Gapless Color-Flavor-Locked Quark Matter

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In neutral cold quark matter that is so dense that the strange quark mass M_s is unimportant, all three quark flavors pair in a color-flavor locked (CFL) pattern, and all nine fermionic quasiparticles have a gap Δ (or 2Δ). We argue that, as the density decreases (or M_s increases), there is a quantum phase transition (at $M_s^2/\mu \approx 2\Delta$) to a new "gapless CFL phase" in which only seven quasiparticles have a gap. There is still an unbroken $\mathrm{U}(1)_{\tilde{Q}}$ gluon/photon, but, unlike CFL, gapless CFL is a $\tilde{Q}$ conductor with gapless (charged) quasiquarks and a nonzero electron density at zero temperature, so its low energy effective theory and astrophysical properties are qualitatively new. At the transition, the dispersion relations of both gapless quasiparticles are quadratic, but for larger M_s^2/μ, one becomes conventionally linear while the other remains quadratic, up to tiny corrections.

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We know a lot about the properties of cold quark matter at sufficiently high baryon density from first principles. Quarks near their Fermi surfaces pair, forming a color superconductor [1]. In this Letter we study how the favored pairing pattern at zero temperature depends on the strange quark mass M_s, or equivalently on the quark chemical potential μ, using the pairing ansatz [2]

$$\langle \psi_a^\alpha C\gamma_5 \psi_b^\beta \rangle \sim \Delta_1 \epsilon^{\alpha\beta 1}\epsilon_{ab1} + \Delta_2 \epsilon^{\alpha\beta 2}\epsilon_{ab2} + \Delta_3 \epsilon^{\alpha\beta 3}\epsilon_{ab3}. \tag{1}$$

Here ψ_a^α is a quark of color $\alpha = (r, g, b)$ and flavor $a = (u, d, s)$; the condensate is a Lorentz scalar, antisymmetric in Dirac indices, antisymmetric in color (the channel with the strongest attraction between quarks), and consequently antisymmetric in flavor. The gap parameters Δ_1, Δ_2, and Δ_3 describe down-strange, up-strange, and up-down Cooper pairs, respectively.

To find which phases occur in realistic quark matter, one must take into account the strange quark mass and equilibration under the weak interaction, and impose neutrality under the color and electromagnetic gauge symmetries. The arguments that favor (1) are unaffected by these considerations, but there is no reason for the gap parameters to be equal once $M_s \neq 0$. Previous work [3–6] compared the color-flavor-locked (CFL) phase (favored in the limit $M_s \to 0$ or $\mu \to \infty$), and the two-flavor (2SC) phase (favored in the limit $M_s \to \infty$). In this Letter we show that, in fact, a transition between these phases does not occur. Above a critical M_s^2/μ, the CFL phase gives way to a new "gapless CFL phase," not to the 2SC phase. The relevant phases are

$$\Delta_3 \simeq \Delta_2 = \Delta_1 = \Delta_{\mathrm{CFL}} \quad \text{CFL}, \tag{2}$$

$$\Delta_3 > 0, \qquad \Delta_1 = \Delta_2 = 0 \quad \text{2SC}, \tag{3}$$

$$\Delta_3 > \Delta_2 > \Delta_1 > 0 \quad \text{gapless CFL}. \tag{4}$$

To impose color neutrality, it is sufficient to consider the $\mathrm{U}(1)_3 \times \mathrm{U}(1)_8$ subgroup of the color gauge group generated by the Cartan subalgebra $T_3 = \mathrm{diag}(\frac{1}{2}, -\frac{1}{2}, 0)$ and $T_8 = \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3})$ in color space [4]. We introduce chemical (color-electrostatic) potentials μ_3 and μ_8 coupled to the color charges T_3 and T_8, and an electrostatic potential μ_e coupled to Q_e, which is the *negative* of the electric charge $Q = \mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$. ($\mu_e > 0$ corresponds to a density of electrons.) The neutrality condition on μ_e, μ_3, μ_8 is

$$\frac{\partial \Omega}{\partial \mu_e} = \frac{\partial \Omega}{\partial \mu_3} = \frac{\partial \Omega}{\partial \mu_8} = 0. \tag{5}$$

Any condensate of the form (1) is neutral with respect to a "rotated electromagnetism" generated by $\tilde{Q} = Q - T_3 - \frac{1}{2} T_8$, so $U(1)_{\tilde{Q}}$ is never broken, but, depending on the values of the Δ_i, the rest of the gauge group may be spontaneously broken. We emphasize that this does not affect the neutrality condition (5): a macroscopic sample must be neutral under *all* gauge symmetries [4].

Previous model-independent calculations [4] have compared the free energy of the CFL phase with that of the 2SC and unpaired phases. In the CFL phase, $\mu_e = \mu_3 = 0$ and $\mu_8 = -M_s^2/(2\mu)$ to leading order in M_s/μ. To this order, the CFL phase has a lower free energy than either 2SC or neutral unpaired quark matter for

$$\frac{M_s^2}{\mu} < 4\Delta_{\mathrm{CFL}}. \tag{6}$$

In this Letter we show that the CFL phase becomes unstable already at a *lower* value of M_s^2/μ.

We shall present solutions to the gap equations for Δ_1, Δ_2, and Δ_3 below. First, however, we give a model-independent argument for the instability of the CFL phase above some critical M_s^2/μ. In the condensate (1), the (gs, bd), (bu, rs), and (rd, gu) quarks pair with gap

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parameters Δ_1, Δ_2, and Δ_3, respectively, while the (*ru*, *gd*, *bs*) quarks pair among each other involving all the Δ's. The gap equations for the three Δ's are coupled, but we can, for example, analyze the effect of a specified Δ_1 on the *gs* and *bd* quarks without reference to the other quarks. The leading effect of M_s is like a shift in the chemical potential of the strange quarks, so the *bd* and *gs* quarks feel "effective chemical potentials" $\mu_{bd}^{\rm eff} = \mu - \frac{2}{3}\mu_8$ and $\mu_{gs}^{\rm eff} = \mu + \frac{1}{3}\mu_8 - \frac{M_s^2}{2\mu}$. In the CFL phase $\mu_8 = -M_s^2/(2\mu)$ [4], so $\mu_{bd}^{\rm eff} - \mu_{gs}^{\rm eff} = M_s^2/\mu$. The CFL phase will be stable as long as the pairing makes it energetically favorable to maintain equality of the *bd* and *gs* Fermi momenta, despite their differing chemical potentials [7]. It becomes unstable when the energy gained from turning a *gs* quark near the common Fermi momentum into a *bd* quark (namely M_s^2/μ) exceeds the cost in lost pairing energy $2\Delta_1$. So the CFL phase is stable when

$$\frac{M_s^2}{\mu} < 2\Delta_{\rm CFL}. \tag{7}$$

For larger M_s^2/μ, the CFL phase is replaced by some new phase with unpaired *bd* quarks, which from (6) cannot be neutral unpaired or 2SC quark matter because the new phase and the CFL phase must have the same free energy at the critical $M_s^2/\mu = 2\Delta_{\rm CFL}$.

For a more detailed analysis, we use a Nambu–Jona-Lasinio (NJL) model with a pointlike four-quark interaction with the quantum numbers of single-gluon exchange, as in the first paper in Ref. [1] but with chemical potentials μ_e, μ_3, and μ_8 introduced as in Ref. [4]. Whereas in nature, the conditions (5) are enforced by the dynamics of the gauge fields whose zeroth components are μ_e, μ_3, and μ_8 [4], in an NJL model Eqs. (5) must be imposed [4]. The model has two parameters, the four-fermion coupling G and a three-momentum cutoff Λ, but we quote results in terms of the physical quantity Δ_0 (the CFL gap at $M_s = 0$), since varying Λ by 20% while tuning G to keep Δ_0 fixed changes all of our results by at most a few percent. We use $\Lambda = 800$ MeV in all results that we quote.

We make the ansatz (1) for the diquark condensate in the quark propagator, using the Nambu-Gorkov formalism, and then evaluate the free energy Ω. We shall present the details of our calculation elsewhere, but the formalism is as in Ref. [10], with the additional constraints of electrical and color neutrality. To simplify the analysis, we neglect the color and flavor symmetric contributions [2] that respect the same symmetries and are known to be small [2,3], set the light quark masses to zero, and treat the constituent strange quark mass M_s as a parameter. We incorporate M_s only via its leading effect, a shift $-M_s^2/2\mu$ in the effective chemical potential for the strange quarks. This requires that M_s^2/μ^2 be small, and neglects the dependence of the gap parameters on the Fermi velocity of the strange quark, meaning that we find $\Delta_3 = \Delta_2 = \Delta_1$ in the CFL phase instead of finding Δ_3 larger than the other two by a few percent [5]. We work to leading nontrivial order in Δ_1, Δ_2, Δ_3, μ_e, μ_3, and μ_8, since these are all small compared to μ. Finally, we neglect the effects of antiparticles. None of these approximations precludes a qualitative understanding of the new phase we shall describe.

We calculate the free energy Ω and solve six coupled integral equations, the neutrality conditions (5), and the gap equations $\partial\Omega/\partial\Delta_1 = \partial\Omega/\partial\Delta_2 = \partial\Omega/\partial\Delta_3 = 0$. Our solutions depend on three parameters: μ, M_s, and Δ_0. We always take $\mu = 500$ MeV, which is reasonable for the center of a neutron star. We quote results only for $\Delta_0 = 25$ MeV, which is within the plausible range [1] and ensures that the transition (7) occurs where M_s^2/μ^2 corrections are under control. Although we have obtained our results by varying M_s at fixed μ, we typically quote results in terms of the important combination M_s^2/μ. Note that in nature M_s increases with decreasing μ.

In Fig. 1, we show the gaps as a function of M_s^2/μ, for $\Delta_0 = 25$ MeV. A phase transition occurs at a critical M_s^c that, in our model calculation with $\mu = 500$ MeV, lies between $M_s = 153$ MeV and $M_s = 154$ MeV. Below $(M_s^2/\mu)_c$, i.e., at high enough density, we have the CFL phase. At $M_s = 153$ MeV, $\Delta_1 = \Delta_2 = \Delta_3 = 23.5$ MeV and $M_s^2/\mu = 46.8$ MeV: the model-independent prediction (7) is in good agreement with our model calculation. For $M_s^2/\mu > (M_s^2/\mu)_c$, i.e., at densities below those where the CFL phase is stable, we find the gapless CFL (gCFL) phase with $\Delta_3 > \Delta_2 > \Delta_1 > 0$ and all the gaps changing much more rapidly with M_s^2/μ. We have checked that, upon varying Δ_0, the critical M_s^2/μ changes quantitatively as predicted by (7) and our results are otherwise qualitatively unchanged.

By evaluating the free energy, we have confirmed that the CFL $\rightarrow$ gCFL transition at $(M_s^2/\mu)_c$ is not first order, and found a first order gCFL $\rightarrow$ unpaired quark matter transition at $M_s^2/\mu \approx 129$ MeV. The (now metastable) gCFL phase continues to exist up to $M_s^2/\mu \approx 144$ MeV where, as we show below, it ceases to be a solution.

We see from the *bd*-*gs* quasiquark dispersion relations (dashed lines in Fig. 2) that there are gapless excitations at momenta p_1^{bd} and p_2^{bd}. The analysis of Ref. [12]

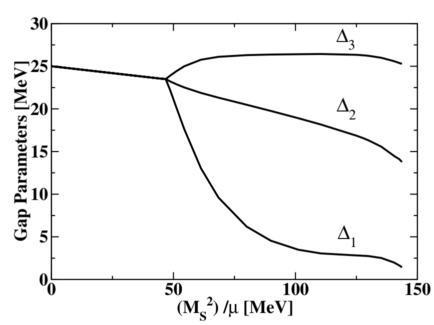

FIG. 1. Gap parameters Δ_3, Δ_2, and Δ_1 as a function of M_s^2/μ for $\mu = 500$ MeV, in a model where $\Delta_0 = 25$ MeV (see text). There is a continuous transition between the CFL phase and the gapless CFL phase.

demonstrates that, as expected from the model-independent argument above, these bound a "blocking region" [11] $p_1^{bd} < p < p_2^{bd}$ in which there are bd quarks but no gs quarks, and thus no pairing. We have confirmed this by explicit calculation of number densities. The volume of the blocking region is zero at the critical point, and grows in the gCFL phase like $(p_2^{bd} - p_1^{bd}) \sim [M_s^2/\mu - (M_s^2/\mu)_c]^{1/2}\Delta_1^{1/2}$. Note that the dispersion relations in the blocking region are nontrivial because the excitations obtained by either adding a gs quark or removing a bd quark mix via the Δ_1 condensate. If we neglect this mixing, the gapless excitations near p_2^{bd} (p_1^{bd}) are bd (gs) quarks and holes. The gapless CFL phase is analogous to the unstable Sarma phase [13], but is rendered stable by the neutrality constraint. This possibility, leading to a gapless phase appearing at a continuous phase transition, was first noted in the two-flavor case [8,9], and it was conjectured that the three-flavor case could be similar [8]. Our model also has a gapless 2SC phase, but it is free-energetically disfavored relative to the gCFL phase except perhaps very near where the gCFL and unpaired quark matter free energies cross at $M_s^2/\mu = 129$ MeV: a calculation with fewer approximations is required to resolve a possible small gapless 2SC "window."

The $\tilde{Q}$-neutral gs and bd quarks are not the only sources of gapless modes in the gCFL phase: there are also gapless $\tilde{Q}$-charged modes, associated with the bu and rs quarks, making the gCFL phase a $\tilde{Q}$ conductor rather than an insulator. Electrons play a crucial role in this, but let us first understand the quark matter on its own, setting the electron mass to infinity and in so doing keeping the gCFL phase a $\tilde{Q}$ insulator.

In the absence of electrons, both the CFL and gCFL phases have a degenerate set of neutral free energy minima over a range of $\mu_{\tilde{Q}} = -\frac{4}{9}(\mu_e + \mu_3 + \frac{1}{2}\mu_8)$, with the two orthogonal chemical potentials and the three gap parameters fixed. The limits of this range (giving μ_e rather than $\mu\tilde{Q}$) are shown in Fig. 3. Because Ω is independent of $\mu_{\tilde{Q}}$ in this range, the material is a $\tilde{Q}$ insulator. At the upper limit in $\mu\tilde{Q}$ (lower limit in μ_e), the bu and rs quarks, which have $\tilde{Q} = +1$ and $\tilde{Q} = -1$, respectively, start to unpair: they develop a blocking region of unpaired bu quarks bounded by gapless modes, meaning that the $\tilde{Q}$-neutrality condition cannot be satisfied. At the lower limit in $\mu\tilde{Q}$ (upper limit in μ_e) an analogous dielectric breakdown occurs with the rd-gu pairs breaking and a blocking region of unpaired gu quarks with $\tilde{Q} = -1$ developing. The solid curves in Fig. 3 thus define the "band gap" for $\tilde{Q}$-charged fermionic excitations. In the region between the curves, $\tilde{Q}$-insulating CFL or gCFL quark matter exists, but outside that region no neutral solution exists. At $M_s^2/\mu = 144$ MeV, which is so large that the gCFL phase is anyway already metastable with respect to unpaired quark matter, the two boundaries cross, meaning that no gCFL solution can be found.

In the real world there are electrons, which we take to be massless. Consequently, in the CFL phase $[M_s^2/\mu < (M_s^2/\mu)_c]$, neutrality requires $\mu_e = 0$, so no electrons are present and the material remains $\tilde{Q}$ neutral and a $\tilde{Q}$ insulator, as before [14]. However, in the gCFL phase, $[M_s^2/\mu > (M_s^2/\mu)_c]$, $\mu_e = 0$ is below the allowed range. In this case, the true solution lies "just below" the lower curve in Fig. 3, where the bu and rs quarks have become gapless, allowing a small density of unpaired bu ($\tilde{Q} = +1$) quarks to cancel the charge of the electrons. The density of electrons is $\mu_e^3/(3\pi^2)$, and the density of unpaired bu quarks is $(p_2^{bu})^3 - (p_1^{bu})^3/(3\pi^2)$, so they cancel when $(p_2^{bu} - p_1^{bu}) = \mu_e^3/3\bar{p}^2$, where $\bar{p}$ is the average of the momenta p_1^{bu} and p_2^{bu} that bound the bu blocking region. At $M_s^2/\mu = 80$ MeV, where $\mu_e = 14.6$ MeV at the lower curve in Fig. 3, this implies $(p_2^{bu} - p_1^{bu}) = 0.0046$ MeV. (To resolve $p_2^{bu} - p_1^{bu}$, we solved the equations assuming 200 and 500 "flavors" of massless electrons.) Because $(p_2^{bu} - p_1^{bu})$ is *so* small, at the true

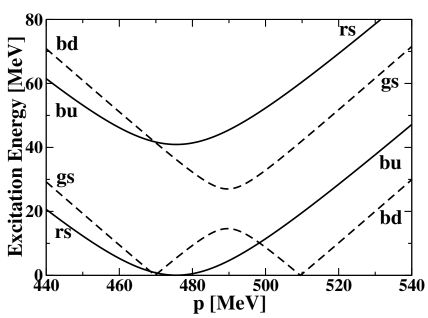

FIG. 2. Dispersion relations at $M_s^2/\mu = 80$ MeV for gs and bd quarks (dashed lines) and for bu and rs quarks (solid lines). There are gapless gs-bd modes at $p_1^{bd} = 469.8$ MeV and $p_2^{bd} = 509.5$ MeV, which are the boundaries of the bd-filled "blocking" [11,12] or "breached pairing" [8] region. One bu-rs mode is gapless with an almost exactly quadratic dispersion relation. The five quark quasiparticles not plotted all have gaps, throughout the CFL and gCFL phases.

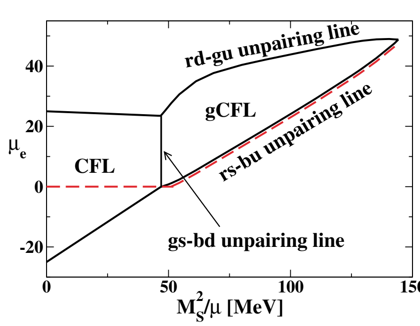

FIG. 3 (color online). The upper and lower curves bound the region of μ_e where CFL or gCFL solutions are found, if electrons are ignored. Between the curves the quark matter is a $\tilde{Q}$ insulator. Taking electrons into account, the correct solution has $\mu_e = 0$ for $M_s^2/\mu < (M_s^2/\mu)_c$ in the CFL phase (dashed line), and has μ_e below but *very* close to the lower curve for $M_s^2/\mu > (M_s^2/\mu)_c$ in the gCFL phase (see text).

$\tilde{Q}$-neutral solution μ_e is *very* close to the lower curve in Fig. 3, and the gaps are almost unaffected by the inclusion of electrons. However, the effect of including electrons is profound: because μ_e cannot be zero, gCFL quark matter is deformed slightly away from being a $\tilde{Q}$ insulator, so that it can carry a positive $\tilde{Q}$ charge to compensate the negatively charged electrons. The gCFL phase is therefore a $U(1)_{\tilde{Q}}$ conductor. The quantum phase transition at $M_s^2/\mu = (M_s^2/\mu)_c$ is a "metal-insulator transition," with electron density $n_e \sim (\mu_e^2 - m_e^2)^{3/2}$ as the most physically relevant order parameter. The phase transition is continuous but higher-than-second order $[dn_e/d(M_s^2/\mu)$ is continuous].

The phenomenology of gCFL quark matter in compact stars is dominated by the modes with energy less than or of order the temperature, which is in the range keV to hundreds of keV. For the *gs-bd* quasiparticles (Fig. 2), the gapless $\tilde{Q} = 0$ quasiparticles at p_1^{bd} and p_2^{bd} have conventional linear dispersion relations, except for $M_s^2/\mu \to (M_s^2/\mu)_c$ where $p_2^{bd} - p_1^{bd} \to 0$. For the *bu-rs* quarks, the dispersion relation is strictly speaking also quadratic only at the quantum critical point, but the gapless points separate so slowly in the gCFL phase that this dispersion relation remains very close to quadratic. For example, at $M_s^2/\mu = 80$ MeV the maximum in the quasiparticle energy between p_1^{bu} and p_2^{bu} is $(p_2^{bu} - p_1^{bu})^2/(8\Delta_2) = (\mu_e^6/72\bar{p}^4\Delta_2^2) \sim 0.13$ eV, which is negligible at compact star temperatures. The requirement of $\tilde{Q}$-neutrality naturally forces the gapless *bu-rs* dispersion relation to be (very close to) quadratic, without requiring fine-tuning to a critical point.

The low energy effective theory of the gCFL phase must incorporate gapless fermions that have number densities $\sim\mu^2\sqrt{\Delta_2 T}$ (the gapless quarks with quadratic dispersion relation), $\sim\mu^2 T$ (the gapless quarks with linear dispersion relations), and $\sim\mu_e^2 T$ (the electrons). In contrast, the (pseudo)Goldstone bosons present in both the CFL and the gCFL phases have number densities at most $\sim T^3$. This means the gCFL phase has very different phenomenology. It will be interesting to compute the cooling of a compact star with a gCFL core, because neutrino emission requires conversion between quasiparticles with linear and quadratic dispersion relations.

Although we have studied the gCFL phase in a model, the qualitative features that we have focused on appear robust, and we have also offered a model-independent argument for the instability that causes the transition. It remains a possibility, however, that the CFL gap is large enough that baryonic matter supplants the CFL phase before $M_s^2/\mu > 2\Delta$. Assuming that the gCFL phase does replace the CFL phase, it is also possible that gaps are small enough that a third phase of quark matter could supplant the gCFL phase at still lower density, before the transition to baryonic matter. In our analysis with $\Delta_0 = 25$ MeV, this third phase would be the unpaired quark matter at $M_s^2/\mu > 129$ MeV but, unlike our central results, this is model dependent. Other possibilities include the gapless 2SC phase [9], a three-flavor extension of the crystalline color superconducting phase [11,12] or weak pairing between quarks with the same flavor.

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[14] The band gap for $\tilde{Q}$-charged bosonic excitations in the CFL phase is narrower than that for fermionic excitations, but still includes $\mu_e = 0$. The CFL phase at $\mu_e = 0$ can exhibit a $\tilde{Q}$-neutral kaon condensate. See P. F. Bedaque and T. Schäfer, Nucl. Phys. A **697**, 802 (2002); D. B. Kaplan and S. Reddy, Phys. Rev. D **65**, 054042 (2002); and Refs. [1]. The gCFL phase has the same (pseudo)Goldstone bosons as the CFL phase, since the same symmetries are broken. However, the pseudo-Goldstone bosons receive new contributions to their squared masses, of order differences between Δ_i^2's, which stabilize against meson condensation.