i)$.

Obviously, the more restrictive the assumptions about X are, the stronger the nominal constraints will be, so it is crucial to be clear on what these assumptions are. For instance, Table I shows that parametrizations A, B, and C all tacitly assume that $X(z) \geq 0$, i.e., that the dark energy density cannot be negative, hence ruling out by fiat the possibility that the Universe can recollapse in a big crunch. Note that even *arbitrary* function $w(z)$ has this hidden assumption built in. Many scalar field models indeed prohibit the dark energy density from being negative, as well as predicting that $X'(a) \leq 0$, since fields usually roll down potentials, not up. However, we wish to avoid such theoretical prejudice, since "dark energy" could be a manifestation of something completely different, such as modified gravity [10].

To minimize such theoretical bias, we use parametrizations E and F from Table I; these are fairly model-independent reconstructions of the dark energy function $X(z)$, assuming merely that $X(z)$ is a sufficiently smooth function that it can be modeled with a cubic spline out to some redshift $z_{\max}$ and by a constant-w power law thereafter. We choose $z_{\max}$ to avoid sparse SN Ia data and parametrize X by its values at N equispaced spline points at $z_{\max}/N, 2z_{\max}/N, \ldots, z_{\max}$. $X(z)$ is matched smoothly on to $(1+z)^{3(1+w_i)}$ at $z > z_{\max}$. This specifies $X(z)$ uniquely once we require $X(z)$ and $X'(z)$ to be everywhere

TABLE I. Parametrizations used for the dark energy function $X \equiv \rho_X(z)/\rho(0)$ in terms of the cosmic scale factor $a = (1+z)^{-1}$.

Parametrization	n	Parameters	Definition
A) Constant equation of state w	1	w	$X = a^{-3(1+w)}$
B) Affine $w(z)$	2	w_0, w_0'	$X = a^{-3(1+w_0-w_0')}e^{3w_0'(a^{-1}-1)}$
C) Affine $w(a)$	2	w_1, w_a	$X = a^{-3(1+w_1+w_a)}e^{3w_a(a-1)}$
D) Forever regular	2	w_i, f_∞	$X = f_\infty + (1-f_\infty)a^{-3(1+w_i)}$
E) 3-parameter spline	3	w_i, $X(z_1)$, $X(z_2)$	Cubic spline in z for $z \leq z_2$, $X = X(z_2)[(1+z)/(1+z_2)]^{3(1+w_i)}$ for $z \geq z_2$
F) 4-parameter spline	4	w_i, $X(z_1)$, $X(z_2)$, $X(z_3)$	Cubic spline in z for $z \leq z_3$, $X = X(z_3)[(1+z)/(1+z_3)]^{3(1+w_i)}$ for $z \geq z_3$

continuous and set $X(0) = 1$, $X'(0) = [X(z_1) - 1]/z_1$. We choose $z_{\max} = 1.4$, as there are only two SNe Ia at higher redshifts. Since $X(z)$ is only very weakly constrained beyond $z > z_{\max}$, we impose a prior of $w_i \geq -2$ to avoid an unbounded parameter space. Changing the prior to $w_i \geq -20$ or changing the functional form of $X(z)$ at $z > z_{\max}$ (to an exponential, for example) has little impact on the reconstructed $X(z)$. We also find our results to be rather robust to data details. Including the "silver" sample from Ref. [5] does not change our results qualitatively, and replacing the CMB shift parameter we used ($R = 1.716 \pm 0.062$) by $R = 1.710 \pm 0.137$ (from WMAP data alone [6]) broadens the 68% confidence envelope by less than 20%.

Figure 1 also shows the constraints on the dark energy function $X(z)$ corresponding to parametrizations A and D from Table I, imposing the priors $w_i \geq -2$ and $f_\infty \geq 0$ for D. For comparison with the results of Ref. [5], we also studied parametrization B, with a weak prior $w_0' \geq -20$ to avoid an unbounded parameter space (MCMC tacitly assumes uniform prior on the parameters, so if the parameter space is unbounded, the MCMC will drift off in the unbounded direction and never converge).

As has been emphasized [22–24], SN Ia data are sensitive only to the smooth, overall shape of $X(z)$. This is because the error bars on sharp features on a scale Δz are proportional to $(\Delta z)^{-3/2}$ due to the derivative involved in going from comoving distance $r(z)$ to dark energy function X [23]; reconstructing $w_X(z)$ is still harder, the requirement that one effectively take the second derivative of noisy data [14] giving the error scaling as $(\Delta z)^{-5/2}$ [23]. Figure 1 shows that as we allow more small-scale freedom by parametrizing $X(z)$ by one, two, three, and four parameters, the allowed bands become thicker. However, the broader bands generally encompass the narrower ones, showing no hint in the data that the true $X(z)$ has funny features outside of the one- and two-parameter model families. Indeed, all bands are seen to be consistent with the simplest model of all: the 0-parameter "vanilla" model $X(z) = 1$ corresponding to Einstein's cosmological constant.

In other words, faced with the fact that an analysis using parametrization A implies $w \approx -1$ (we obtain $w = -0.91^{+0.13}_{-0.15}$ combining SN Ia, CMB, and LSS), readers hoping for something more interesting than vanilla may correctly argue that these constraints are dominated by accurate measurements at lower redshift and may fail to reveal hints of an upturn in $X(z)$ at $z \gtrsim 1$ because parametrization A incorrectly assumes that $(\log a, \log X)$ is a straight line. Our more general parametrizations close this loophole by allowing $X(z)$ much greater freedom, and the fact that none of them provide any hint yet of nonvanilla dark energy behavior therefore substantially strengthens the case for a simple cosmological constant, $X(z) = 1$.

What is the ultimate fate of the Universe? If for any of our models ρ_X eventually goes negative so that total density drops to zero at some time $t_{\rm turn}$, then the expansion reverses and a big crunch occurs at $t = 2t_{\rm turn}$; this applies only if X is uniquely determined by the cosmic scale factor (equivalently z) as in Table I and not for many scalar field models [25]. The cosmic time $t = \int da/\dot{a} = \int H^{-1} d\ln a$, and if this asymptotes to a finite value as $a \to \infty$, then a cataclysmic big rip [26] occurs at this time. This is equivalent to $w(z) < -1$ at $z = -1$, so parametrizations A, B, and C rip if $w < -1$, $w_0 - w_0' < -1$, and $w_a > 0$, respectively.

Predictions for the future need to be taken with a large grain of salt, since they are obviously highly model dependent. For instance, parametrizations A, B, and C cannot crunch, whereas E and F cannot rip. Simply combining all MCMC models from all our parametrizations, we find that 95% of them last at least another 49 Gyr, 25% ending in a big crunch, 8% ending in a big rip, and 67% quietly expanding forever.

Caveats and potential pitfalls.—When interpreting dark energy constraints such as those that we have presented, two crucial caveats must be borne in mind: potential SN Ia systematic errors and potential false assumption about other physics. We refer the reader to Refs. [3,5] for thorough discussions of the former and focus on the latter.

The SN Ia, CMB, and LSS measurements we have used involve only $X(z)$, Ω_m, and $\Omega_{\rm tot}$. Because of degeneracies between these three quantities, the inferences about $X(z)$ therefore depend strongly on the assumptions about the two cosmological parameters Ω_m and $\Omega_{\rm tot}$. Yet it is all too common to constrain dark energy properties using prior information about Ω_m and $\Omega_{\rm tot}$ that, in turn, hinges on assumptions about the dark energy, usually the vanilla assumption $X(z) = 1$, a pitfall emphasized by, e.g., [24].

We have assumed flat space, $\Omega_{\rm tot} = 1$, as have virtually all recent publications measuring dark energy properties (usually using parametrizations A, B, or C). It is well known that this assumption is crucial: introducing $\Omega_{\rm tot}$ as a free parameter to be marginalized over has such a dramatic effect on luminosity distances that essentially no interesting constraints can be placed on $X(z)$ at the present time, not even assuming the highly restrictive parametrization A.

We now turn to the issue of dark energy independent constraints on Ω_m. As emphasized by Ref. [24], assumptions about Ω_m make a crucial difference as well. As an example, Fig. 2 shows the constraints on (w_0, w_0') for parametrization B. The left panel illustrates that the constraints from SN Ia alone are much weaker than those obtained by imposing a strong prior $\Omega_m = 0.27 \pm 0.04$ as was done in Fig. 10 of Ref. [5]. Although this prior coincides with the measurement of Ω_m from WMAP and 2dFGRS [6], it should *not* be used here since it assumes $X(z) = 1$. The right panel in Fig. 2 shows the effect of including CMB information self-consistently (via the R parameter) in our constraints. We see that w_0 values as low as -3 remain allowed, as expected given

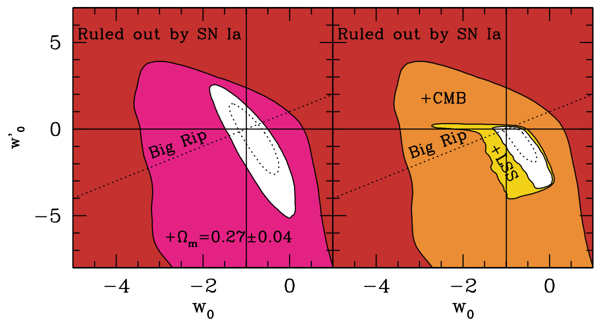

FIG. 2 (color online). How constraints on w_0 and w_0' depend on assumptions and data used. Darker shaded regions are ruled out at 95% confidence by SNe Ia alone; lighter shaded regions are ruled out when adding other information as indicated. 68% contours are dotted. Models above the dotted line end in a big rip. The 157 SNe Ia (Riess sample) have been flux averaged with $\Delta z = 0.05$.

the above-mentioned weak Ω_m constraints, and that additional information (in this case from LSS) is needed to tighten things up. This panel also illustrates the hazard of poor dark energy parametrizations: the seemingly impressive upper limit on w_0' tells us nothing whatsoever about dark energy properties via SN Ia but merely reflects that the unphysical exponential blowup $X \propto e^{3w_0'z}$ would violate the CMB constraint.

Conclusions.—In conclusion, we have reported the most accurate measurements to date of the dark energy density ρ_X as a function of time, assuming a flat universe. We have found that, in spite of their constraining power, the spectacular new high-z supernova measurements of Ref. [5] provide no hints of departures from the vanilla model corresponding to Einstein's cosmological constant. This is good news in the sense of simplifying the rest of cosmology but dims the prospects that nature will give us quantitative clues about the true nature of dark energy by revealing nonvanilla behavior. The apparent constancy of $\rho_X(z)$ also makes attempts to explain away dark energy by blaming systematic errors appear increasingly contrived, further strengthening the evidence that dark energy is real and hence a worthy subject of study. Future experiments [27] can dramatically shrink the error bars in Fig. 1 and, therefore, hold great promise for illuminating the nature of dark energy.

A Fortran code is available [28] that uses flux-averaging statistics to compute the likelihood of an arbitrary dark energy model (given the SN Ia data from Ref. [5]).

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