Comment on "Interferometric Detection of Optical Phase Shifts at the Heisenberg Limit"

Some time ago, Holland and Burnett [1] discussed a method of performing Heisenberg-limited (HL) phase shift measurements by driving a Mach-Zehnder interferometer (MZI) with a pair of Fock states of equal photon number: $|N\rangle \otimes |N\rangle$. According to [1], the uncertainty in the phase shift measurement approaches $\Delta\varphi_{\rm HL} = 1/(2N)$ asymptotically for large N. The authors arrived at this conclusion through the phase difference distribution of the state *inside* the MZI, which was deemed localized with a width given by $1/(2N)$ radians. Furthermore, the authors assumed that the phase shift itself could be detected by the usual method of subtracting the photocurrents at the output beam splitter of the MZI, i.e., measuring $2\hat{J}_3 = \hat{a}^\dagger_{\rm out}\hat{a}_{\rm out} - \hat{b}^\dagger_{\rm out}\hat{b}_{\rm out}$.

It turns out, however, that balanced detection is, in principle, not a functional method for interferometry with the given class of input states. The reason is apparent at once when we rewrite $2\hat{J}_3$ via the internal modes of the MZI, $\hat{a}^\dagger\hat{b}e^{i(\varphi-\vartheta)} + \hat{a}\hat{b}^\dagger e^{-i(\varphi-\vartheta)}$, where φ is the phase difference between the arms of the device and ϑ is a degree of freedom of the output beam splitter. This senses only single-photon transitions and because the twin Fock states produce even numbers of photons inside the MZI [2], $\langle\hat{J}_3\rangle = 0$. Hillery *et al.* [3] reexamined the original detection method, pointing out that there are, in fact, *two* narrow peaks, separated by π radians, in the phase difference distribution. More recently, the proposal of Ref. [1] was adapted to phase resolution measurements between two components of a Bose-Einstein condensate, with [4] and without [5] taking into account the problem of vanishing $\langle\hat{J}_3\rangle$.

Clearly, higher-order quantum transitions are required for a working scheme. Kim *et al.* [6] suggested as an alternative the next order moment, i.e., $\hat{S} = 4\hat{J}_3^2$; however, while this method attains the HL of phase sensitivity for high N, the signal-to-noise ratio (SNR) for the measurement of $\hat{S}$ itself, $\langle\hat{S}\rangle/\Delta S$, is rather low. In fact, from [3], SNR $\approx \sqrt{2}$, and this assumes ideal measurements.

In a recent paper [7], we reconsidered the Holland-Burnett method in the context of parity measurements performed on one of the output beams. Using the parity operator $\hat{O} = \exp(i\pi\hat{N}_{\rm out})$, where $\hat{N}_{\rm out}$ is the number operator of the output mode being monitored, we showed that $\langle\hat{O}\rangle = P_N(\cos 2\varphi)$, where $P_N(x)$ is a Legendre polynomial. Our numerical calculations indicate that $\Delta\varphi = \Delta O/|\partial\langle\hat{O}\rangle/\partial\varphi| \rightarrow \Delta\varphi_{\rm HL} = 1/(2N)$ for large N near $\varphi \approx 0$, with favorable SNR, $\langle\hat{O}\rangle/\Delta O$. The parity method encodes HL phase shifts because it inherently depends on all higher-order moments of photodetection. The measurement of parity could be accomplished by several possible techniques including photon counting with improved detectors with sensitivity at the level of a single photon. One also could possibly use homodyning to reconstruct the Wigner function near the origin of phase space, the Wigner function being the expectation value of the displaced parity operator [8,9].

The parity operator approach we advocate is an outgrowth of our researches on interferometry with maximally entangled states [7,10] where, again, one has $\langle\hat{J}_3\rangle = 0$. The ideas discussed here have been adapted to the problem of phase resolution between two Bose-Einstein condensates [11]. HL interferometry with parity detection was recently demonstrated for two maximally entangled trapped ions [12].

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