Evolution of the Dark Matter Distribution at the Galactic Center

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Annihilation radiation from neutralino dark matter at the Galactic center (GC) would be greatly enhanced if the dark matter were strongly clustered around the supermassive black hole (SBH). The existence of a dark matter "spike" is made plausible by the observed, steeply rising *stellar* density near the GC SBH. Here the time-dependent equations describing gravitational interaction of the dark matter with the stars are solved. Scattering of dark matter particles by stars would substantially lower the dark matter density near the GC SBH over 10 Gyr, due both to kinetic heating and to capture of dark matter particles by the SBH. This evolution implies a decrease by several orders of magnitude in the observable flux of annihilation products compared with models that associate a steep, dark matter spike with the SBH.

Neutralinos in supersymmetry are likely candidates for the nonbaryonic dark matter [1,2]. If neutralinos make up a large fraction of the dark matter in the Galactic halo, pair annihilations will produce an excess of photons which may be observed in gamma ray detectors [3]. The Galactic center (GC) is a promising target for such searches, since the dark matter density is predicted to rise as $\rho \sim r^{-1}$ at the centers of dark matter halos [4]. In addition, the GC contains a supermassive black hole (SBH) with mass $M_{\bullet} \approx 10^{6.5} M_{\odot}$ [5]. "Adiabatic growth" models, in which the SBH remains stationary as it grows, predict the formation of a steep power-law density profile around the SBH, a "spike," and an increase by many orders of magnitude in the amplitude of the neutralino annihilation signal [6].

The bulges of galaxies like the Milky Way are believed to have formed via mergers of preexisting stellar systems. If the latter contained SBHs, a merger would result in the formation of a binary SBH [7]. The density of stars and dark matter around a binary SBH drops rapidly as the binary ejects matter via the gravitational slingshot [8].

Evidence for the scouring effect of binary SBHs is seen at the centers of the brightest galaxies [9,10], where the *stellar* density profiles are nearly flat and sometimes even exhibit a central minimum [11]. However, in fainter elliptical galaxies and in the bulges of spiral galaxies like the Milky Way, steeply rising stellar densities are observed: $\rho_{\star} \sim r^{-\gamma}$, $1.5 \lesssim \gamma \lesssim 2.5$. In these galaxies, the most recent mergers may have taken place before the era in which SBHs formed, allowing the stellar density near the SBH to remain high. Since stars and dark matter respond similarly to the presence of a SBH, galaxies with steeply rising stellar densities are the most plausible sites for steeply rising dark matter densities and hence for the detection of annihilation radiation [12].

Stars near the SBH would also interact with the dark matter via gravitational scattering [13,14]. Here, the time-dependent equations describing the scattering of dark matter particles off of stars in the presence of a SBH are solved. Scattering decreases the density of a dark matter spike by kinetic heating, and by driving particles into the SBH. The result, after 10 Gyr, is the virtual dissolution of the dark matter spike in a galaxy like the Milky Way. This result suggests that enhancements in the dark matter density around the GC SBH would be modest whether or not the Milky Way bulge has experienced the scouring effects of a binary SBH.

Let r_h be the radius of gravitational influence of the SBH, with $M_*(r < r_h) = 2M_{\bullet}$. For the GC SBH, $r_h \approx 1.67$ pc [15]. After growth of the SBH (assumed to remain fixed with respect to the bulge), the dark matter density is approximately

$$\rho(r) = \rho(r_b) \times (r/r_b)^{-\gamma_{\rm sp}}, \qquad r \lesssim r_b \qquad (1a)$$
$$\times (r/r_b)^{-\gamma_c}, \qquad r \gtrsim r_b, \qquad (1b)$$

where $\gamma_{\rm sp} = 2 + 1/(4 - \gamma_c)$, $\rho \propto r^{-\gamma_c}$ is the dark matter density before growth of the SBH, and $r_b \approx 0.2 r_h$ [16].

An estimate of the local heating rate of dark matter particles due to gravitational encounters with stars is the change per unit time of $\epsilon = \frac{1}{2} m v_{\rm rms}^2$, the mean kinetic energy of the dark matter particles. Assuming Maxwellian velocity distributions for the stars and dark matter,

$$\frac{d\epsilon}{dt} = \frac{8(6\pi)^{1/2} G^2 \rho_{\star} m \ln\Lambda}{(v_{\rm rms}^2 + v_{\star,\rm rms}^2)^{3/2}} (\epsilon_{\star} - \epsilon), \qquad (2)$$

where $\epsilon_{\star} = \frac{1}{2} m_{\star} v_{\star,\rm rms}^2$, $\rho_{\star}$ is the stellar mass density, and $\ln\Lambda$ is the Coulomb logarithm [17]. Taking the limit $m \ll m_{\star}$ and assuming $v_{\rm rms} \approx v_{\star,\rm rms}$, appropriate shortly after the dark matter spike forms, the local heating time becomes

$$T_{\text{local}} \equiv \left| \frac{1}{\epsilon} \frac{d\epsilon}{dt} \right|^{-1} = \frac{0.0814 v_{\text{rms}}^3}{G^2 m_\star \rho_\star \ln\Lambda} \quad (3a)$$

$$\approx 1.8 \times 10^9 \text{ yr} \left(\frac{r}{1 \text{ pc}} \right)^{-0.1}; \quad (3b)$$

the latter expression uses the observed stellar mass density near the GC SBH, $\rho_\star \approx 3.2 \times 10^5 M_\odot \text{pc}^{-3}(r/1\text{pc})^{-\gamma}$, $\gamma = 1.4 \pm 0.1$ [15], and $v_{\star,\text{rms}} \approx 1.12(GM_\bullet/r)^{1/2}$, with $M_\bullet = 3 \times 10^6 M_\odot$. The Coulomb logarithm was set to $\ln\Lambda = \ln(0.4N)$, with $N \approx 6 \times 10^6$ the number of stars within r_h [18]. The time to heat the dark matter is nearly independent of radius and shorter by a factor ~ 5 than the age (~ 10 Gyr; [19]) of the stellar bulge.

The change with time of the dark matter density can be computed from the Fokker-Planck equation describing the evolution of $f(\mathbf{r}, \mathbf{v}, t)$, the mass density of dark matter particles in phase space, due to gravitational interactions with stars [20]. We assume that f is isotropic in velocity space, $f = f(E, t)$, with $E \equiv -v^2/2 + \phi(r)$ the binding energy per unit mass of a dark matter particle, and $\phi(r) = -\Phi(r)$, with $\Phi(r)$ the gravitational potential due to the SBH and the stars. The kinetic equation describing the evolution of $f(E, t)$ due to scattering off of stars with masses $\gg m$ is

$$4\pi^2 p(E) \frac{\partial f(E, t)}{\partial t} = -\frac{\partial F_E}{\partial E} - F_{lc}(E, t), \quad (4a)$$

$$F_E(E, t) = -D_{EE}(E) \frac{\partial f}{\partial E}, \quad (4b)$$

$$D_{EE}(E) = 64\pi^4 G^2 \ln\Lambda$$

$$\times \left[q(E) \int_{-\infty}^{E} dE' h_\star(E') \right.$$

$$\left. + \int_{E}^{\infty} dE' q(E') h_\star(E') \right]. \quad (4c)$$

Here $p(E) = 4\sqrt{2} \int_0^{r_{\max}(E)} dr r^2 \sqrt{\phi(r) - E} = -\partial q/\partial E$ is the phase space volume accessible per unit of energy, with $\phi(r_{\max}) = E$. The dark matter heating rate is determined by $h_\star(E) = \int m_\star f_\star(E, m_\star) dm_\star$, with $f_\star(E, m_\star) dm_\star$ the mass density of stars in phase space in the interval $m_\star$ to $m_\star + dm_\star$. If the distribution $n(m_\star) dm_\star$ of stellar masses is assumed independent of energy (i.e., distance from the black hole), then $h_\star(E) = \tilde{m}_\star f_\star(E)$, with $f_\star(E)$ the total mass density of stars in phase space and

$$\tilde{m}_\star = \frac{\int n(m_\star) m_\star^2 dm_\star}{\int n(m_\star) m_\star dm_\star}. \quad (5)$$

$F_{lc}(E)$ is the flux of stars that are scattered from low angular momentum orbits into the SBH and is discussed in more detail below. Equations (4) assume that small-angle scatterings dominate the evolution of f and that the gravitational potential changes on a time scale long compared with T_{local}.

Neglecting F_{lc}, the total energy $\mathcal{E} = \int_{E_1}^{E_2} N(E) E dE$ of dark matter particles in the energy range $E_1 < E < E_2$

changes with time according to

$$\frac{d\mathcal{E}}{dt} = -\int_{E_1}^{E_2} dE E \frac{\partial F_E}{\partial E} \quad (6a)$$

$$= -[EF_E]_{E_1}^{E_2} - [fD_{EE}]_{E_1}^{E_2} - \quad (6b)$$

$$\int_{E_1}^{E_2} dE N(E) Q(E), \quad (6c)$$

$$Q(E) = 16\pi^2 G^2 \tilde{m}_\star \ln\Lambda \int_0^E dE' f_\star(E'). \quad (6d)$$

The third term in Eq. (6c), which is always negative, represents heating of the dark matter. We, accordingly, define the (nonlocal) time scale for heating of the dark matter particles to be

$$T_{\text{heat}}^{-1} \equiv \frac{\int_{E_1}^{E_2} dE N(E) Q(E)}{\int_{E_1}^{E_2} dE E N(E)}. \quad (7)$$

This expression may be used to estimate the dissolution time of a dark matter spike. Assume that both stars and dark matter particles initially have power-law density profiles near the SBH: $\rho(r, t = 0) \propto r^{-\gamma_{\text{sp}}}$, $\rho_\star(r) \propto r^{-\gamma}$, $r \lesssim r_h$, and that $\phi(r) = GM_\bullet/r$. The isotropic distribution function corresponding to an $r^{-\gamma}$ density profile in an r^{-1} potential is $f(E) \propto E^{\gamma-3/2}$. Setting $E_1 = \phi(r_h) = GM_\bullet/r_h$ and $E_2 \rightarrow \infty$, T_{heat} becomes

$$T_{\text{heat}} = A(\gamma, \gamma_{\text{sp}}) \frac{M_\bullet}{\tilde{m}_\star} \left(\frac{GM_\bullet}{r_h^3} \right)^{-1/2} \frac{1}{\ln\Lambda}, \quad (8a)$$

$$A(\gamma, \gamma_{\text{sp}}) = \frac{1}{2} \sqrt{\frac{\pi}{3}} \frac{(\gamma - 1/2)(7/2 - \gamma - \gamma_{\text{sp}})}{(3 - \gamma)(2 - \gamma_{\text{sp}})} \frac{\Gamma(\gamma + 1/2)}{\Gamma(\gamma + 1)}. \quad (8b)$$

When $\gamma = 3/2$, equal within the uncertainties with the slope of the stellar cusp around the Milky Way SBH [15], the coefficient $A(\gamma, \gamma_{\text{sp}})$ in Eqs. (8) is independent of γ_{sp} and

$$T_{\text{heat}} = \frac{4\sqrt{3}}{27} \frac{M_\bullet}{\tilde{m}_\star} \left(\frac{GM_\bullet}{r_h^3} \right)^{-1/2} \frac{1}{\ln\Lambda} \quad (9a)$$

$$= 1.25 \times 10^9 \text{ yr} \times \left(\frac{M_\bullet}{3 \times 10^6 M_\odot} \right)^{1/2} \left(\frac{r_h}{2 \text{ pc}} \right)^{3/2}$$

$$\times \left(\frac{\tilde{m}_\star}{M_\odot} \right)^{-1} \left(\frac{\ln\Lambda}{15} \right)^{-1}. \quad (9b)$$

The effective stellar mass $\tilde{m}_\star$ that appears in Eq. (9a) depends, via Eq. (5), on the mass function $n(m_\star)$ of stars in the GC stellar cusp. While $n(m_\star)$ is not strongly constrained by observations, either at the high or low mass ends, it is sometimes assumed (e.g., [15]) to be a power law with Salpeter [21] index, $n(m_\star) \propto m^{-(1+\alpha)}$, $\alpha \approx 1.35$. Setting $m_{\star,\min} = 0.08 M_\odot$ and $m_{\star,\max} = (5)10(20)M_\odot$ then yields $\tilde{m}_\star \approx (0.8)1.2(1.8)M_\odot$.

We take T_{heat} as defined in Eqs. (9) as our unit of time in what follows, with $\tau \equiv t/T_{\text{heat}}$. The age of the majority of the stars near the GC is $\gtrsim 10$ Gyr [19], although some

much younger ($t \lesssim 10^7$ yr) stars are present in the cusp at distances $\lesssim 0.1$ pc from the SBH [22,23]. Setting $t = 10$ Gyr, $r_h = 1.67$ pc, and $1 \leq \tilde{m}_\star/M_\odot \leq 2$ gives $10 \lesssim \tau \lesssim 20$. If the young stellar population is continually replenished, $m_\star$ and τ could be larger, implying a higher mean rate of dark matter heating.

Diffusion in energy will cause a modest loss of stars to the SBH, $\dot{M} = -F_E(E_2)$, with $E_2 \approx c^2$. A much greater capture rate is implied by scattering of dark matter particles on low angular momentum (eccentric) orbits into the SBH [24]. The loss rate is given approximately by [25]

$$F_{lc}(E) \approx 4\pi^2 P(E) J_c^2(E) \overline{\mu}(E) R \frac{\partial f}{\partial R} \qquad (10a)$$

$$\approx S(E)f(E), \qquad (10b)$$

$$S(E) = 4\pi^2 P(E) J_c^2(E) \overline{\mu}(E) [\ln R_0(E)^{-1}]^{-1}. \qquad (10c)$$

Here $R \equiv J^2/J_c^2(E)$ is a scaled angular momentum variable, with $J_c(E)$ the angular momentum of a circular orbit of energy E; P is the period of a radial orbit; $\overline{\mu}$ is the orbit-averaged angular momentum diffusion coefficient $\langle(\Delta R)^2\rangle/2R$; R_0 is the value of the angular momentum
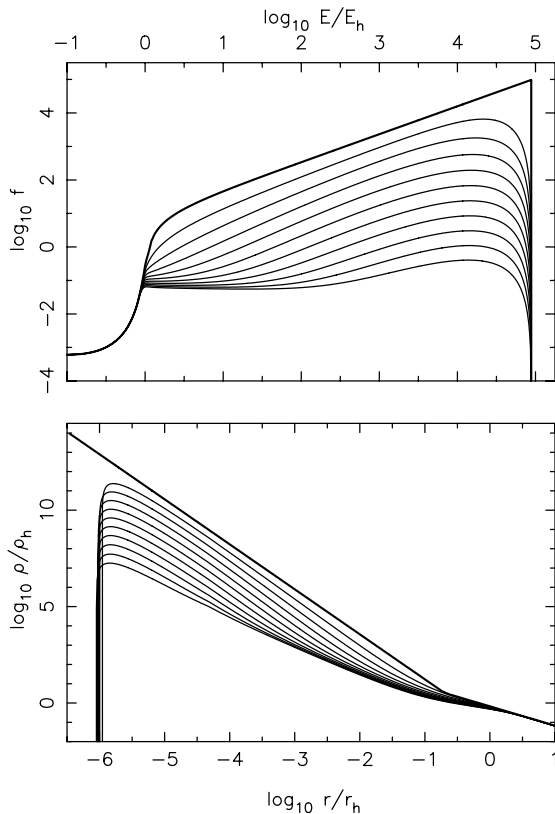

FIG. 1. Evolution of the dark matter phase space density $f(E)$ and mass density $\rho(r)$ due to gravitational interactions with stars around the GC SBH. The initial dark matter density is given by Eqs. (1), with $\gamma_{sp} = 7/3$ and $\gamma_c = 1$. Times shown are $\tau = 0$ (thick curves) and $\tau = 2, 4, ..., 20$, where τ is the time in units of T_{heat} [Eq. (5)]; $10 \leq \tau \leq 20$ corresponds roughly to the age of the Galactic bulge.

variable at which f drops to zero due to the competing effects of capture and diffusion. The final line of Eqs. (10) assumes $f \sim \ln[R/R_0(E)]$ near the loss cone [25,26]. Cohn and Kulsrud [26] give expressions for R_0 as a function of E based on solutions to the R-dependent Fokker-Planck equation; we adopt their expressions here. The angular momentum diffusion coefficients used by these authors, for modeling systems containing a single stellar mass, may be shown to remain unchanged when the scattered objects (here dark matter) have masses much less that those of the scatterers (stars). The rate of the loss of stars predicted by Eqs. (10) depends only weakly on the radius of the capturing sphere, which we set to $2GM_\bullet/c^2$. The diffusion coefficient $\overline{\mu}$ is of the order T_{heat}^{-1}, hence $F_{lc}(E) \approx N(E)/T_{heat}(E) \ln R_0^{-1}$.

The detailed evolution of the dark matter density around the GC SBH was computed by integrating Eq. (4a) forward in time. The stellar density was modeled via Dehnen's [27] density law, $\rho_\star(r) \propto (r/r_0)^{-\gamma}(1 + r/r_0)^{\gamma-4}$, with $\gamma = 1.4$ and r_0 chosen to match the observed stellar density at $r \lesssim r_h$ [15]. Figure 1 shows the evolution of the dark matter density assuming $\gamma_{sp} = 7/3$ and $\gamma_c = 1$, the values corresponding to a spike that developed in response to adiabatic growth of a SBH in a dark matter halo with $\gamma_c = 1.0$. Figure 1 shows that scattering of dark matter particles by stars causes the dark matter density to drop and the spike to flatten, within a radius $\sim r_h$ where the heating time is shorter than the age of the bulge. Figure 2 shows the dark matter density at $\tau = 10$ for a range of initial spike profiles, $1.0 \leq \gamma_{sp} \leq 2.75$. The mean density within r_h drops by several orders of magnitude when $\gamma_{sp} \approx 2$.

In the absence of scattering into the SBH, Eqs. (4) have the time-independent solution $f(E) = $ constant, $\rho \sim r^{-3/2}$ [14]. As Figs. 1 and 2 show, there is a tendency to evolve toward this characteristic profile, although a
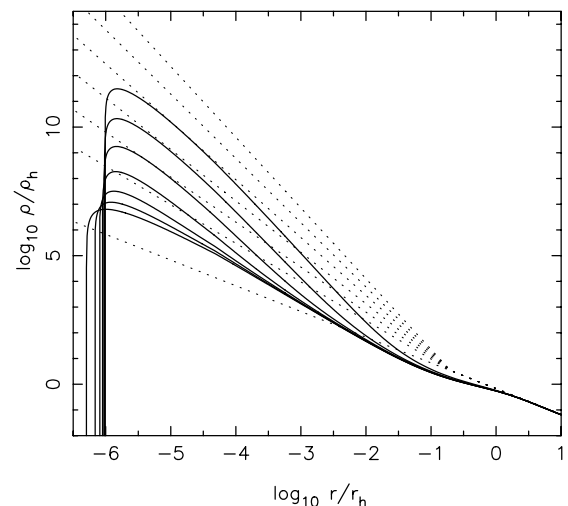

FIG. 2. Dark matter density at $\tau = 0$ (dotted lines) and $\tau = 10$ (solid lines) given an initial density that satisfies Eqs. (1), with $\gamma_c = 1$ and $\gamma_{sp} = (1.00, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75)$.
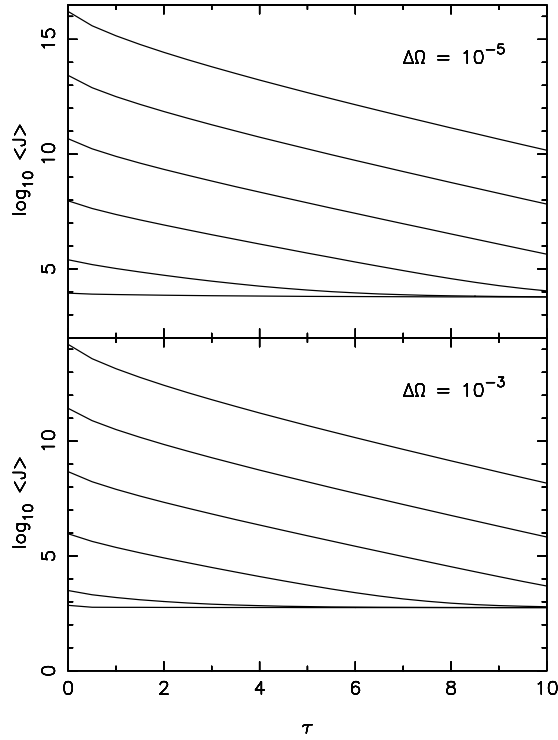
FIG. 3. Evolution of $\langle J \rangle$, a dimensionless measure of the cuspiness of the dark matter spike [Eq. (11)], for two values of the solid angle $\Delta \Omega$ of a detector centered on the SBH. Curves are shown for $\gamma_{sp} = 1.50$ (lower), 1.75, 2.00, 2.25, 2.50, and 2.75 (upper). The dark matter density was normalized to an initial value of $100 M_\odot$ pc^{-3} at $r = r_h$; $\langle J \rangle$ scales as $\rho^2(r_h)$.

number of factors keep it from being precisely reached, including the finite evolution time: the presence of the loss term F_{lc} and the fact that $f = $ constant can only hold true over a finite range of energies given the boundary conditions on f. Nevertheless, at late times ($\tau \gtrsim 20$), the solutions found here are generally well described by $\rho \sim r^{-3/2}$ at radii $10^{-5} \lesssim r/r_h \lesssim 10^{-2}$.

The flux of dark matter annihilation photons along a direction that makes an angle ψ with respect to the GC is proportional to the line-of-sight integral $\int_\psi \rho^2 dl$. Following earlier authors [3], we define the dimensionless form factor $J(\psi) \equiv K \int_\psi \rho^2(l) dl$, $K^{-1} = (8.5 \text{ kpc}) \times (0.3 \text{ GeV/cm}^3)$. Given a photon detector with angular acceptance $\Delta \Omega$ directed toward the GC, the signal is proportional to

$$\langle J \rangle \equiv \frac{1}{\Delta \Omega} \int_{\Delta \Omega} J(\psi) d\Omega. \qquad (11)$$

Figure 3 shows the evolution of $\langle J \rangle$ for $\Delta \Omega = 10^{-5}(10^{-3})$ sr; the first value is the approximate solid angle of the detectors in GLAST [28] and in atmospheric Cerenkov telescopes like VERITAS [29], while the larger angle corresponds approximately to EGRET [30]. The dark matter density was normalized to a fiducial value of $\rho = 100 M_\odot$ pc^{-3} at $r = r_h$; J and $\langle J \rangle$ scale as $\rho^2(r_h)$. We note that $\rho(r_h)$ is uncertain and could be much lower [8,31,32]. Figure 3 shows that the very large initial values

of $\langle J \rangle$ are rapidly diminished as the spike is dissolved; by $\tau = 10$, $\langle J \rangle$ has dropped below $\sim (10^4, 10^3)$, $\Delta \Omega = (10^{-5}, 10^{-3})$ for all $\gamma_{sp} \lesssim 2.5$. These values are similar to what would be predicted for the central regions of a dark matter halo in the absence of a SBH [3].

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