Climbing the Entropy Barrier: Driving the Single- towards the Multichannel Kondo Effect by a Weak Coulomb Blockade of the Leads

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We study a model proposed recently in which a small quantum dot is coupled symmetrically to several large quantum dots characterized by a charging energy E_c. Even if E_c is much smaller than the Kondo temperature T_K, the long-ranged interactions destabilize the single-channel Kondo effect and induce a flow towards a multichannel Kondo fixed point associated with a *rise* of the impurity entropy with decreasing temperature. Such an "uphill flow" implies a *negative* impurity specific heat, in contrast with all systems with local interactions. An exact solution found for a large number of channels allows us to capture this physics and to predict transport properties.

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Simple models of non-Fermi liquids, such as the multichannel Kondo model, have attracted much theoretical interest [1], especially due to their striking properties such as zero-bias anomalies or a finite entropy at zero temperature, $S(T = 0) = \ln(g)$, with a noninteger g [2]. So far, however, their experimental realization has been a challenging task. The advent of nanostructures that can be designed and tuned more easily than solid state systems is a major step towards observing these interesting strong-correlation effects in the laboratory.

The multichannel Kondo model describes a single spin coupled symmetrically to several *independently* conserved conduction electron "channels." Recently Oreg and Goldhaber-Gordon [3] suggested that these channels can be realized by attaching several large quantum dots serving as "leads" to a single small dot. They pointed out that a sufficiently large charging energy, $E_c \gtrsim T_K$, in those leads can suppress all low-energy cotunneling processes between them which would otherwise mix the channels and destroy the multichannel physics. The experimental realization of such a system is a demanding task as the size of the large dots has to be chosen such that the level-splitting Δ_L is sufficiently small and, simultaneously, the charging energy is large enough, $\Delta_L \ll T_K^{\rm multi} \lesssim T_K \lesssim E_c$, where $T_K^{\rm multi}$ is the Kondo temperature of the multichannel Kondo model and T_K refers to the single-channel Kondo temperature for $E_c = 0$. Furthermore, considerable fine tuning, using gate voltages, is required to guarantee that all leads couple equally to the spin residing on the small dot. Pustilnik *et al.* [4] have recently discussed in detail how to achieve such a fine tuning by calculating the conductance for small variations in the coupling to the various channels. The idea that interactions in the leads can lead to multichannel physics was also put forward earlier by Coleman and Tsvelik [5], and received recent additional support [6].

What happens if the charging energy E_c of the leads is much smaller than the Kondo temperature, $E_c \ll T_K$? In this case a single-channel Kondo resonance will develop upon lowering T. According to conventional wisdom, the Fermi-liquid fixed point of the single-channel Kondo effect can never be destroyed by small perturbations (for example a weak magnetic field, $B \ll T_K$, does not prevent the formation of the Kondo resonance). Indeed, the powerful "g theorem" of Affleck and Ludwig [7,8] proves that no small local perturbation can destabilize such a zero-entropy fixed point: The impurity entropy $\ln(g)$ of boundary conformal field theories (to which Kondo models belong) always *decreases* under renormalization group flow. According to this theorem, a flow from the multichannel to the single-channel fixed point is possible, but not vice versa. However, the g theorem does not cover the situation under discussion, where long-range interactions induce a Coulomb blockade in the leads.

The purpose of this Letter is to show that in such a situation tiny charging energies $E_c \ll T_K$ can destroy the single-channel Kondo effect and stabilize a multichannel fixed point. As the multichannel system is characterized by a *finite* residual entropy [$S(T = 0) = \ln\sqrt{2}$ for two channels] this requires a negative impurity specific heat in some T range. A simple argument in favor of such a scenario is that the existence of the single-channel Kondo effect implies resonant tunneling between the leads. The Coulomb blockade in the leads, however, prohibits such a resonance. The Kondo effect cannot overcome the Coulomb blockade, as resonant tunneling in an energy window of width T_K costs a charging energy of order $E_c\langle(\Delta N)^2\rangle \sim E_c(T_K/\Delta_L)$, where ΔN are charge fluctuations induced by the resonant tunneling and Δ_L is the level spacing in the leads. These energy costs are larger than T_K whenever $E_c > \Delta_L$, suggesting that for $E_c \gg \Delta_L$ a multichannel Kondo effect will form.

We consider a model where the spin on the "small" quantum dot (represented by Abrikosov fermions $f_\sigma^\dagger$) couples to several larger dots displaying charging effects:

$$H = \sum_{k\sigma\alpha} \epsilon_k c^\dagger_{k\sigma\alpha} c_{k\sigma\alpha} + \sum_\alpha E_c \left[\sum_{k\sigma} : c^\dagger_{k\sigma\alpha} c_{k\sigma\alpha} : \right]^2 + \frac{J}{NK} \sum_{k,k'} \sum_{\sigma,\sigma'=1}^{N} \sum_{\alpha,\alpha'=1}^{K} f^\dagger_{\sigma'} f_\sigma c^\dagger_{k\sigma\alpha} c_{k'\sigma'\alpha'}, \quad (1)$$

where $: \ldots :$ denotes normal ordering and we assume nondegenerate charge states in the leads. In view of greater generality that will be useful in the following, we suppose an arbitrary number K of interacting leads, and we consider an SU(N) spin in a representation which fulfills the constraint $\sum_{\sigma=1}^{N} f^\dagger_\sigma f_\sigma = N/2$. For $N = 2$, the last term in Eq. (1) describes (up to a potential scattering term) the usual exchange coupling of a spin 1/2 to a symmetric combination of electrons from all leads, $c^\dagger_{k\sigma,s} = (1/\sqrt{K}) \sum_{\alpha=1}^{K} c^\dagger_{k\sigma\alpha}$. Therefore the conventional one-channel Kondo effect develops for $E_c = 0$. Note the symmetric coupling of the spin to all leads; as mentioned above, this requires fine tuning in experiments.

In order to make contact with previous results [3,4], we first use the perturbative renormalization group (RG) to discuss the case of large charging energy $T_K \ll E_c \ll D$, where D is an ultraviolet cutoff set by the bandwidth or the charging energies in the small dot. For cutoffs large compared to E_c, the Coulomb blockade does not modify the RG flow for the dimensionless coupling $j = JN_f$ (N_f is the density of states). For a running cutoff $\Lambda \gg E_c$, one therefore finds within one-loop RG $\partial j / \partial \ln\Lambda = -j^2$ and at the scale $\Lambda = E_c$ the running $j(\Lambda)$ takes the value $j(E_c) = 1/\ln[E_c/T_K]$ with $T_K = De^{-1/j}$ [9] and $j(E_c) \ll 1$ as $E_c \gg T_K$. For $\Lambda \ll E_c$ the charge on each lead is conserved separately and all processes which mix channels are frozen out. Therefore one expects a flow [1] to the K-channel Kondo fixed point with $\partial j / \partial \ln\Lambda = -j^2/K$ with the flow starting at $j(E_c)$. From this we can read off the multichannel Kondo temperature [9] for $T_K \ll E_c \ll D$

$$T_K^{\text{multi}} \approx E_c e^{-K/j(E_c)} \approx T_K (T_K/E_c)^{K-1}. \quad (2)$$

For $E_c > D$, the multichannel equation determines the RG flow alone as channel mixing is suppressed, which leads to the usual result $T_K^{\text{multi}} = De^{-K/j}$. Figure 1 shows the enhancement of T_K^{multi} with decreasing E_c.

To analyze the physics at small E_c, we introduce a phase (or slave rotor) representation [10,11] of the charging energy:

$$H = \sum_{k\sigma\alpha} \epsilon_k a^\dagger_{k\sigma\alpha} a_{k\sigma\alpha} + \sum_\alpha E_c \hat{L}_\alpha^2 + \frac{J}{NK} \sum_{kk'\sigma\sigma'\alpha\alpha'} f^\dagger_{\sigma'} f_\sigma a^\dagger_{k\sigma\alpha} a_{k'\sigma'\alpha'} e^{i\theta_\alpha - i\theta_{\alpha'}}, \quad (3)$$

where we have set $c^\dagger_{k\sigma\alpha} = a^\dagger_{k\sigma\alpha} \exp(i\theta_\alpha)$ and $L_\alpha = -i\partial/\partial\theta_\alpha$. This effective *matrix* of Kondo couplings $J_{\alpha\alpha'}(\theta) = J \exp(i\theta_\alpha - i\theta_{\alpha'})$ allows us to reinterpret the original idea of Oreg and Goldhaber-Gordon. At large E_c the phases θ_α fluctuate wildly in $J_{\alpha\alpha'}(\theta)$ for $\alpha \neq \alpha'$,

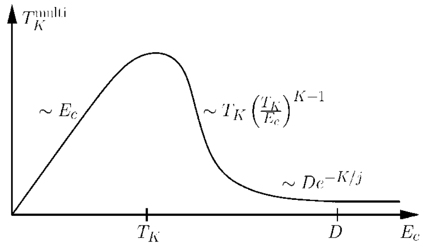

FIG. 1. Schematic plot of the multichannel Kondo temperature T_K^{multi} versus E_c. Note the maximum for $E_c \sim T_K$.

which leads to a flow towards a diagonal coupling at low-energy. In this case, channel number is conserved and a multichannel Kondo effect will develop. However, for E_c equal to zero strictly, the phases are locked and drop from the Kondo coupling in Eq. (3), leaving a model displaying Fermi-liquid properties and complete screening of the spin.

To study the effects of a small $E_c \ll T_K$ within perturbation theory, we expand Eq. (3) around constant θ_α. This expansion is valid as long as $\langle (\theta_\alpha(\tau) - \theta_\alpha(\tau'))^2 \rangle \ll 1$, which allows us to reliably calculate the correction ΔF to the free energy for $T \gg E_c$. After some straightforward but tedious manipulations we obtained

$$\frac{\Delta F}{N} \approx -\frac{K-1}{2K} \int_0^\beta d\tau \langle [\theta(\tau) - \theta(0)]^2 \rangle T(\tau) G_0(\tau) \approx \frac{K-1}{\pi^2 K} \int_T^{T_K} \frac{2E_c}{\omega^2} \omega \approx 2 \frac{K-1}{\pi^2 K} E_c \log\left[\frac{T_K}{T}\right], \quad (4)$$

where $T(\tau)$ is the T matrix of the symmetric channel and $G_0(i\omega_n) = \sum_k 1/(i\omega_n - \epsilon_k)$ the bare local Green's function of the conduction electrons. For the last equality in (4) we used that $\pi N_f \text{Im} T(\omega) = 1$ for $\omega, T \ll T_K$ which follows from Friedel's sum rule. It is therefore valid for $E_c \ll T \ll T_K$. Up to (known) prefactors of order 1, the impurity entropy in this regime is thus given by

$$S(T)/N \approx T/T_K + E_c/T, \quad (5)$$

where the first term arises from the usual Kondo effect and the second from the charge fluctuations described by Eq. (4). This proves that the impurity entropy will show a minimum at the scale $T_S = \sqrt{E_c T_K}$ and implies a *negative* impurity specific heat for $E_c \ll T \ll T_S$ (the specific heat of the total system $\sim T/\Delta_L$ remains positive as we assumed a negligible level spacing $\Delta_L \ll E_c$). Moreover, the entropy per spin reaches values of order 1 for $T \sim E_c$ where the expansion breaks down, opening the possibility of a flow towards a multichannel fixed point. When we take into account that the crossover at E_c takes place deep in the strong-coupling regime, this suggests that the corresponding multichannel Kondo temperature is directly given by E_c, as shown schematically in Fig. 1.

To be able to describe the crossover at E_c and the physics for $T \lesssim E_c \ll T_K$ we need a nonperturbative method. We have found an exactly solvable limit of the Hamiltonian (1) that confirms the previous calculations, and also offers a direct computational tool to describe the crossover from single-channel to multichannel physics as a function of T and charging energy. The idea is to solve the problem by taking both N and K to be large (we recall that we have considered a generalized model with an SU(N) quantum spin in the dot and $K \equiv \gamma N$ interacting leads coupled to it). The technical step is to notice that the Kondo interaction in (3) can be decoupled using a *single* bosonic field $B(\tau)$ conjugate to $\sum_{k\sigma\alpha} f_\sigma^\dagger a_{k\sigma\alpha} \exp(-i\theta_\alpha)$. Integrating out the leads, we obtain the following action in imaginary time:

$$S = \int_0^\beta d\tau \frac{KB^\dagger B}{J} + \sum_\alpha \frac{(\partial_\tau \theta_\alpha)^2}{4E_c} + \sum_\sigma f_\sigma^\dagger (\partial_\tau - \mu) f_\sigma + \mu \frac{N}{2} + \int_0^\beta d\tau \int_0^\beta d\tau' \frac{G_0(\tau - \tau')}{N} \times \sum_{\sigma\alpha} [f_\sigma^\dagger B e^{-i\theta_\alpha}]_\tau [f_\sigma B^\dagger e^{i\theta_\alpha}]_{\tau'}, \quad (6)$$

where μ is a complex Lagrange multiplier used to enforce the constraint on the spin size. We can introduce [12] two fields $Q(\tau, \tau')$ and $\overline{Q}(\tau, \tau')$ to decouple fermions from bosons in the last term of (6). Then the $f_\sigma^\dagger$ and θ_α variables are integrated out to obtain an effective action $S[B, Q, \overline{Q}, \mu]$ which is proportional to N and therefore solved by a saddle point when $N \to \infty$. Using time-translational invariance and particle-hole symmetry (so that $\mu = 0$), we obtain the self-consistent equations

$$G_f(i\omega_n) \equiv \langle f^\dagger(i\omega_n) f(i\omega_n) \rangle = \frac{1}{i\omega_n - B^2 Q(i\omega_n)}, \quad (7)$$

$$G_X(\tau) \equiv \langle e^{i\theta(\tau) - i\theta(0)} \rangle, \quad (8)$$

$$Q(\tau) = \gamma G_0(\tau) G_X(\tau), \quad (9)$$

$$\overline{Q}(\tau) = -B^2 G_0(\tau) G_f(\tau), \quad (10)$$

$$\frac{1}{J} = \int_0^\beta d\tau\, G_0(\tau) G_X(\tau) G_f(\tau), \quad (11)$$

where the condensate B is determined from Eq. (11) and the correlator $G_X(\tau)$ is computed from the action

$$S = \int_0^\beta d\tau \frac{(\partial_\tau \theta)^2}{4E_c} + \int_0^\beta d\tau \int_0^\beta d\tau'\, \overline{Q}(\tau - \tau')\, e^{i\theta(\tau) - i\theta(\tau')}. \quad (12)$$

In principle, the model (12) can be solved very efficiently by Monte Carlo [13], but we have chosen to simplify the numerics by using the spherical limit as an approximation to this rotor model. This is done by introducing a further large M expansion, where we generalize $e^{i\theta} \equiv X$ with the constraint $|X|^2 = 1$ to M fields X_i with $\sum_{i=1}^M |X_i|^2 = M$(see Ref. [11] for details). We then obtain

$$G_X^{-1}(i\nu_n) = \nu_n^2/(4E_c) + \lambda + \overline{Q}(i\nu_n), \quad (13)$$

$$G_X(\tau = 0) = 1. \quad (14)$$

The parameter λ is determined from Eq. (14), reflecting the constraint $|e^{i\theta}|^2 = 1$ in average.

We first investigate the general properties of the system of Eqs. (7)–(11) and (13) and (14). For $E_c = 0$ we recover nicely the usual large-N limit of the single-channel Kondo model [14], since $G_X(\tau) = 1$ follows from Eqs. (13) and (14) in this case. For $E_c \neq 0$, this behavior also holds at $E_c \ll T$, reflecting the fact that the single-channel Kondo fixed point controls the regime $E_c \ll T \ll T_K$. However, at $T = 0$ a low-frequency analysis of our integral equations [11,12] shows the appearance of universal power laws characteristic of the multichannel fixed point [12], $G_f(i\omega) \sim (1/i\omega)|\omega|^{1/(1+\gamma)}$ and $G_X \sim 1/|\omega|^{1/(1+\gamma)}$.

Our equations allow us also to calculate physical quantities at intermediate coupling. Figure 2 shows the impurity entropy, i.e., the difference of the total entropy and the entropy in the absence of the spin. As predicted by Eq. (5), there is a minimum in $S(T)$ at the scale $T_S \simeq \sqrt{E_c T_K}$ and the specific heat is negative for $E_c \lesssim T < T_S$. Close to the low-T fixed point the specific heat is positive. Accordingly, the entropy drops below E_c and for $T \to 0$ reaches the finite value characteristic for the K-channel fixed point [12]. The scaling plot in the inset of Fig. 2 shows that E_c can be identified with the multichannel Kondo temperature $T_K^{\rm multi}$ if $E_c \ll T_K$, as discussed above. For $E_c > T_K$, we find only the free-spin solution with $B = 0$ and no multichannel physics. This is consistent with Eq. (2), which shows that $T_K^{\rm multi}$ drops rapidly to 0 for $K \to \infty$ if $E_c > T_K$. For finite K and low T, multichannel behavior is of course maintained for arbitrarily large E_c.

Although this thermodynamic analysis provides interesting insights into the model considered here and the general question of how long-range interactions can

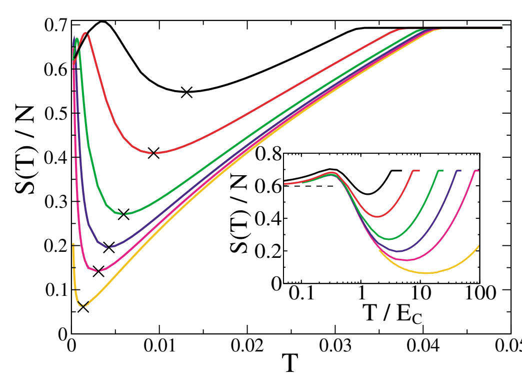

FIG. 2 (color online). Entropy $S(T)$ for $T_K/E_c = 400$, 80, 40, 20, 8, 4 (bottom to top) with $JN_f = 1/4$, $\gamma = 1$ and a flat density of states of half-width $D = 2$ (giving $T_K \approx 0.04$). The minimum of S is located at $T_S \approx 0.7\sqrt{E_c T_K}$ (crosses). For $T \ll T_S$ and $E_c \ll T_K$, the entropy is a function of T/E_c only (see inset), which shows that E_c can be identified with the multichannel Kondo temperature $T_K^{\rm multi}$ in this limit. The dotted line in the inset denotes the exact value for $S(T = 0)$ taken from Ref. [12]. Note that in the regime $0.4E_c < T < T_S$, the impurity specific heat $C = TdS/dT$ is *negative*.

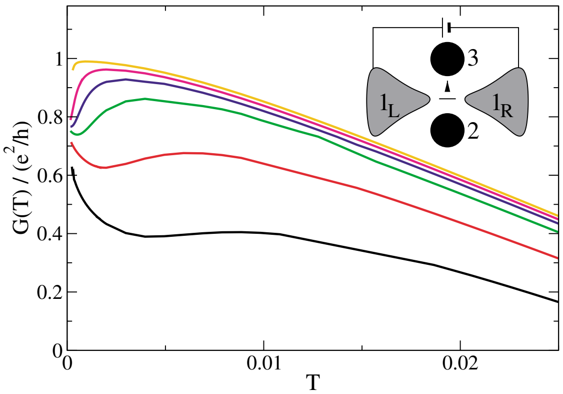

FIG. 3 (color online). Conductance $G(T)$ for $E_c = 0.0001$, 0.0005, 0.001, 0.002, 0.005, 0.01 (top to bottom) as in Fig. 2. For $T \to 0$ and $\gamma = 1$, G approaches the universal value $G = \pi G_0/4$. In contrast to the entropy (inset of Fig. 2), large nonuniversal corrections spoil the one-parameter scaling with T/E_c. Inset: Experimental setup suggested in Ref. [3]. Two leads (1_L and 1_R) are connected to a voltage source, the other "leads" 2, 3 are large quantum dots with a charging energy E_c and small level spacing $\Delta_L \ll E_c$, prohibiting charge transport between 1 and 2, 3. Therefore, a mapping onto our Hamiltonian (1) with $K = 3$ is possible, where channel 1 arises from the even combination of electrons [3,4] in 1_L and 1_R. However, fine tuning is required [4] to obtain symmetric coupling to all leads at lowest energies.

destabilize zero-entropy fixed points, it is almost impossible to measure the entropy of quantum dots. Therefore we have also calculated the conductance through the dot using a setup suggested by Oreg and Goldhaber-Gordon [3], which is sketched in Fig. 3. In a situation where the odd combination of electrons in leads 1_L and 1_R decouples from the dot, the linear conductance can be calculated from the imaginary part of the T matrix [4,15]:

$$G = \frac{Ne^2}{2K\pi\hbar}\int d\omega \frac{\partial n_F(\omega)}{\partial\omega}\mathrm{Im}G_0(\omega)\mathrm{Im}T(\omega), \qquad (15)$$

$$T(\tau) = B^2 G_X(\tau)G_f(\tau). \qquad (16)$$

For $E_c \ll T \ll T_K$, the single-channel Kondo effect results in resonant scattering among the K equivalent leads and therefore in a conductance $G \approx (N/K)G_0 = (1/\gamma)G_0$ with $G_0 = e^2/(2\pi\hbar)$. For $T \ll E_c \approx T_K^{\mathrm{multi}}$, the conductance is governed by the multichannel T matrix which has been calculated from conformal field theory by Parcollet *et al.* [12] for arbitrary K and N. For $K = N = 2$ one obtains $G = G_0$, while in the large N limit one gets $G = G_0\,\pi/(2 + 2\gamma)\tan[\pi/(2 + 2\gamma)]$.

We believe that all the results obtained in the previous large N and K limit are qualitatively valid for the experimentally relevant case $N = 2$ and $K = 2$ or 3. First, this is indicated by our perturbative expansion (4) for $T \gg E_c$ and $E_c \ll T_K$ which proves the existence of the minimum in $S(T)$ for *arbitrary* values of N and K. Second, the Coulomb blockade of the leads makes the multichannel fixed point obviously stable against interlead tunneling. Finally, we have verified our scenario for $K = 2$ in a strong-coupling expansion of the Hamiltonian (1), taking first the limit $J \to \infty$ (and therefore $T_K \to \infty$) and analyzing the resulting model for large $E_c \ll J$. In this limit we recover a two-channel Anderson model which can be mapped via a Schrieffer-Wolff transformation to the two-channel Kondo model.

In conclusion, we have shown that tiny charging energies in the leads, $E_c \ll T_K$, can destabilize the single-channel Kondo effect and induce a flow towards the multichannel Kondo fixed point. While in systems with local interactions the g theorem [7,8] guarantees that the impurity entropy always decreases with decreasing T, in our case it will rise for $E_c \lesssim T \lesssim \sqrt{E_c T_K}$. Our observation that the multichannel Kondo fixed point can be stabilized even by small charging energies should help to realize the experimental setup proposed by Oreg and Goldhaber-Gordon [3]. To obtain a high multichannel Kondo temperature T_K^{multi}, parameters with $E_c \approx T_K$ seem to be the most promising candidate (see Fig. 1).

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