

## Theory of the Kapitza-Dirac Diffraction Effect

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We treat the Kapitza-Dirac diffraction effect observed recently by Batelaan *et al.* using a newly developed nonperturbative quantum-field scattering theory. Our theory shows that an electron beam passing perpendicularly through a focused standing light wave can produce diffraction patterns. Our theory predicts (1) the minimum value of the ponderomotive energy is  $(\hbar\omega)^2/m_e c^2$ , (2) the critical laser intensity above which the first pair of electron diffraction peaks will occur, and (3) the existence of sidebands in the electron spectra separated far from the central band by a momentum of several hundred photons. Our theory provides a unified explanation of the experimental results of Bucksbaum *et al.* and Batelaan *et al.*

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Recently, Batelaan *et al.* [1] observed a resolved diffraction pattern formed by electrons passing through a standing-wave light beam. The origin of their experiment is the suggestion first proposed by Kapitza and Dirac (KD) in 1933 that electrons could pass through and be reflected by a standing-wave light [2], with incident and reflection angles subject to Bragg's law, analogous to diffraction in x-ray crystallography. After the invention of the laser, several experimental attempts were made to observe this effect, but without success [3,4]. The deep splitting of photoelectron angular distributions in standing-wave multiphoton ionization observed by Bucksbaum *et al.* [5] was the only evidence supporting KD's conjecture. (Previous studies on the KD effect were summarized in a review article [6].)

Early papers on the KD effect theory [7–10] are largely based upon perturbative quantum mechanics, with a classical treatment of the light fields. Recently, Guo [11] analyzed the treatment of Guo and Drake [12] of the splitting observed by Bucksbaum *et al.* [5] and predicted Bragg-like electron scattering in a standing light wave. The treatment of Guo and Drake [12] is based on a non-perturbative quantum electrodynamic scattering theory proposed by Guo, Åberg, and Crasemann (GAC) [13], where the Volkov states play a role as intermediate states.

The recently observed electron diffraction pattern [1] appears similar to a classical thin-slit interference pattern, with the signals dropping sharply away from the center. The momentum transfer between two neighboring peaks is the momentum of two photons. In contrast, Bucksbaum's observed angular splitting corresponds to 500–2000 transferred photons, but without observing diffraction peaks in the middle. The question is as follows: What is the relationship, if any, between the observation of Batelaan and that of Bucksbaum?

The full ponderomotive energy of the electrons in the laser intensity used in Batelaan's experiment may cause a splitting as the one observed by Bucksbaum *et al.* [5]. A simple estimate shows that the two splitting peaks of Bucksbaum are located at the position of the 7th and 8th of Batelaan's diffraction peaks, corresponding to  $2u_p = 0.000567$ . This indicates that Bucksbaum's splitting peaks must be located at the two far sides of Batelaan's diffraction peaks. According to the analysis presented here, the diffraction patterns will not always resemble those from a thin slit. With an increased laser intensity, the envelope of the diffraction pattern must show a crescent line shape: the two far sides may have stronger diffraction signals than those in the middle. The patterns, as a function of intensity, will eventually pass over to Bucksbaum's splitting.

In this Letter, following the nonperturbative scattering approach [11–13] with a refined treatment of ponderomotive energy, we develop a KD diffraction theory. The transition rate in momentum space  $dW/d^3\mathbf{P}_f$  is

$$\frac{dW}{d^3\mathbf{P}_f} = \frac{4}{T} |\Omega_{fi} - \delta_{fi}|^2,$$

where  $T$  is total interaction time and  $\Omega_{fi}$  is the Møller operator matrix element

$$\Omega_{fi} = \sum_{(\mu, \varepsilon_\mu = \varepsilon_i)} \langle \phi_f | \Psi_\mu \rangle \langle \Psi_\mu | \phi_i \rangle,$$

where  $|\Psi_\mu\rangle$  are quantum-field Volkov states.

In the experiments of Bucksbaum *et al.* [5] and Batelaan *et al.* [1], the standing waves were made by two opposite propagating laser beams. When the two beams are circularly polarized with the same spatial angular momentum, the quantum-field Volkov states and their energy eigenvalues were derived as [11]

$$\Psi_\mu = V e^{-1/2} e^{i[\mathbf{P}_\mu - \mathbf{k}(N_{a_1} - N_{a_2})] \cdot \mathbf{r}} \sum_{j=-n_1}^{\infty} |n_1 + j, n_2\rangle_c X_{-j}(z),$$

$$\mathcal{E}_\mu = \frac{\mathbf{P}_\mu^2}{2m_e} + \sum_{i=1,2} (n_i + 1/2)\omega + 2u_p\omega, \quad (1)$$

where  $\mathbf{P}_\mu$  is the electron on-mass-shell momentum with  $\mu$  denoting different Volkov states,  $\mathbf{k}$  stands for the photon momentum;  $N_{a_1}$  and  $N_{a_2}$  are number operators of the original two photon modes in standing wave, while  $|n_1, n_2\rangle_c$  are the Fock states of the photon operators  $c_1 = 2^{-1/2}(a_1 + a_2)$ ,  $c_2 = 2^{-1/2}(a_1 - a_2)$ . The  $c_1$  and  $c_2$  photons can be called antinode and node photons, respectively. The phased Bessel functions are defined by

$$X_j(z) \equiv J_j(|z|) e^{ij \arg(z)}, \quad \text{with } z = \frac{2\sqrt{2}|e|\Lambda}{m_e\omega} \mathbf{P}_\mu \cdot \boldsymbol{\epsilon},$$

where  $\boldsymbol{\epsilon} = 2^{-1/2}(\boldsymbol{\epsilon}_x + i\boldsymbol{\epsilon}_y)$  denotes the circular polarization vector and  $2\Lambda$  is the classical amplitude for the vector potential of each photon mode, and  $2u_p\omega \equiv 2U_p$  is the total ponderomotive energy of the two traveling laser modes. This wave function has a unique advantage that its momentum phase contains an operator difference,  $\mathbf{k}(N_{a_1} - N_{a_2})$ , allowing an arbitrarily large momentum transfer between the two laser modes [12]. It has no classical-field correspondence.

The Møller operator matrix element reads

$$\Omega_{fi} = \frac{(2\pi)^3}{V_e} \delta(\mathbf{P}_i - \mathbf{P}_f + \Delta_f \mathbf{k} - \Delta_i \mathbf{k}) \mathcal{X}_q(z), \quad (2)$$

where  $\mathcal{X}_q(z) = \sum_j X_{-j}(z)^* X_{q-j}(z) F_q$ , and the factor  $F_q$  is given by [11]

$$F_q \equiv \sum_{n_2} \langle m_1, m_2 | n_1 + s, n_2 \rangle_c \langle n_1 + j, n_2 |_{c_1} l_1, l_2 \rangle$$

$$= \frac{1}{\pi} \left\{ \frac{\sin[(\Delta_f + \Delta_i)\pi/2]}{\Delta_f + \Delta_i} + \frac{\sin[(\Delta_f - \Delta_i)\pi/2]}{\Delta_f - \Delta_i} \right\}$$

$$\times \delta_{l_1 + l_2 - m_1 - m_2, q}, \quad (3)$$

in which  $\Delta_i \equiv l_2 - l_1 \ll l_1 + l_2 \approx m_1 + m_2$ ,  $\Delta_f \equiv m_2 - m_1 \ll l_1 + l_2$  are the initial and final photon-number differences between the two traveling modes. The  $\delta$  symbol gives the value of the net transferred photon number  $q \equiv j - s$  with  $j$  being the absorbed antinode-photon number when the electron enters the radiation field and  $s$  the emitted antinode-photon number when it leaves the field. The relation  $q = \Delta_f - \Delta_i + 2(l_2 - m_2)$  shows that  $q$  has to be odd for nonvanishing  $F_q$ .

In the entry process, the energy conservation required by the scattering theory and the momentum conservation specified by the  $\delta$  function are

$$\frac{\mathbf{P}_\mu^2}{2m_e} + (2u_p - j)\omega = \frac{\mathbf{P}_i^2}{2m_e}, \quad \mathbf{P}_\mu = \mathbf{P}_i - \Delta_i \mathbf{k}.$$

The quadratic equation for  $\Delta_i$  is

$$\Delta_i^2 - 2\Delta_i |\mathbf{P}_i| \cos\theta_i / \omega + 2m_e(2u_p - j) / \omega = 0.$$

Thus, the existence condition for  $\Delta_i$  is

$$(2u_p - j)\omega \leq \frac{\mathbf{P}_i^2}{2m_e} \cos^2\theta_i,$$

which leads to  $2u_p - j < 0$  for a perpendicular entry where  $\theta_i = \pi/2$ . To enter the field, the electron has to absorb at least one antinode photon such that  $j = 1, 2, 3, \dots$ . The absorbed photons contribute  $2u_p\omega$  to the ponderomotive energy and  $(j - 2u_p)\omega$  to the electron kinetic energy. The electron is accelerated, from the relation  $|\mathbf{P}_\mu| > |\mathbf{P}_i|$ , with a large deflection due to the extra antinode-photon absorption corresponding to a background photon-number change  $\Delta_i = \pm\sqrt{2m_e\omega^{-1}(j - 2u_p)}$ . In the experiment of Batelaan *et al.*,  $2u_p$  is about 0.000567, which leads  $\Delta_i \approx \pm\sqrt{2m_e/\omega} \approx \pm 663$ . In the 4-momentum space, the absorption of one photon in the energy direction will cause a photon-number change  $\sqrt{2m_e/\omega}$  in the  $\mathbf{k}$  direction. For  $j = 2, 3, \dots$ , the background photon-number change is even much greater. The electron with such large deflection may exit the field directly to form two  $j$ th-order sidebands in the electron spectrum, or exit with a  $s$ -photon emission to form two  $q$ th-order (noting  $q \equiv j - s$ ) sidebands in contrast to the central band observed by Batelaan *et al.*

The momentum and energy conservation relations between the initial and the final states are

$$\frac{\mathbf{P}_i^2}{2m_e} = \frac{\mathbf{P}_f^2}{2m_e} + (s - j)\omega, \quad \mathbf{P}_i = \mathbf{P}_f - (\Delta_f - \Delta_i)\mathbf{k}, \quad (4)$$

$$(\Delta_f - \Delta_i)^2 + 2(\Delta_f - \Delta_i)|\mathbf{P}_i| \cos\theta_i / \omega - 2m_e q / \omega = 0,$$

which determines the value of  $\Delta_f$  directly from parameters of the initial state.

From Eq. (4), in the  $j = s$  case, we have either  $\Delta_f = \Delta_i$  the penetration case or  $(\Delta_f - \Delta_i) = -2|\mathbf{P}_i| \omega^{-1} \cos\theta_i$  the reflection case satisfying the Bragg's law. In the  $j = 1, s = 0$  case with a tiny  $2u_p$  (say, about 0.0005),  $\Delta_f - \Delta_i$  satisfies almost the same algebraic equation satisfied by  $\Delta_i$ , which leads to  $\Delta_f = 0$  and makes each sideband irresolvable as one bright line.

From the preceding discussion we conclude that in the case of strictly antiparallel standing light wave, an electron beam passing through the standing wave (injected perpendicularly) will not produce diffraction peaks in the central band; it will, however, produce sidebands. The sidebands are located at two sides, far away from the central position. For nonperpendicularly injected electron beam, Bragg's scattering angles are enforced to guarantee a stimulated emission [2].

By analyzing either the exit or the entry process, we can find out the minimum quantum of the ponderomotive number  $u_p$ . Consider the entry process: The energy conservation for electron entry without an extra absorption of (antinode) photon is  $(\mathbf{P}_i^2/2m_e) - (\mathbf{P}_\mu^2/2m_e) = 2u_p\omega$ . To

form a Volkov state in which the electron momentum is perpendicular to the photon momentum  $\mathbf{k}$ , the electron has to enter the field with the Bragg's angle. The minimum background photon-number change is 2. By momentum conservation  $\mathbf{P}_i^2 - \mathbf{P}_\mu^2 = (\Delta_i \mathbf{k})^2$  ( $\Delta_i = \pm 2, \pm 4, \pm 6, \dots$ ) where  $\Delta_i$  is the change of the background photon number. Evidently, it can only be an even number without extra antinode-photon absorption and emission. Combining the above two equations, we have the value of the ponderomotive parameter for one laser beam

$$u_p(d) = d^2 \frac{\omega}{m_e}, \quad (d = |\Delta_i|/2 = 1, 2, 3, \dots).$$

When  $d = 1$ , one gets the minimum quantum of the ponderomotive parameter,  $u_p(1) = \omega/m_e$ , which corresponds to the first pair of diffraction peaks in the experiment of Batelaan *et al.* If this pair is the only pair observed, one gets the critical laser-beam intensity

$$I_c = \frac{\omega^4}{2\pi e^2}$$

using the formula  $I = u_p m_e \omega^3 / 2\pi e^2$ . When the laser-beam intensity falls below this critical value, no electron diffraction peaks can be formed.

Now we turn our attention to KD diffraction in a standing-wave focusing to a thin waist. To simplify the description, in our model system we choose two standing waves (four photon modes) to interact with the electron. We also assume that there is no interaction between the two standing waves through the electron. The incident electron beam is assumed to be perpendicular to the first pair of standing waves, while the second pair of standing waves is oriented at a slight angle  $\delta$  to the first one. The latter is labeled with a prime. In this case, the quantum-field Volkov solutions and their energy eigenvalues are

$$\Psi_\mu = V_e^{-1/2} e^{i[\mathbf{P}_\mu - \mathbf{k}(N_{a_1} - N_{a_2}) - \mathbf{k}'(N'_{a_1} - N'_{a_2})] \cdot \mathbf{r}} \sum_{j, j'} |n_1 + j, n_2\rangle_c \times X_{-j}(z) |n'_1 + j', n'_2\rangle_c X_{-j'}(z'), \quad (5)$$

$$\mathcal{E}_\mu = \frac{\mathbf{P}_\mu^2}{2m_e} + (n_1 + 1/2)\omega + (n_2 + 1/2)\omega + 2u_p\omega + (n'_1 + 1/2)\omega + (n'_2 + 1/2)\omega + 2u'_p\omega,$$

where  $\mathbf{k}' = \mathbf{k} - \delta\omega \mathbf{e}$  with  $|\mathbf{k}'| = |\mathbf{k}| = \omega$ ,  $\mathbf{e}$  being a unit vector, and  $\delta \ll 1$ .

The Møller operator matrix element reads

$$\Omega_{fi} = \frac{(2\pi)^3}{V_e} \delta(\mathbf{P}_i - \mathbf{P}_f + \Delta' \mathbf{k}' - \Delta \mathbf{k}) \mathcal{X}_q(z) \mathcal{X}_{q'}(z'), \quad (6)$$

with  $\Delta' \equiv \Delta'_f - \Delta'_i$  and  $\Delta \equiv \Delta_i - \Delta_f$ .

Using the momentum conservation between the initial and the intermediate states specified by the momentum delta function and the definition of the diffraction order  $d$ , we obtain the following equation set:

$$\begin{aligned} (\Delta_i + \Delta'_i)^2 + 2\Delta'_i \delta |\mathbf{P}_i| / \omega + 2m_e [(2u_p^{\max} - j - j') / \omega] &= 0, \\ (\Delta_f + \Delta'_f) - (\Delta_i + \Delta'_i) &= 2d \quad (d = 0, \pm 1, \pm 2, \dots), \end{aligned} \quad (7)$$

where the incident angle of the electron beam  $\theta_i$  is set as  $\pi/2$  for the perpendicular entry case. We consider an incident electron in the entry process, which absorbs one extra antinode photon from the first pair of standing waves, then emits another extra antinode-photon into the second pair of standing waves. Thus, we have  $j = 1$ ,  $j' = 0$  in the entry process and  $s = 0$ ,  $s' = 1$  in the exit process. One should note that absorption or emission of an antinode photon will cause several hundred photon-number changes between the two traveling modes. In this equation set, with choosing two independent integer variables  $\Delta'_i = 0, \pm 2, \pm 4, \dots$  and  $\Delta_f = 0, \pm 2, \pm 4, \dots$ , we have two equations to solve for two unknown variables  $\Delta_i$  and  $\Delta'_f$ . With given  $\Delta'_i$  and  $\Delta_f$ , the solutions are

$$\Delta_i = \pm \sqrt{2m_e \omega^{-1} (1 - 2u_p^{\max}) - 2\Delta'_i \delta \omega^{-1} |\mathbf{P}_i|} - \Delta'_i, \quad (8)$$

$$\Delta'_f = 2d + \Delta_i + \Delta'_i - \Delta_f,$$

where  $\Delta_i$  may take the closest odd numbers. From the overall energy conservation  $|\mathbf{P}_f|^2 / 2m_e = |\mathbf{P}_i|^2 / 2m_e + (q + q')\omega$ , where  $q \equiv j - s = 1$  and  $q' \equiv j' - s' = -1$ , we have the exact relation  $|\mathbf{P}_f| = |\mathbf{P}_i|$ . From the overall momentum conservation given by the Møller operator matrix element, and neglecting the  $\delta^2$  term, we have an approximate relation  $|\mathbf{P}_f| \cos \theta_f = 2\omega d$  to determine  $\cos \theta_f$ . Combining the overall energy and momentum conservations, we find that the angle  $\delta$  is uniquely determined by the diffraction order via the relation

$$\delta = \frac{2\omega d^2}{|\mathbf{P}_i| \Delta'}, \quad |\delta| \leq \delta_{\max}. \quad (9)$$

With a given  $\Delta'_i$  and  $\Delta_f$ , the angular distribution, when  $q = 1$ ,  $q' = -1$ , and  $\theta_i = \pi/2$ , is

$$\begin{aligned} \frac{dW}{d\Omega} &= \frac{4}{T} \frac{(2\pi)^3}{V_e} |X_{-1}(z_i) X_0(z_f) X_0(z'_i) X_{-1}(z'_f) F_1 F_{-1}|^2 \\ &\times \delta(\varphi_f - \varphi_i) \delta\left(\cos \theta_f - \frac{2\omega d}{|\mathbf{P}_f|}\right). \end{aligned} \quad (10)$$

The total transition rate is obtained by summing over all the possible  $\Delta'_i$  and  $\Delta_f$ . In our calculations, the angle between the two pairs of standing waves changes, according to Eq. (9), with the diffraction order and value of  $\Delta'$ , while  $\delta_{\max}$  sets the constraint for  $\Delta'$  which should be large enough for a fixed diffraction order to satisfy the inequality. For a Gaussian beam,  $\delta_{\max} = \lambda / \pi w_0$ , where  $w_0$  is the minimal radius of the laser beam. In the experiment of Batelaan *et al.*,  $\delta_{\max} = 800 \text{ nm} / (\pi 62.5 \text{ } \mu\text{m}) = 0.004$ . The computational results are shown in two figures. Figure 1 is our theoretical reproduction of the diffraction peaks observed by Batelaan *et al.* The diffraction

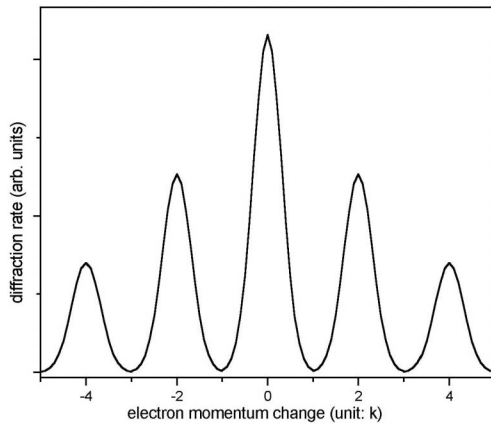


FIG. 1. The calculated KD diffraction pattern for laser intensity  $I = 2 \times 10^{14}$  W/m<sup>2</sup>, laser wave length 532 nm. The electrons are of kinetic energy 380 eV and are vertical incident, and the electron beam is assumed to be a Gaussian distributed spatially.

peaks weakened away from the center, while the central peak is the penetration peak without diffraction. Figure 2 is a calculation for the laser beam with higher intensity. The envelope of the diffraction patterns, without considering the central penetrating peak, shows a crescent line shape. The two maxima on the two sides of the crescent line corresponds to Bucksbaum's splitting.

In summary, we conclude the following: (1) For a strictly parallel standing light wave, a perpendicular incident electron beam cannot diffract due to energy-momentum conservation; while a nonperpendicular incident electron beam can diffract according to Bragg's law. (2) For a focusing standing light wave with a thin waist, a perpendicular incident electron beam can diffract to form the central band of the diffraction pattern as observed by Batelaan *et al.* (3) The ponderomotive number has a minimum  $u_p(1) = \hbar\omega/m_e c^2$  corresponding to the first pair of electron diffraction peaks, and the other smaller ponderomotive numbers  $u_p(d) = d^2 \hbar\omega/m_e c^2$  ( $d = 2, 3, 4, \dots$ ) correspond to the other pairs of the diffraction peaks. (4) There is a critical laser intensity,  $I = \hbar^2 \omega^4 / (2\pi e^2 c)$ , below which no diffraction peaks can be formed. (5) With increased laser intensities, the envelope of electron diffraction peaks, in the central band, will show a crescent line shape. Batelaan's diffraction peak corresponds to a decay of a partial ponderomotive energy  $U_p = u_p(d)\hbar\omega$  while Bucksbaum's splitting corresponds to a full ponderomotive decay. Here the "ponderomotive decay" means when an electron leaves the field, the ponderomotive energy turns back to the light field. (6) In addition to the central band observed by Batelaan *et al.*, we predict that the electron diffraction pattern will have sidebands located several hundred photon momenta away from the central position. They should be observable as a bright signal, but irresolvable in finer peaks.

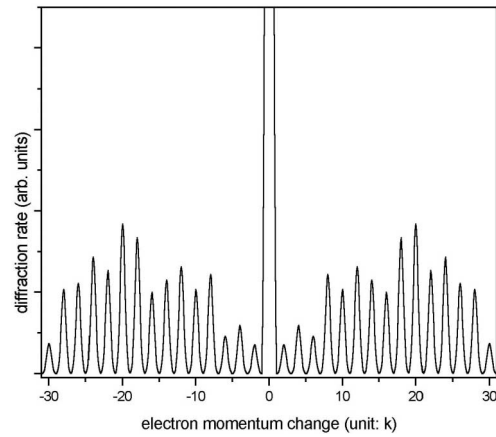


FIG. 2. The calculated KD diffraction pattern for laser intensity  $I = 2 \times 10^{15}$  W/m<sup>2</sup>, and the other conditions are the same as those in Fig. 1.

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*Note added.*—The most recent experimental results of the KD effect of the Bragg's scattering type [14] will be discussed in our future publications.

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