Return to Return Point Memory

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We describe a new class of systems exhibiting return point memory (RPM), different from those discussed before in the context of ferromagnets. We show numerically that one-dimensional random Ising antiferromagnets have exact RPM when evolving from a large field, but not when started at finite field, unlike the ferromagnetic case. This implies that the standard approach to understanding ferromagnetic RPM will fail for this case. We also demonstrate RPM with a set of variables that keeps track of spin flips at each site. Conventional RPM for the spins is a projection of this result, suggesting that spin flip variables might be a more fundamental representation of the dynamics. We also present a mapping that embeds the antiferromagnetic chain in a two-dimensional ferromagnet, and prove RPM for spin-exchange dynamics in the interior of the chain with this mapping.

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Many uses have been envisaged for nanomagnetic structures; for example, as patterned media for disk storage and as a possible means of information processing [1]. Such structures are easy to fabricate, with the shape, size, and separation of the individual magnetic units chosen as desired. For instance, in recent experiments [1–3], a large number of magnetic cylinders with a diameter of order 100 nm are fabricated in a linear or rectangular geometry, and a variety of techniques are used to probe the magnetization, either of the whole system or of individual nanomagnets. The interactions between individual nanomagnets are very important, and much of the experimental effort is in elucidating the nature of these interactions and their consequences for the system as a whole, particularly when an external magnetic field is applied.

Among the diversity of hysteresis behavior seen in magnetic systems, an important organizing principle is the surprising phenomenon of "return point memory" (RPM) [4]. This is demonstrated in hysteresis loops for ferromagnets, Fig. 1, where an applied field H is lowered from saturation to $H_{\min}$, raised by an intermediate amount to $H_{\max}$, and then lowered again to $H_{\min}$. In systems with RPM, the final state of the system is identical to when it first reaches $H_{\min}$. Generalizations are shown in Fig. 1. A well-known demonstration of this phenomenon is Barkhausen noise [6], where the noise observed in changing H of a ferromagnet is highly reproducible under repeated cycling of the field [7].

RPM was elegantly proved [5] for a broad class of ferromagnetic models (with zero temperature dynamics) by building on the earlier no-passing theorem for charge density waves [8]. However, many systems *do not* exhibit RPM. For example, spin glass Hamiltonians with both ferromagnetic and antiferromagnetic bonds violate return point memory [9] and instead exhibit subharmonic limit cycles under the application of a periodic field [10]. A two-dimensional antiferromagnetic model, where each spin is a Preisach hysteron [11], shows a phase transition from a RPM to a non-RPM state as the antiferromagnetic coupling is increased [12]. Even for ferromagnetic systems, dipolar interactions can cause RPM to break down [13]. Therefore, from a theoretical point of view, it is of interest to try to find the conditions for a system to exhibit RPM.

In this Letter, we demonstrate the existence of a new class of systems that satisfies RPM even though it violates the conditions of the ferromagnetic proof [5]. This is shown by examining fully antiferromagnetic Ising chains in one dimension, with zero temperature (deterministic) dynamics identical to those used for ferromagnetic systems. We find that, when started from a large H and fully saturated magnetization, the system always satisfies RPM. However, unlike the ferromagnetic case, if one

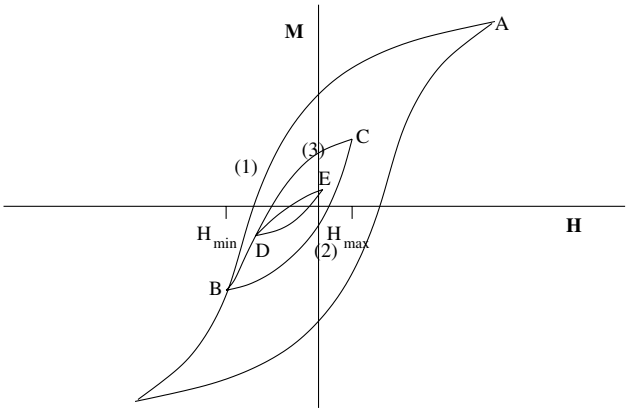

FIG. 1. Schematic of hysteresis loop for a ferromagnet, showing RPM. RPM is seen on the trajectory ABCB, or ABCDED, or in general when H backtracks to a value that does not cross the previous extremum (e.g., E cannot cross C for the path ABCDED to show RPM). For zero temperature single spin flip dynamics, it can be proved [5] that the full spin configuration on branch (3) is bounded by (1) and (2), whence RPM follows.

starts in a random state that is stable at some large field H_s, and lowers the field to $H_{\min}$, then raising the field to $H_{\max} < H_s$ and returning it to $H_{\min}$ changes the state. From a practical perspective, this is not a severe restriction since, if H is saturated in the distant past, RPM is valid for any subsequent evolution of $H(t)$ [14]. However, the ferromagnetic proof [5] relies on a partial ordering of spin configurations that is preserved by the dynamics (called "no-passing"), which must apply to an arbitrary initial state. Since this is not true for the antiferromagnetic chain, the proof of RPM must be qualitatively different.

Even for states descended from the saturated state, for which RPM is satisfied, we find that no-passing is violated for the spin configuration: In Fig. 1, the spin configuration on branch (3) is not bounded above and below by (1) and (2), respectively. However, we have been able to construct a new "spin flip" variable that does satisfy no-passing when starting from a high field. (This is not true starting from a random configuration; as discussed in the previous paragraph, no such variable can exist.) No-passing for the spin flip variable implies that it also satisfies RPM. (The converse is not true, as seen with the spin configuration.) Since the spin configuration can be obtained as a projection of the flip state, this version of RPM is stronger, suggesting that this new variable may be a more fundamental way of understanding these systems.

We also investigate whether, going beyond single spin flip dynamics [15], it is possible for the antiferromagnetic chain to show RPM in the same sense as the ferromagnetic case. We could show that this is the case with spin-exchange dynamics [16] that conserve magnetization except at the ends. We did this by embedding this 1D antiferromagnetic problem in a 2D *ferromagnetic* system which has single spin flip dynamics and therefore shows return point memory. However, this mapping fails for single spin dynamics in an interesting way: because under this mapping single spin flips become nonlocal and the standard proof [5] does not apply [17]. Therefore it is not just the Hamiltonian, but the dynamics as well, that determine whether or not a system satisfies RPM.

We consider the random antiferromagnetic Ising model:

$$\mathcal{H} = -\sum_i [J_i s_i s_{i+1} + (h_i + H) s_i], \qquad (1)$$

where the bonds J_i and local fields h_i are independent random variables. All the J_i's are negative, and the h_i's are equally likely to be positive and negative. H is the externally applied field. Initially, H is large and positive, and all the spins point up. Thereafter, the field is changed adiabatically. At any field, a spin is flipped if doing so reduces the energy $\mathcal{H}$ of the system. This spin flip can make other spins unstable, in which case the process is repeated until there are no more spins to flip. If several spins are unstable, the one which reduces the energy the most is flipped. Because an avalanche propagates out from the original site, and the left and right propagating directions are disjoint, the same results would be obtained if all unstable spins were flipped simultaneously or randomly. For all the numerical results reported in this Letter, $\sim 10^7$ random choices of $\{J_i, h_i\}$ were tested.

Figure 2 shows a typical hysteresis loop, with random bond disorder but no random fields ($h_i = 0$). The bonds are drawn from a distribution uniform over $[-1, 0]$. Return point memory is seen at $H = -1.4$ the hysteresis loop. RPM is also found when the h_i's are drawn from a distribution uniform over $[-1, 1]$, if $J_i = -1$ for all i. In both cases, although it cannot be shown in the figure, RPM exists for the full spin configuration rather than just M. However, if the J_i's *and* h_i's are drawn from distributions uniform over $[-1, -1 + \delta J]$ and $[-\delta h, \delta h]$, respectively, we find that RPM fails if $\delta h \gtrsim 0.01$ and $\delta J \gtrsim 0.01$. Therefore either δh or δJ must be zero for RPM. The results are the same for open and periodic boundary conditions [18]. (Even when δh and δJ are both nonzero, the deviation from RPM is quite small, and hard to detect if one averages the hysteresis loop over realizations of randomness. A similar phenomenon was observed earlier for Sherrington-Kirkpatrick spin glasses [19].)

An important difference between the ferromagnetic and antiferromagnetic cases is that a spin at a single site can flip several times while the magnetic field is varied monotonically. As the field is lowered, a spin can flip down; if its neighbors have already flipped down, they can then be pushed back up by the new spin flip. As a result of this, M does not vary monotonically with H. This can be

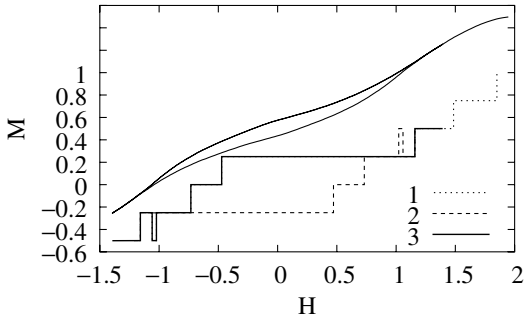

FIG. 2. Hysteresis curve for a one-dimensional Ising chain of length 8, where the bonds are all antiferromagnetic and of random strength. The lower curve shows the hysteresis curve for a single realization of randomness, with H lowered from ∞ to -1.4 (curve 1), raised to 1.4 (curve 2), and then lowered to -1.4 again (curve 3). The magnetization M changes contrary to H on (2) at $H \approx 1.05$ and on (3) at $H \approx -1$; apart from this excursion, (3) coincides with (1) from $-1.4 \leq H \leq 1.4$. The upper plot, shifted vertically by $M = 0.5$ for clarity, is a similar graph for a chain of length 64 000. The analogs of curves (1) and (3) for this graph are so close that they are indistinguishable in the plot, but there is actually a small gap between them. However, even for this case, return point memory (at $H = -1.4$) is exact.

seen in the plot for a single realization of randomness in Fig. 2. In more detail, it is possible to observe that (i) an avalanche that starts from a site and destabilizes both its neighbors is possible only for $\downarrow\downarrow\uparrow\downarrow\downarrow$ going to $\downarrow\uparrow\downarrow\uparrow\downarrow$ (or its mirror image), where the initial site is in the middle. The next nearest neighbors are stabilized, and the avalanche covers only three sites. (ii) An avalanche that starts from a site and destabilizes only one neighbor is possible only for a configuration $\uparrow\downarrow\downarrow$ going to $\downarrow\uparrow\downarrow$ (or its mirror image), where the initial site is at the end. The avalanche covers only two sites. Thus, as H is varied, the chain evolves through single spin flips, two-site avalanches with $\Delta M = 0$, and three-site avalanches which have $\Delta M = 1$ for decreasing H and $\Delta M = -1$ for increasing H. These results and more have been proven earlier with random field disorder (without bond disorder), for the major hysteresis loop [20]; the full shape of the hysteresis loop was found analytically [20]. We have extended the results of [20] to prove (ii) for the major loop and (i) for the entire hysteresis curve [22].

Motivated by the retrograde variation of M with H, we construct an alternative representation of the dynamics in terms of spin flips. Initially, when all the spins point up, the flip variable is zero at each site. Thereafter, each time a spin at site i is reversed, the flip variable l_i is increased by 1 if this happens when the field H is increasing, and decreased by 1 if this happens when H is decreasing. Clearly, along any branch of the hysteresis loop, while H varies monotonically, so must each l_i. Also, $s_i = 1 - 2[l_i \bmod 2]$, and if two configurations α and β satisfy the condition that $l_i^\alpha - l_i^\beta$ is even for all i, they correspond to the same spin state. In our numerical simulations, we find that, for the cases when RPM is valid, it also holds for the flip configuration. Since the configuration $\{s_i\}$ is a projection of $\{l_i\}$, this is a stronger result than RPM, and suggests that the dynamics in terms of $\{l_i\}$ is fundamental to random antiferromagnetic chains.

With $m_i = \sum_{j \neq i} l_j$, we find that no-passing is satisfied: If H is decreased from $H_{\max}$ to $H_{\min}$, increased to $H_{\max}$ and then returned to $H_{\min}$, for any H and any site i the value of m_i on the third segment of this path is bounded below and above by the corresponding m_i's on the first and second segments (see Fig. 1). This is not true for the spin variables s_i [23]. However, as emphasized earlier, RPM is not satisfied if one starts from an arbitrary initial state at some H instead of the saturated state, so that unlike the ferromagnetic case [5] the proof of RPM must take into account the ancestry of a state.

To see whether RPM is influenced by the dynamics used for the model, we now consider spin-exchange dynamics [16] instead of single spin flip [15]. A pair of oppositely oriented neighboring spins are exchanged if this is energetically favorable. Since this does not change the overall magnetization, in order for there to be a response to a magnetic field, we allow single spin flips at the two ends of the chain (only open chains are considered).

This problem can be solved by embedding the antiferromagnetic chain in a two-dimensional ferromagnetic model. We first consider the case when there is only random bond disorder. Figure 3 shows a two-dimensional square lattice of spins, rotated by an angle $\pi/4$. Ferromagnetic bonds connect next nearest neighbors, but (without random fields) not nearest neighbors. As shown in the figure, the vertical bonds are all zero, and the horizontal bonds are identical within each vertical strip. At the top and bottom boundaries, the boundary conditions force all the spins to be up and down, respectively. Free boundary conditions are used on the side walls. Thus, in its ground state, there is one horizontal domain wall across the system. As shown in the figure, we adopt a convention in which the domain wall consists of line segments oriented at $\pm\pi/4$, i.e., along the principal directions of the square lattice. The mapping from the two-dimensional system to the one-dimensional chain is as follows: If any line segment of the two-dimensional domain wall is oriented at $\pi/4$ or $-\pi/4$, the corresponding spin in the antiferromagnetic chain is 1 or -1, respectively. The two-dimensional ground state corresponds to alternating spins in the chain, as is appropriate when H is zero.

For a general shape of the domain wall, whenever two successive segments point in the same direction, a (horizontal) bond is broken, whereas this does not happen when they point in opposite directions. By choosing the

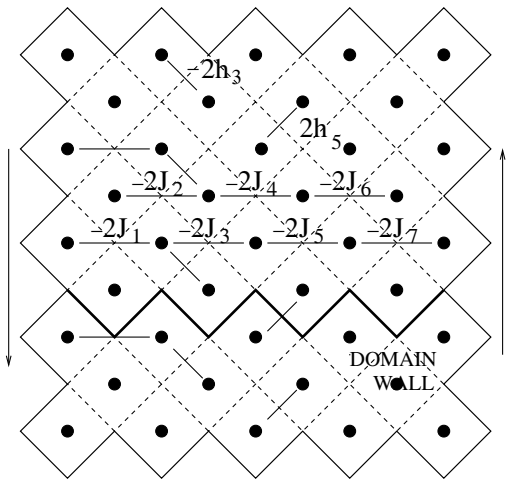

FIG. 3. Two-dimensional lattice with ferromagnetic bonds. The dashed lines are to guide the eye; the spins are at the black dots. The spins are forced to be up and down at the top and bottom boundaries, respectively. The domain wall in between maps to a one-dimensional spin chain. The case shown corresponds to a chain of (eight) alternating spins. The horizontal and diagonal bonds in the two-dimensional lattice correspond to the random bonds and fields, respectively, of the chain; for the system shown, $h_3 < 0$ and $h_5 > 0$. All bonds in a vertical column are the same; for clarity, only some are shown. An external field H on the chain is equivalent to a field at the side edges, increasing as shown with a gradient H.

horizontal bond strengths to be $-2J_1, -2J_2, -2J_3, \ldots$, correlated vertically, the energy of the antiferromagnetic chain is increased by $-2J_i$ when spins i and $i+1$ point in the same direction compared to when they are opposite, as desired for an antiferromagnetic chain. The magnetic field H couples to $\sum_i s_i$ for the chain, which is equivalent to the difference in height between the ends of the two-dimensional domain wall. This is equivalent to a magnetic field H on the rightmost column of the two-dimensional system, with the left end of the domain wall tethered. It is also possible to generalize the model to include random bond disorder for the chain: Nearest neighbor bonds of strength $2|h_i|$ are introduced in the ith column, oriented at $\pi/4$ if h_i is positive and $-\pi/4$ if h_i is negative.

With this construction, all bonds are ferromagnetic for the two-dimensional system. Further, the fields at the side boundaries vary monotonically with H. Further, spin-exchange for the chain is equivalent to single spin flips in the two-dimensional lattice. The results of Ref. [5] can therefore be invoked. We conclude that, with these dynamics, RPM is valid for all configurations, and is valid for simultaneous random field and random bond disorder. Neither of these statements is valid for single spin flip dynamics for the chain; the two-dimensional analog of spin flip at a site on the chain is to move the entire domain wall to the right of the site up or down by one unit if the spin flips up or down.

In this Letter, we have shown that the hysteresis loop for random Ising antiferromagnetic chains at zero temperature exhibit return point memory (RPM). For spin flip dynamics, the result is history dependent, being valid only for configurations that start from saturated magnetization and a large magnetic field. This is unlike the result for ferromagnets, where the result is valid for all configurations, indicating that the mechanism for RPM is different from the ferromagnetic case. (Also, RPM is valid only if either random field or random bond disorder is present, but not both, a restriction that does not apply to ferromagnets.) For spin-exchange dynamics, we have proven RPM by mapping to a two-dimensional ferromagnetic model, and have therefore shown that it is as general: valid for all configurations, and with simultaneous random field and bond disorder. This implies that RPM depends on the Hamiltonian and the dynamics used. These results do not generalize to higher dimensional antiferromagnets.

These results make trying to understand experiments on one-dimensional nanomagnetic chains worthwhile [1–3]. They often have an effective anisotropy due to their shape that makes their behavior Ising-like. However, more work needs to be done to see how to map these continuous dynamics onto the ones used here.

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