Force on a Charged Test Particle in a Collisional Flowing Plasma

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The force on a charged test particle embedded in a flowing (electron-ion) plasma is calculated using the linear dielectric response formalism. This approach allows us to take into account ion-neutral collisions self-consistently. The effect of collisions on the ion drag force is analyzed. It is shown that collisions can play a major role and can enhance the force substantially.

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The main forces acting on a charged particle embedded in a weakly anisotropic (unmagnetized) plasma have an electrostatic nature. They are usually exerted due to self-consistent large-scale electric fields and the plasma fluxes induced by these fields. The combination of the momentum transfer due to the relative plasma flow ("drag force") and the electrostatic force determines equilibrium plasma configurations. Knowledge of the drag force is especially important in complex (dusty) plasmas, when equilibrium states as well as dynamics of charged microparticles are considered. In this Letter, we calculate the force on a charged test particle embedded in a flowing (electron-ion) plasma using the linear dielectric response formalism. This approach allows us to take into account ion-neutral collisions self-consistently and also to retrieve the potential distribution around the particle. We analyze how the collisions change the flow-induced force on the particle and apply the obtained results to evaluate the ion drag force on charged microparticles in complex plasmas.

We start by calculating the potential around a pointlike test particle. The plasma flows with very small relative velocity $\mathbf{u}$ (much smaller than the ion thermal velocity $v_{T_i} = \sqrt{T_i/m_i}$, e.g., ambipolar drift in bulk plasmas). The equivalent problem is to calculate a potential around the particle moving through the plasma with the velocity $-\mathbf{u}$. The potential is [1]

$$\varphi(\mathbf{R}) = \int \frac{4\pi eZe^{i\mathbf{kR}}}{k^2\varepsilon(-\mathbf{ku}, k)}\frac{d\mathbf{k}}{(2\pi)^3}, \tag{1}$$

where $\mathbf{R}$ is the coordinate with respect to the particle center and Z is the particle charge number (positive or negative).

The plasma permittivity is $\varepsilon = 1 + \chi_e + \chi_i$, with the electron susceptibility $\chi_e \simeq (k\lambda_{\mathrm{D}e})^{-2}$. The ion susceptibility is [2]

$$\chi_i(\omega, k) = \frac{1}{(k\lambda_{\mathrm{D}i})^2}\left[\frac{1+\mathcal{F}(\xi)}{1+\frac{i\nu}{\omega+i\nu}\mathcal{F}(\xi)}\right], \qquad \xi = \frac{\omega + i\nu}{\sqrt{2}kv_{T_i}}. \tag{2}$$

Here $\lambda_{\mathrm{D}e,i} = \sqrt{T_{e,i}/4\pi e^2 n}$ is the electron or ion Debye length with n the unperturbed (electron or ion) density, and ν is the effective frequency of the ion-neutral collisions. The power series for the dispersion function is $\mathcal{F}(\xi) \simeq -2\xi^2 + i\sqrt{\pi}\xi$ (for $|\xi| \ll 1$), and the asymptotic series is $\mathcal{F}(\xi) \simeq -1 - \frac{1}{2}\xi^{-2}$ (for $|\xi| \gg 1$) [3]. We assume the flow is along the z axis. Introducing the ion mean-free path $\ell = v_{T_i}/\nu$ and the *thermal* Mach number $M_T = u/v_{T_i}$, we substitute $\omega = -k_z u$ in Eq. (2) and expand it into a series over small M_T. Then the first two expansion terms for the plasma permittivity are

$$\varepsilon(\mathbf{k}) \simeq 1 + \frac{1}{(k\lambda)^2} \times \begin{cases} \left(1 - iM_T\frac{k_z}{k^2\ell}\right), & M_T \ll k\ell \ll 1, \\ \left(1 - iM_T\sqrt{\frac{\pi}{2}}\frac{k_z}{k}\right), & k\ell \gg 1. \end{cases} \tag{3}$$

Here $\varepsilon(\mathbf{k}) \equiv \varepsilon(-k_z u, k)$ and λ is the linearized Debye length, $\lambda^{-1} \equiv \sqrt{\lambda_{\mathrm{D}i}^{-2} + \lambda_{\mathrm{D}e}^{-2}}$, which is assumed to be very close to the ion Debye length (since T_e is usually much larger than T_i). In the hydrodynamic limit ($k\ell \ll 1$), the plasma polarization along the flow is due to the ion collisions with neutrals. In the opposite weakly collisional limit ($k\ell \gg 1$) it is because of the ion Landau damping. Note that for $k\ell \ll M_T$ the permittivity scales as $\propto (M_T k_z)^{-1}$, or even $\propto k^{-2}$ (depending on k).

One can divide the potential in Eq. (1) into the "screened" and the "wake" parts: $\varphi = \varphi_0 + \varphi_{\mathrm{w}}$. The isotropic screened potential $\varphi_0(R) = (eZ/R)\exp(-R/\lambda)$ is determined by $\varepsilon(0, k) \equiv \varepsilon_0(k) \simeq 1 + (k\lambda)^{-2}$, so that the distortion due to the plasma flow is

$$\varphi_{\mathrm{w}}(\mathbf{R}) = \frac{eZ}{2\pi^2}\int\left[\frac{1}{\varepsilon(\mathbf{k})} - \frac{1}{\varepsilon_0(k)}\right]\frac{e^{i\mathbf{kR}}}{k^2}d\mathbf{k}. \tag{4}$$

Let us consider the potential profile along the z axis. We normalize the wave number and coordinate by the screening length, $k\lambda \to k$ and $z/\lambda \to z$, and, substituting Eq. (3) in Eq. (4), finally obtain for the wake potential:

$$\varphi_{\rm w}(z) \simeq -\frac{2eZM_T}{\pi\lambda z^2}\left[\frac{\lambda}{\ell}\int_0^{\lambda/\ell}\frac{\sin kz - kz\cos kz}{k(1+k^2)^2}dk + \sqrt{\frac{\pi}{2}}\int_{\lambda/\ell}^{k_{\max}}\frac{\sin kz - kz\cos kz}{(1+k^2)^2}dk\right]. \tag{5}$$

We extended the integration down to $k = 0$, because the contribution of $k \lesssim (\lambda/\ell)M_T$ to the integral is proportional to M_T^3 [since $\varepsilon^{-1}(\mathbf{k}) \propto M_T k_z$ in this range] and thus can be neglected. Of course, condition $(\lambda/\ell)M_T \ll 1$ should also be satisfied.

First, we set the upper limit $k_{\max}$ in the second integral Eq. (5) equal to infinity. At large distances ($|z| \gg 1$) Eq. (5) has the following asymptotic behavior: In the strongly collisional regime ($\lambda/\ell \gg 1$) the main contribution is due to the first term and the potential scales as $\propto -Zz/|z|^3$ [4], whereas in the absence of collisions ($\lambda/\ell \to 0$) the second (Landau damping) term yields $\propto -Z/z^3$ [5]. In a close vicinity of the particle ($|z| \to 0$), the contribution of the first term is $\propto -Zz$, but the second one yields a logarithmic divergence of the electric field, since $\varphi_{\rm w} \propto -Zz\ln|z|$ [6].

The wake potential $\varphi_{\rm w}$ represents the plasma polarization along the z axis. Let us discuss the behavior of $\varphi_{\rm w}$. The logarithmic divergence of the wake field at small z is, of course, unphysical. Formally, this occurs because the expression in the second integral in Eq. (5)—the ion Landau damping term—scales as $\propto k^{-1}$ at $1 \ll k \lesssim |z|^{-1}$. Physically, the wake electric field diverges because large values of k imply the contribution from a close vicinity of the test charge, where plasma perturbations are so strong that the linear response approach is no longer valid. For ions, this vicinity is approximately a sphere with the Coulomb radius $R_{\rm C} = e^2|Z|/T_i$. Linear theory applied inside this sphere yields absurd results: For instance, the total ion density becomes negative for $Z > 0$. Hence, the necessary condition for the linear approximation to be used is to have the whole range of interaction with the particle—the screening length—be much larger than the range of strong interaction—the Coulomb radius, i.e., $\lambda \gg R_{\rm C}$. Then the upper limit of the integration in Eq. (5) can be set approximately equal to $k_{\max} = \lambda/R_{\rm C}$, since the integral has "logarithmic convergence" at $k \gtrsim 1$.

The polarization along the plasma flow induces the electric force on the particle. Equation (5) yields the force $F = -eZ(d\varphi_{\rm w}/dz)|_{z=0} > 0$. Note that it is always pointed in the direction of the flux, and both contributions—collisional and the Landau damping—have the same sign. Substituting $k_{\max} = \lambda/R_{\rm C} \gg 1$ in Eq. (5), we derive at the "logarithmic accuracy":

$$F \simeq \frac{M_T}{3\pi}\left(\frac{eZ}{\lambda}\right)^2\left[\mathcal{K}(\lambda/\ell) + \sqrt{2\pi}\ln\frac{\lambda}{R_{\rm C}}\right], \tag{6}$$

$$\mathcal{K} = \frac{\lambda}{\ell}\arctan\frac{\lambda}{\ell} + \left(\sqrt{\frac{\pi}{2}} - 1\right)\frac{(\lambda/\ell)^2}{1+(\lambda/\ell)^2} - \sqrt{\frac{\pi}{2}}\ln\left(1+\frac{\lambda^2}{\ell^2}\right). \tag{7}$$

The function $\mathcal{K}$ represents the contribution of collisions. Figure 1 shows that for $\ell \gtrsim \lambda$ the "collisional function" scales as $\mathcal{K} \sim (\lambda/\ell)^4 \ll 1$ and is small compared to the Landau damping part (constant logarithmic term). When $\ell < \lambda$, we have $\mathcal{K} \sim \lambda/\ell \gtrsim 1$ and the collisional contribution can prevail, enhancing the force.

In order to ensure that the approximations we made above to derive Eqs. (6) and (7) do not affect the validity of the obtained analytical results, a series of numerical tests was performed. Using the tabulated dispersion function $\mathcal{F}(\xi)$, the electric force on the particle was obtained by direct numerical integration of Eq. (1) for different values of parameters λ/ℓ, M_T, and $k_{\max}$. Figure 2 shows the comparison of the analytical expression, Eq. (6), with the numerically calculated force. For the sake of convenience, the force is normalized to $\frac{1}{3\pi}M_T(eZ/\lambda)^2$, so that the analytical curves do not depend on M_T. One can see that the agreement between analytical and numerical results is fairly good [small offset $\lesssim 15\%$ at $\lambda/\ell \ll 1$ is due to the logarithmic accuracy of Eq. (6), i.e., because constant terms $O(1)$ were omitted compared to the logarithmic term]. It is noteworthy that the analytical formula, being formally derived for very small M_T, remains sufficiently accurate for relatively large Mach numbers. Deviation from the numerical data occurs at $\lambda/\ell \gtrsim 1$, and it is stronger for larger M_T because of the condition $(\lambda/\ell)M_T \ll 1$ imposed after Eq. (5). Hence, inequality $M_T \lesssim \ell/\lambda$ can be considered as a practical range of Mach numbers where

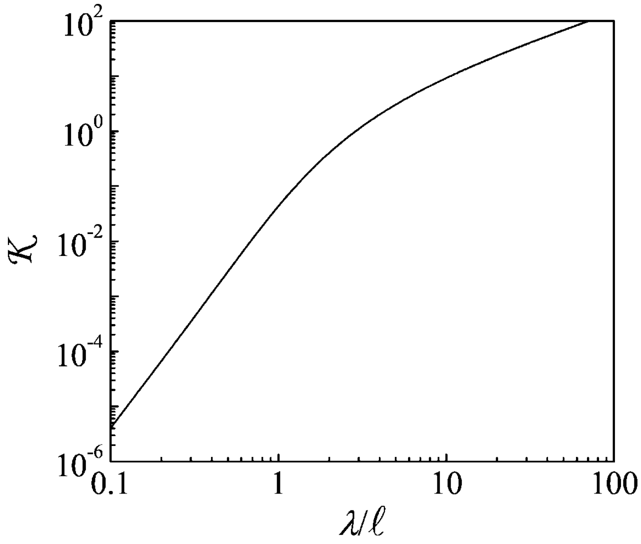

FIG. 1. Function $\mathcal{K}$ from Eq. (7) versus the ratio of the screening length to the ion mean-free path, λ/ℓ. The function characterizes the contribution of the ion-neutral collisions to the ion drag force [Eq. (6)].

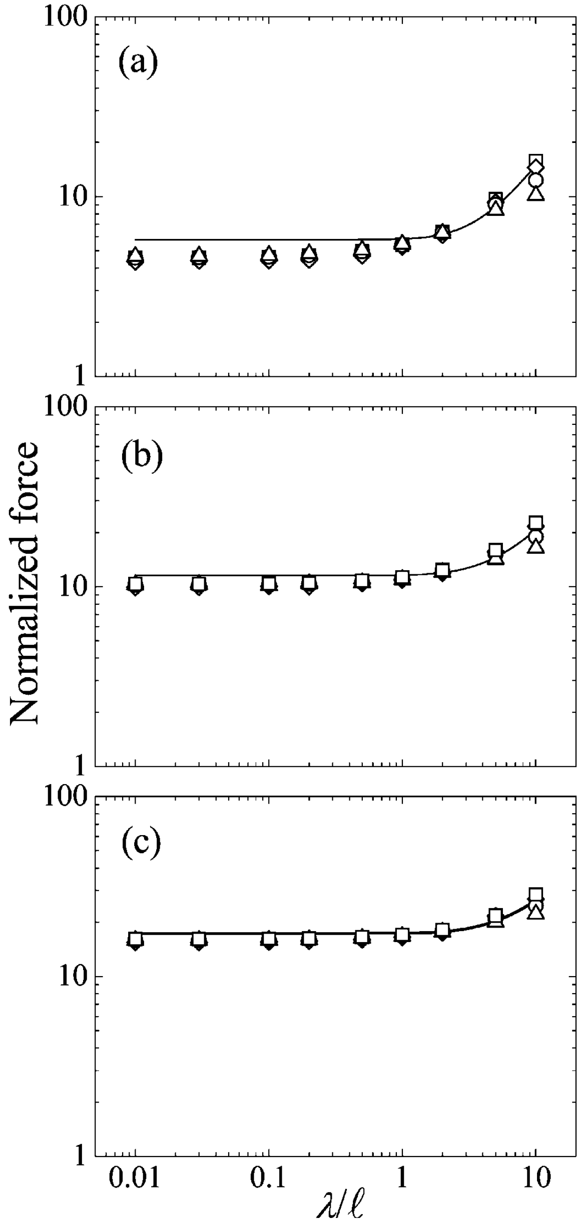

FIG. 2. Comparison of the analytical expression for the ion drag force [Eq. (6), solid line] with the numerically calculated force. The results are normalized and plotted as functions of the collisional parameter λ/ℓ for different values of the Mach number $M_T = u/v_{T_i}$ and the integration limit $k_{\max}$ (see text). Figures (a), (b), and (c) correspond to $k_{\max} = 10$, 10^2, and 10^3, respectively, and symbols represent $M_T = 0.01$ (□), 0.1 (◇), 0.2 (○), and 0.3 (△).

Eq. (6) is valid. One can also see that the dependence of the force on particular choice for the upper integration limit is indeed very weak (logarithmic, provided $k_{\max} \gg 1$).

In fact, the Landau damping contribution to the force in Eq. (6) is physically identical to the so-called "ion drag force": The Landau damping terms coincide with the well-known expression for the (collisionless) ion drag obtained in the limit of large Coulomb logarithm, $\ln(\lambda/R_C) \gg 1$ (e.g., Refs. [1,7]). The force is usually calculated assuming the unperturbed (isotropic) potential φ_0 around a particle, and the momentum transfer is derived from the trajectory deflection. The polarization potential φ_w derived here is essentially determined by a sum over these trajectories with the "unperturbed" ion density along them. Both approaches are obviously identical as long as only linear effects are considered [8,9]. In terms of the ion trajectories, the (nonlinear) polarization at small R is equivalent to the strong deflection of ions which occurs at $R \lesssim R_C$. The contribution of these ions to the force is logarithmically small—the resulting momentum transfer is $\sim \ln(\lambda/R_C)$ times smaller than the total force [7,8]. The latter justifies the choice of the upper cutoff $k_{\max} = \lambda/R_C$ in Eq. (5). Thus, it is natural to refer to the Landau damping term in Eq. (6) as the "collisionless ion drag," and to the collisional term as the "collisional correction." Collisions do not play a role while the ion mean-free path exceeds the screening length and function $\mathcal{K}$ is small; but in the opposite case, when $\ell \ll \lambda$, they become crucial. Figure 2, where the normalized force is plotted [i.e., terms in the brackets in Eq. (6)], clearly shows this transition, from the collisionless asymptote, $\sqrt{2\pi}\ln(\lambda/R_C)$, to the collisional one, $\frac{1}{2}\pi(\lambda/\ell) + \sqrt{2\pi}\ln(\ell/R_C)$. Note that the collisions diminish the argument of the Coulomb logarithm from λ/R_C to ℓ/R_C; when they become too frequent ($\ell \lesssim R_C$), the kinetic effects—the Landau damping—completely disappear and only hydrodynamic effects play a role. Then the wake potential and the corresponding force on the particle are determined by the first integral in Eq. (5). The upper limit of integration can be set equal infinity in this case, since the integral converges.

The reason why the collisions enhance the ion drag force can be understood in terms of the ion trajectories: Every collision with neutrals "eliminates" the angular momentum (with respect to the particle) the ion had before the collision due to the drift. Hence, the more collisions the ions experience while passing the particle within the Debye sphere, the more "radial" their trajectories become. This "collisionally induced" radial motion (superimposed on the drift) causes the ion trajectories to focus closer to the particle. The "focusing center" for the trajectories is obviously located downstream for the negatively charged particle. (If the particle is positively charged the ions are defocused, which simply means that the focusing center is upstream.) The focusing implies the local increase of the ion density and, thus, induces an additional electric field which increases the force.

Let us consider how the ion drag force changes when a particle has a finite size a. The correction occurs because a certain fraction of ions is absorbed on a particle: In terms of the ion orbits, the ions having an impact parameter smaller than the so-called "absorption radius," $\rho_{\rm abs}$, transfer their momentum in direct collisions with the particle [10]. The rest of the ions are scattered due to the electrostatic interaction and their contribution to the force is calculated as for a pointlike particle. The absorption radius depends on the ratio λ/ℓ and attains the maximum value $\rho_{abs} \sim R_C$ at $\lambda/\ell \sim 1$, because of the ion trapping inside the Coulomb sphere [11–13]. Hence, the ratio of the momentum transfer due to absorption to that of the electrostatic scattering does not exceed $\sim \ln^{-1}(\lambda/R_C)$. Therefore, the influence of the particle size on the ion drag is small and can be neglected as long as the Coulomb logarithm is large.

One can use the obtained results to calculate the ion drag force on a microparticle in complex plasmas. The advantage of Eq. (6) compared to the expressions used thus far is that the former allows us to take into account the role of ion-neutral collisions self-consistently. The (negative) particle charge is determined by the parameter $\gamma = e^2|Z|/aT_e$, which is always between a "few tenths" and a "few" (even when ion-neutral collisions are taken into account, e.g., Refs. [13,14]). We also introduce the electron-to-ion temperature ratio $\tau = T_e/T_i$, which is usually in the range $10 \lesssim \tau \lesssim 100$. Then Eq. (6) can be conveniently rewritten in terms of the plasma and particle parameters as follows:

$$F = \tfrac{4}{3}\tau^2\gamma^2 a^2 n m_i v_{T_i} u(\mathcal{K} + \sqrt{2\pi}\ln\Lambda), \qquad (8)$$

where the argument of the Coulomb logarithm is $\Lambda \equiv \lambda/R_C = (\tau\gamma)^{-1}(\lambda/a)$. As we discussed above, in the "collisionless" regime ($\ell \gtrsim \lambda$), function $\mathcal{K}$ can be neglected and Eq. (8) reduces to the standard expression for the ion drag force [1,7]. In the opposite "strongly collisional" regime ($\ell \ll \lambda$), when the Coulomb logarithm is small compared to $\mathcal{K}$, the force is solely determined by the collisional ion focusing, as shown in Fig. 2. Note that the transition between these two regimes is accompanied by change in the force dependence on pressure p: The drift velocity is usually determined by the ion mobility, with $u/v_{T_i} \simeq eE\ell/T_i \propto p^{-1}$ (subthermal ion drift, $u \lesssim v_{T_i}$, is often observed in the bulk plasma, where it is due to the ambipolar diffusion in a weak electric field E). Therefore, $F \propto p^{-1}$ in the collisionless regime (lower pressures). In the strongly collisional case the ion drag does not explicitly depend on p, because $\mathcal{K} \propto \ell^{-1} \propto p$ for $\ell \lesssim \lambda$. Note, however, that the other plasma parameters (i.e., n, γ, and τ) are generally some functions of p, and this can yield an additional dependence of the force on pressure.

In conclusion, let us discuss applicability of the obtained results for typical experimental conditions. The expression for the ion drag force, Eq. (8), is valid when the condition $\Lambda \gg 1$ is satisfied. The argument of the Coulomb logarithm scales as $\Lambda \propto a^{-1}n^{-1/2}T_e^{-1}T_i^{3/2}$. Therefore, one can immediately conclude that our approach may be applied in the bulk region of discharges for sufficiently rarefied plasmas with high ion (or low electron) temperatures, and/or for small (submicron) particles. For example, in Ar plasma at pressure $p = 1$ mbar with typical discharge parameters: $T_e = 1$ eV, $T_i = 0.03$ eV, and $n = 10^8$ cm^{-3}, the (ion) screening length is $\lambda \simeq 130$ μm and the normalized particle charge is $\gamma \simeq 1$ [13]. For half-micron particles ($2a = 0.5$ μm), we then derive $\Lambda \simeq 16$, i.e., Eq. (8) is well applicable. Also, the role of collisional focusing is already substantial for this example: The ion mean-free path is $\ell \simeq 15$ μm which yields $\lambda/\ell \simeq 9$ and, thus, the resulting collisional correction $\mathcal{K} \simeq 7$ (see Fig. 1) is equal to the collisionless contribution $\sqrt{2\pi}\ln\Lambda \simeq 7$.

In complex plasmas, however, one often has to deal with the situation when $\Lambda \lesssim 1$ (e.g., relatively big particles and/or high plasma density). Then the linear dielectric response formalism is no longer applicable: The range of the "nonlinear" ion-grain interaction, R_C, exceeds the screening length. For this situation, the ion drag force was derived recently by calculating trajectories for scattered ions [7,15,16]. Unfortunately, it is unclear how collisions with neutrals can be included self-consistently in this case. Their role for $\Lambda \lesssim 1$ still needs to be studied. Also, the influence of a finite particle size might be crucial under these circumstances, e.g., due to the change in the momentum transfer caused by the ion absorption [15,16], because of the effect of trapped ions [12,13], etc. Investigation of all these problems requires development of sophisticated numerical methods.

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