Observation of Half-Quantum Defects in Superfluid ^{3}He-B

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In the course of high-precision measurements of the relation between the superflow current J through a weak link in ^{3}He-B and the difference in order parameter phase between each side of the link φ in a flexible wall Helmholtz resonator equipped with a rotation pickup loop, we have observed the signature of a stable textural defect that sustains a change of the phase by π across it. "Cosmiclike" solitons, proposed by Salomaa and Volovik and hitherto thought unstable, can constitute such a defect.

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Experimental studies of Josephson supercurrents in superfluid ^{3}He have revealed a number of interesting features [1]. Quasisinusoidal relations between the current J through a weak link and the difference of the order parameter phase ϕ on either side $\varphi = \phi^R - \phi^L$ have been observed close to the superfluid transition temperature T_c in a way quite reminiscent of s-wave superconductors [2]. Other features pertain more directly to the p-wave character of superfluid ^{3}He.

In particular, a number of different determinations of $J(\varphi)$ have been found to exist in the B phase in various types of weak links, arrays of pinholes [3], and finite size apertures [4], which result from different possible arrangements of the order parameter near the link. This order parameter is characterized by a rotation $R_{\mu j}$ by angle θ about unit vector $\hat{\mathbf{n}}$. In the case of interest here, θ is fixed by the dipolar coupling to its bulk value, the Leggett angle $\theta_L = 104°$; the orientation of $\hat{\mathbf{n}}$ represents the important parameter. The combined effects of walls, applied magnetic fields, and bending forces, together with the particular conditions of a given cooldown through the transition temperature, determine the $\hat{\mathbf{n}}$ textures, the topological defects ($\hat{\mathbf{n}}$ solitons) that may appear [5], and the determination of $J(\varphi)$. These features have been studied theoretically by Yip [6] and by Viljas and Thuneberg [7–9], among others.

Besides $\hat{\mathbf{n}}$ solitons, there may also exist domain walls, not stabilized by topology, separating two regions of ^{3}He-B with different $\hat{\mathbf{n}}$ vectors. These planar structures have been studied by Salomaa and Volovik [10] in the framework of the Ginzburg-Landau theory and correspond to a local minimum of the interfacial free energy. These objects, which can sustain an order parameter phase difference of π, are analogous to nontopological solitons already known in cosmology [11].

In the course of series of high-precision measurements of $J(\varphi)$ in ^{3}He-B, in which $\hat{\mathbf{n}}$ textures are controlled by magnetic fields, we have observed unexpected shifts by π of the current-phase relation, which we report here [12].

The experimental setup, shown in Fig. 1, is similar to that of Ref. [13] but for details in the pickup loop geometry and for the weak link. Two different weak links have been used over a two year period, a $0.13 \times 3.0\ \mu\text{m}^2$ slit and an array of 198 microholes 0.08 μm in diameter and 2 μm apart, nanofabricated in a 0.1 μm thick SiN window. The nuclear demagnetization cryostat can be oriented about its vertical axis so that the flux of the Earth rotation vector $\mathbf{\Omega}_\oplus$ picked up by the superfluid loop can be varied. In this manner, the phase difference across the weak link φ can be precisely controlled.

Temperature is measured with a Pt NMR spectrometer calibrated at T_c and by a lanthanum cerium magnesium nitrate (LCMN) thermometer. The latter can be operated in a feedback mode to control the temperature of the cell by regulating the demagnetization current. Two pairs of Helmholtz coils produce magnetic fields of up to 340 G either parallel, $H_\parallel$, or perpendicular, $H_\perp$, to the flow direction through the weak link.

The Helmholtz resonator geometry is shown approximately to scale in Fig. 1. In applied $H_\parallel \gtrsim 50$ G, $\hat{\mathbf{n}}$ should be approximately normal to the plane of the weak link, either parallel or antiparallel on both sides of the link. In $H_\perp \gtrsim 50$ G, $\hat{\mathbf{n}}$ should be at an angle with both $H_\perp$ and $\hat{\mathbf{z}}$, which is likely to differ between the upper and lower chambers of the resonator.

The phase difference φ across the weak link is related to the pressure difference by the Josephson ac relation $\hbar\dot{\varphi} = 2m_3\delta P/\rho$, ρ being the density and m_3 the atomic mass. The current through the Josephson link is quite generally a 2π-periodic function $J(\varphi)$ of the phase difference, with different determinations coming from different orientations of $\hat{\mathbf{n}}$. The time evolution of φ is governed by a resonance equation [14],

$$\ddot{\varphi} + \eta\dot{\varphi} + \Omega_1^2[\varphi + \Lambda J(\varphi)] = \Omega_1^2(\varphi_x + \varphi_{\text{drive}}), \quad (1)$$

which, when the weak link contribution $J(\varphi)$ is neglected, describes a purely harmonic motion at angular

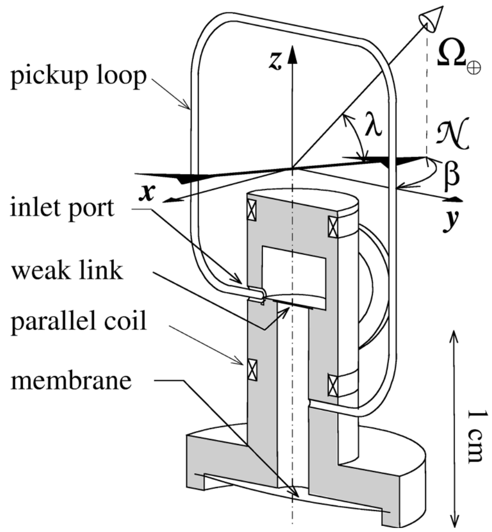

FIG. 1. Schematic view of the cell, approximately to scale for the inner parts, except for the loop, which is made of two turns of 0.4 mm internal diameter capillary (only one turn is shown) with total area of $5.90 \pm 0.10\ \mathrm{cm}^2$ from caliper measurements. The lower chamber of the resonator is a cylindrical duct, 1 mm in diameter, and connects the weak link to the flexible diaphragm and to one end of the pickup loop; the upper chamber, a squat cylinder, is connected to the other end of the loop and to an inlet toward the main superfluid bath in which the resonator is immersed. The "parallel" coils produce $H_{\parallel}$ parallel to the flow through the weak link; only one coil for $H_{\perp}$ is shown. The cryostat is rotated about the vertical axis z by angle β from the North, shown by a compass needle; λ is the latitude, $48°43'$, at Saclay.

frequency Ω_1. The nonlinear mass current through the weak link adds a term $\Lambda J(\varphi)$ with $\Lambda = 2\pi l_1/\kappa_3 \rho_s s_1$, l_1 being the length of the loop, s_1 its cross section, ρ_s the superfluid density, and κ_3 the quantum of circulation in ^{3}He. The damping parameter η is assumed negligible in the following.

The electrostatic drive applied to the membrane is represented by the term φ_{drive}; φ_x is the dc component of the residual phase difference along the loop and is made up of two distinct contributions,

$$\varphi_x = A \sin\beta + \varphi_b. \tag{2}$$

The first arises from the motion of the laboratory frame with respect to the distant stars along with the rotation of the Earth, $\mathbf{\Omega}_{\oplus}$, as discussed in Ref. [13]: $A = (2\pi/\kappa_3)\, 2\Omega_{\oplus} S_{\mathrm{loop}} \cos\lambda$, S_{loop} being the area of the superfluid pickup loop (the rotation antenna), λ the latitude, and β the angle specifying the orientation of the loop with respect to the North (see Fig. 1). The second contribution is the residual phase bias φ_b, which arises from the presence in the loop of stray currents, or, possibly, from textural defects as will be discussed below.

The resonator rest point, that is, the value φ_0 of φ at complete standstill, is obtained from Eq. (1) by removing all the time-dependent terms:

$$\Lambda J(\varphi_0) = \varphi_x - \varphi_0. \tag{3}$$

When β is changed by reorienting the cryostat about the vertical axis, the phase difference φ_x varies by an accurately controlled amount; φ_0 and J adjust according to Eq. (3).

The resonator natural frequency ω for small amplitude motion about the rest point is obtained by linearizing Eq. (1) and taking the Fourier transform:

$$\frac{\omega^2}{\Omega_1^2} = 1 + \Lambda \frac{dJ(\varphi)}{d\varphi}\bigg|_{\varphi=\varphi_0}. \tag{4}$$

The measurement of ω and Ω_1 yields the derivative of the current with respect to the phase. Since ω^2 is a positive quantity, $dJ(\varphi)/d\varphi|_{\varphi_0} \geq -\Lambda^{-1}$. When this condition is not fulfilled, the resonator rest point becomes unstable and settles to another, stable, operating point also satisfying Eq. (3): $(\varphi_x - \varphi)/\Lambda$ defines the "load line" of current generator $J(\varphi)$.

The raw data consist of sets of values of the small-signal frequency ω at different values of the cryostat angle β. Each data point takes approximately 4 min to collect. Data acquisition, cryostat rotation, temperature regulation, and dilution fridge monitoring are performed by a network of computers. The frequency Ω_1 is measured independently by driving the resonator to very large amplitudes. Both A and Ω_1 are subsequently finely adjusted in the analysis so that $J(\varphi)$ is exactly, as discussed below, an odd function with period 2π. The final value of A, $0.842 \times 2\pi$, is 1.5% smaller than the value estimated from measured dimensions.

Examples of these raw data at 0.2 bar for the two (out of at least four) most commonly met determinations of $J(\varphi)$ that set in when cooling through T_c in $H_{\parallel}$ are given in Fig. 2(a). A given determination, i.e., a given textural state, survives slow temperature cycling (while remaining below T_c) and moderate mechanical perturbations. More determinations can also be reached by varying the magnetic field amplitude and direction at low temperature.

In this Letter, we concentrate on the following features of the data: (i) the extrema of $\omega(\beta)$ occur approximately at the same values of β from one cooldown to another; (ii) most of the $\omega(\beta)$ curves come in pairs, such as (●),(○) and (▲),(△) in Fig. 2(a), with nearly the same minimum and maximum values, but with the angular positions of these minima and maxima interchanged.

To explore further these peculiarities, we proceed to derive $J(\varphi)$ from the raw $\omega(\beta)$. Taking the derivative of Eqs. (2) and (3) with respect to φ $(=\varphi_0)$ and eliminating $dJ/d\varphi$ with the help of Eq. (4) yields $d\beta/d\varphi$. The integration of $d\varphi/d\beta$ with respect to β gives:

$$\varphi = \varphi_i + A \int_0^{\beta} \frac{\Omega_1^2}{\omega^2} \cos\beta' d\beta'. \tag{5}$$

The current through the weak link is then obtained

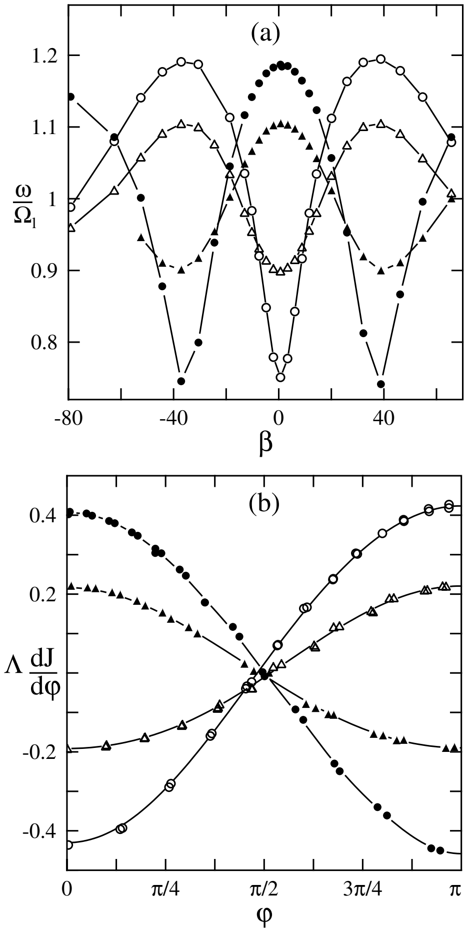

FIG. 2. (a) Small signal frequency normalized by Ω_1 versus cryostat angle in degrees. (b) Derivative of the current-phase relation, multiplied by Λ, versus phase difference in radians, for four different data sets observed in a field of $\sim$170 G applied parallel to the flow through the array weak link at $0.82T_c$, $P = 0.2$ bar, pertaining to the two most frequently obtained determinations of $J(\varphi)$ and their π-shifted counterparts.

straightforwardly from Eq. (3):

$$J = \Lambda^{-1}\{\varphi_b + A\sin\beta - \varphi(\beta)\}. \tag{6}$$

The integration constant φ_i in Eq. (5) fulfills the requirement that $J = 0$ for $\varphi = 0$, which stems from the relation $J(-\varphi) = -J(\varphi)$ imposed by time reversal symmetry [8]. The periodicity by 2π further imposes that $J(\varphi) = -J(2\pi - \varphi)$, which yields that J also displays odd symmetry about $\varphi = \pi$. Thus, J is an odd function about $\pm n\pi$, $dJ(\varphi)/d\varphi$ is an even function and has extrema for $\varphi = \pm n\pi$, and so does $dJ(\varphi)/d\beta$ since $d\varphi/d\beta \neq 0$ in $-\pi/2 < \beta < \pi/2$. In this range of cryostat angles, there are at least two values, $\beta_{\min}$ and $\beta_{\max}$, for which the resonator frequency ω lies either at a minimum or at a maximum, corresponding to either $\varphi = 0$ or $\varphi = \pi$ modulo 2π. If φ_i is a satisfactory integration constant, so is $\varphi_i + \pi$; the current, as given by Eq. (6), is determined only to a shift of φ by π.

Assigning $\varphi = 0$ to the extrema of $\omega(\beta)$ close to $\beta = 0$, as discussed below, results in the current-phase relations shown in Fig. 2(b). The similarity between the curves (○) and (●) on the one hand, and (△) and (▲) on the other, becomes striking: they fall onto one another after being shifted by π along the φ axis. In moderate applied magnetic fields, only a small (≤ 5 in $H_{\parallel}$, ≤ 8 in $H_{\perp}$, more in $H = 0$) number of different determinations of $J(\varphi)$ are observed, so that the above association in pairs becomes quite conspicuous after a series of cooldowns.

We rule out that there would be pairs of different determinations of $J(\varphi)$ shifted by π such as $J_1(\varphi) = J_2(\varphi + \pi)$. No particular symmetry requires this to be the case and an exact match is improbable, not to mention exact matching of quite a few of the observed $J(\varphi)$'s. We conclude that an actual π shift in the bias occurs in some of the cooldowns through T_c. These π shifts are robust features; they survive temperature cycling, as long as $T < T_c$, and moderate mechanical perturbations.

By manipulating the magnetic field, we are able to change the current-phase characteristics. This change is evidence for a drastic rearrangement of the order parameter texture in the vicinity of the junction. This rearrangement never resulted in a change of the π shift. We interpret this observation as a strong indication that this extraneous phase difference of π takes place in the superflow loop and not in the weak link. Hence, we argue that these π shifts reveal the existence of defects that support a difference of π in the order parameter phase.

Vortices are known to give rise to phase biases depending on where they are pinned in the superfluid flow path. However, pinning forces are very weak in superfluid ^{3}He and a vortex does not readily get pinned on smooth walls, and, if it would, the odds are against repeatedly finding a bias very close to π. Other known objects that sustain a π phase difference are the cosmiclike solitons proposed by Salomaa and Volovik [10]. However, their stability is not guaranteed in bulk superfluid [15] and it is not known how they evolve when confined by the walls of the cell.

Shifts by π occur more frequently in $H_{\parallel}$ at 0.2 bar (as well as for rapid cooldowns). This field direction has two effects: (i) a local maximum of the magnetic field occurs at the level of the bottom Helmholtz coil where a domain of A phase should appear when crossing the superfluid transition, which vanishes at lower temperature, possibly leaving a wall defect behind; (ii) flare-out textures of $\hat{\mathbf{n}}$ form in the cylindrical chambers of the resonator (Fig. 1), creating regions where $\hat{\mathbf{n}}$ is already strongly bent and more likely to pop up a defect. The cosmiclike soliton would thus form in the lower resonator chamber. However, experiments conducted at 10 bars yield results opposite to those at 0.2 bar: in $H_{\parallel}$ far fewer π shifts are observed, while in $H_{\perp}$ they occur in 5 out of 21 cooldowns.

Observations that have been interpreted as π shifts in a loop have also been reported by Simmonds [16] in a

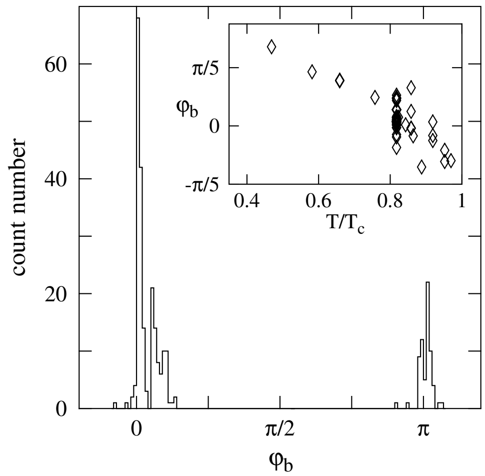

FIG. 3. Histogram of the phase bias φ_b at $T = 0.82\,T_c$, $P = 0.2$ bar for 274 cooldowns at various applied fields, for the array weak link. The inset shows the temperature variation of the non-π-shifted φ_b.

two-weak-link device. However, it does not seem possible in this work to ascertain whether these observations, sometimes accompanied by changes in $J(\varphi)$, come from a particular combination of the supercurrents through the two weak links or from the connecting loop.

We now justify the assignment of the true zero of φ made above. For s-wave superconductors and superfluid ^{4}He, the (Josephson) free energy F_J has a minimum at $\varphi = 0$, the current, $J = (2m_3/\hbar)\partial F_J/\partial\varphi$, goes to zero, and its derivative, $\partial J/\partial\varphi$, is positive; the resonator frequency ω goes through a maximum at $\varphi = 0$. The assignment is straightforward but cannot be carried over to p-wave superfluids. For p waves, the free energy F_J still has a minimum at $\varphi = 0$ when the order parameter matrix is the same on both sides of the junction: this situation corresponds to the highest critical current. But it can also be, for other order parameter arrangements, that F_J is at a maximum for $\varphi = 0$. Then, $J(\varphi)$ has negative slope at $\varphi = 0$ and ω is at a minimum [6–9].

Our choice for $\beta(\varphi = 0)$ is based on the following. The highest current determination is likely to be the s-wave-like determination, which has positive slope at $\varphi = 0$ [8]. This favors (●) in Fig. 2. Independently, we assume that the most frequently observed state of the loop is soliton free: (△) can be continuously tracked when rotating the field from $H_\perp$ to $H_\parallel$ and back but has *not* been observed to occur in cooldowns in $H_\perp$; only (▲) is observed, and is therefore favored. Both choices of (▲) and of (●) lead to the same $\beta(\varphi = 0)$.

These observations became possible only in the present cell because the bias, given by $\varphi_b = -A\sin\beta(\varphi)|_{\varphi=0}$, is not random as in previous cells: φ_b falls preferentially close to two values that differ by π (to experimental accuracy), as already seen in Fig. 2 and as shown for a large number of cooldowns by the histogram in Fig. 3. The bias depends on T in a reproducible manner, as shown in the inset, probably because of a residual heat current between the resonator chambers. Increasing the current in the magnetic field coils increases this temperature dependence. Estimates shows that even a tiny heat leak into the resonator can create a superfluid countercurrent that produces a velocity circulation of the correct order of magnitude. Hence, φ_b comes in part from the thermomechanical effect; another part is frozen in at the superfluid transition and is occasionally shifted by π.

In summary, we have repeatedly observed in a large number of cooldowns under various applied magnetic fields an anomalous shift of the phase bias by π across two types of weak links, a slit and an array of microholes. This robust feature survives field and temperature changes as long as the ^{3}He remains superfluid; it is not associated with the weak link and reveals the appearance of π-shifting defects elsewhere in the resonator along the superfluid flow path. Cosmiclike solitons, if they can live in the restricted geometry of the cell, could constitute such defects.

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