Possibility of Observable Amount of Gravitational Waves from Inflation

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(Received 19 January 2004; published 19 May 2004)

The curvaton and inhomogeneous reheating scenarios for the generation of the cosmological curvature perturbation on large scales represent an alternative to the standard slow-roll scenario. The basic assumption of these mechanisms is that the initial curvature perturbation due to the inflaton field is negligible. This is usually attained by lowering the energy scale of inflation, thereby concluding that the amount of gravitational waves produced during inflation is highly suppressed. We show that the curvaton and inhomogeneous reheating scenarios are compatible with a level of gravity-wave fluctuations which may well be observed in future satellite experiments.

DOI: 10.1103/PhysRevLett.92.201303 PACS numbers: 98.80.Cq, 04.30.Db

Inflation [1,2] has become the dominant paradigm for understanding the initial conditions for structure formation and for cosmic microwave background (CMB) anisotropy. In the inflationary picture, primordial density and gravity-wave fluctuations are created from quantum fluctuations "redshifted" out of the horizon during an early period of superluminal expansion of the Universe, where they are "frozen" [3,4]. Perturbations at the surface of last scattering are observable as temperature anisotropy in the CMB, which was first detected by the Cosmic Background Explorer (COBE) satellite [5,6]. The last and most impressive confirmation of the inflationary paradigm has been recently provided by the data of the Wilkinson Microwave Anisotropy Probe (WMAP) mission, which has marked the beginning of the precision era of the CMB measurements in space [7]. The WMAP collaboration has produced a full-sky map of the angular variations of the CMB, with unprecedented accuracy. WMAP data confirm the inflationary mechanism as responsible for the generation of curvature (adiabatic) superhorizon fluctuations.

Despite the simplicity of the inflationary paradigm, the mechanism by which cosmological adiabatic perturbations are generated is not yet established. In the standard slow-roll scenario associated with one-single field models of inflation, the observed density perturbations are due to fluctuations of the inflaton field itself when it slowly rolls down along its potential. When inflation ends, the inflaton ϕ oscillates about the minimum of its potential $V(\phi)$ and decays, thereby reheating the Universe. As a result of the fluctuations, each region of the Universe goes through the same history but at slightly different times. The final temperature anisotropies are caused by the fact that inflation lasts different amounts of time in different regions of the Universe, leading to adiabatic perturbations. Under this hypothesis, the WMAP data set already allows one to extract the parameters relevant for distinguishing among single-field inflation models [8].

An alternative to the standard scenario is represented by the curvaton mechanism [9–12] where the final curvature perturbations are produced from an initial isocurvature perturbation associated with the quantum fluctuations of a light scalar field (other than the inflaton), the curvaton, whose energy density is negligible during inflation. The curvaton isocurvature perturbations are transformed into adiabatic ones when the curvaton decays into radiation, much after the end of inflation. Recently, another mechanism for the generation of cosmological perturbations has been proposed [13–15], dubbed the inhomogeneous reheating scenario. It acts during the reheating stage after inflation if superhorizon spatial fluctuations in the decay rate of the inflaton field are induced during inflation, causing adiabatic perturbations in the final reheating temperature in different regions of the Universe.

Contrary to the standard picture, the curvaton and the inhomogeneous reheating mechanisms exploit the fact that the total curvature perturbation (on uniform density hypersurfaces) ζ can change on arbitrarily large scales due to a nonadiabatic pressure perturbation which may be present in a multi-fluid system [16–20]. While the entropy perturbations evolve independently of the curvature perturbation on large scales, the evolution of the large-scale curvature is sourced by entropy perturbations.

The generation of gravity-wave fluctuations is a generic prediction of an accelerated de Sitter expansion of the universe. Gravitational waves, whose possible observation might come from the detection of the B mode of polarization in the CMB anisotropy [21], may be viewed as ripples of spacetime around the background metric $g_{\mu\nu} = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$, where a is the scale factor and t is the cosmic time. The tensor h_{ij} is traceless and transverse and has 2° polarizations, $\lambda = \pm$. Since gravity-wave fluctuations are (nearly) frozen on superhorizon scales, a way of characterizing them is to compute their spectrum on scales larger than the horizon.

 0031-9007/04/92(20)/201303(4)$22.50

In the standard slow-roll inflationary models where the fluctuations of the inflaton field ϕ are responsible for the curvature perturbations, the power spectrum of gravity-wave modes is

$$\mathcal{P}_T(k) = \frac{k^3}{2\pi^2}\sum_\lambda |h_{\mathbf{k}}|^2 = \frac{8}{M_p^2}\left(\frac{H_*}{2\pi}\right)^2\left(\frac{k}{aH_*}\right)^{-2\epsilon}, \quad (1)$$

where $M_p = (8\pi G)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the Planck scale. Here $\epsilon = (\dot{\phi}^2/2M_p^2 H_*^2)$ is a standard slow-roll parameter, and $H_* = \dot{a}/a$ indicates the Hubble rate during inflation. On the other hand, the power spectrum of curvature perturbations in slow-roll inflationary models is given by

$$\mathcal{P}_\zeta(k) = \frac{1}{2M_p^2\epsilon}\left(\frac{H_*}{2\pi}\right)^2\left(\frac{k}{aH_*}\right)^{n_\zeta - 1}, \quad (2)$$

where $n_\zeta \simeq 1$ is the spectral index. Since the fractional changes of the power spectra with scales are much smaller than unity, one can safely consider the power spectra as roughly constant on the scales relevant for the CMB anisotropy and define a tensor-to-scalar amplitude ratio

$$r = \frac{\mathcal{P}_T}{16\,\mathcal{P}_\zeta} = \epsilon. \quad (3)$$

The present WMAP data set allows one to extract the upper bound $r \lesssim 10^{-1}$ [8]. Since the scale of inflation in slow-roll models of inflation is fixed to be

$$V^{1/4} \simeq 6.7 r^{1/4} \times 10^{16}\ \text{GeV}, \quad (4)$$

in order to match the observed amplitude of CMB anisotropy, one can already infer an upper bound on the energy scale of inflation of about 4×10^{16} GeV. The corresponding upper bound on the Hubble rate during inflation H_* is about 4×10^{14} GeV. What is more relevant though is the width of the potential [22]. The slow-roll paradigm gives, using the definition of ϵ and Eq. (3),

$$\frac{1}{M_p}\left|\frac{d\phi}{dN}\right| = \sqrt{2}r^{1/2}, \quad (5)$$

where $d\phi$ is the change in the inflaton field in $dN = Hdt \simeq d\ln a$ Hubble times. While the scales corresponding to the relevant multipoles in the CMB anisotropy are living the horizon, $\Delta N \simeq 4.6$ and therefore the field variation is $(\Delta\phi/M_p) \simeq (r/2 \times 10^{-2})^{1/2}$ [23]. This is a minimum estimate because inflation continues for some number N of e-folds of the order of 50. The detection of gravitational waves requires, in general, variation of the inflaton field of the order of the Planck scale [22]. This conclusion sounds fairly pessimistic since slow-roll models of inflation are generically based on four-dimensional field theories, possibly involving supergravity, where higher-order operators with powers of (ϕ/M_p) are disregarded. This assumption is justified only if the inflaton variation is small compared to the Planck scale. It is therefore difficult to construct a satisfactory model of inflation firmly rooted in modern particle theories having supersymmetry as a crucial ingredient and with large variation of the inflaton field. There is a strong theoretical prejudice against the likelihood of the observation of gravity-wave detection within slow-roll models of inflation where the curvature perturbation is due to the fluctuations of the inflaton field itself.

What about the expected amplitude of gravity-wave fluctuations in the other scenarios where the curvature perturbation is generated through the quantum fluctuations of a scalar field other than the inflaton? Consider, for instance, the curvaton scenario [10]. During inflation, the curvaton energy density is negligible and isocurvature perturbations with a flat spectrum are produced in the curvaton field σ, $\langle\delta\sigma^2\rangle^{1/2} = (H_*/2\pi)$, where σ_* is the value of the curvaton field during inflation. After the end of inflation, the curvaton field oscillates during some radiation-dominated era, causing its energy density to grow and thereby converting the initial isocurvature into curvature perturbation. Indeed, after a few Hubble times, the curvaton oscillation will be sinusoidal, except for the Hubble damping. The energy density ρ_σ will then be proportional to the square of the oscillation amplitude and will scale as the inverse of the locally defined comoving volume corresponding to matter domination. On the spatially flat slicing, corresponding to uniform local expansion, its perturbation has a constant value $\delta\rho_\sigma/\rho_\sigma \simeq (\delta\sigma/\sigma)_*$. The curvature perturbation ζ is supposed to be negligible when the curvaton starts to oscillate, growing during some radiation-dominated era when $\rho_\sigma/\rho \propto a$. After the curvaton decays, ζ becomes constant. In the approximation that the curvaton decays instantly, it is then given by $\zeta \simeq (2\gamma/3)(\delta\sigma/\sigma)_*$, where $\gamma \equiv (\rho_\sigma/\rho)_D$ and the subscript D denotes the epoch of decay. The corresponding spectrum is [10]

$$\mathcal{P}_\zeta^{1/2} \simeq \frac{2\gamma}{3}\left(\frac{H_*}{2\pi\sigma_*}\right). \quad (6)$$

The curvaton scenario—as well as the inhomogeneous reheating scenario—liberates the inflaton from the responsibility of generating the cosmological curvature perturbation and therefore avoids slow-roll conditions. Its basic assumption is that the initial curvature perturbation due to the inflaton field is negligible.

Common lore is to assume that the energy scale of the inflaton potential is too small to match the observed amplitude of CMB anisotropy, that is,

$$V^{1/4} \ll 6.7 r^{1/4} \times 10^{16}\ \text{GeV}, \quad (7)$$

corresponding to $H_* \ll 10^{14}$ GeV. Therefore, while certainly useful to construct low scale models of inflation [24], it is usually thought that the curvaton mechanism (as well as the inhomogeneous reheating scenario) predicts an amplitude of gravitational waves which is far too small to be detectable by future satellite experiments aimed to observe the B mode of the CMB polarization.

This conclusion would be discouraging if true. It would imply that all future efforts in measuring tensor modes in the CMB anisotropy are of no use because both the slow-roll standard scenario (for theoretical reasons) and the alternative mechanisms for the production of the curvature perturbation predict a tiny level of gravity-wave fluctuations.

There is, however, an obvious way out to such a pessimistic scenario. Within the curvaton mechanism (and the inhomogeneous reheating scenario), the curvature perturbations of the inflaton field may be highly suppressed, not because the energy scale of is tiny, but because the inflaton field is well anchored at the false vacuum with a mass much m_ϕ larger than the Hubble rate during inflation. Suppose that the inflaton potential is of the form

$$V(\phi) = V_0 + \frac{m_\phi^2}{2}(\phi - \phi_0)^2 + \cdots, \tag{8}$$

where ϕ_0 is the location of the minimum around which $m_\phi \gg H_*$. Under these circumstances, slow-roll conditions are badly violated since $\eta = (m_\phi^2/3H_*^2) \gg 1$, and the fluctuations of the inflaton field on superhorizon scales read [25]

$$|\delta\phi_{\mathbf{k}}|^2 \simeq \frac{\pi}{4}\frac{e^{-\pi\nu}}{a^2}\frac{1}{aH}\left[\frac{[1+\coth(\pi\nu)]^2\sinh(\pi\nu)}{\pi\nu} + \frac{\nu}{\pi\sinh(\pi\nu)}\right], \tag{9}$$

where $\nu = m_\phi/H_*$. The resulting power spectrum, after properly substructing the zero-point fluctuations, is highly suppressed,

$$\mathcal{P}_{\delta\phi}(k) = \left(\frac{H_*}{2\pi}\right)^2\left(\frac{k}{aH_*}\right)^3 e^{-2m_\phi^2/H_*^2}. \tag{10}$$

Thus, the embarrassing constraint (7) is circumvented, and the curvature perturbations due to the inflaton field may be small even if the Hubble rate during inflation is sizable (close to the present upper bound of $\sim 4 \times 10^{14}$ GeV).

We conclude that within the curvaton scenario (and the inhomogeneous reheating scenario) gravity-wave fluctuations may well be observed in future satellite experiments. This conclusion does not imply embarrassingly large field variations in units of the Planck scale as in the slow-roll scenario. A future detection of the B mode of CMB polarization will not automatically disprove the curvaton and the homogeneous reheating scenarios. This is our main observation.

Let us elaborate on this point. Since the amplitude of curvature perturbation $\mathcal{P}_\zeta^{1/2}$ must match the observed value 5×10^{-5}, from Eq. (6) one infers that

$$\sigma_* \simeq 2\gamma \times 10^3 H_*. \tag{11}$$

On the other hand, a large signal-to-noise ratio for the detection of the B mode of CMB polarization requires $H_* \gg 2 \times 10^{12}$ GeV [23] (notice that for $H_* \lesssim 10^{11}$ GeV, secondary effect dominates over that of primordial tensors [26]),

$$\sigma_* \gg 20H_* \gtrsim 10^{-5}M_p, \tag{12}$$

where in the last inequality we have made use of the current WMAP bound on non-Gaussianity which requires $\gamma \gtrsim 9 \times 10^{-3}$ [27]. If future WMAP data on non-Gaussianity strengthen the bound on γ, the lower bound on σ_* will be correspondingly tighter. Similarly, the present upper bound on the Hubble rate, $H_* \lesssim 4 \times 10^{14}$ GeV [8], implies

$$\sigma_* \lesssim 2 \times 10^3 H_* \lesssim 0.3M_p, \tag{13}$$

where we have taken into account that γ has to be smaller than unity.

Curvaton scenarios able to predict a detectable background of gravity-wave fluctuations are characterized by large values of the Hubble rate and σ_*, $10^{-5} \ll \sigma_*/M_p \lesssim 0.3$. A working example of a model of inflation manifesting these properties is the recently proposed stringy version of old inflation [28]. The model does not require any slow-roll inflaton potential and is based on string compactifications with warped metric. Warped factors are quite common in string theory compactifications and arise, for example, in the vicinity of D-brane sources. Similarly, string theory has antisymmetric forms whose fluxes in the internal directions of the compactification typically introduce warping. From the inflationary point of view, the basic property of the setup is that it admits in the deep IR region of the metric the presence of p anti-D3-branes. These antibranes generate a positive vacuum energy density. For sufficiently small values of p, the system sits, indeed, on a false vacuum state with positive vacuum energy density. The latter is responsible for the accelerated period of inflation.

In terms of the four-dimensional effective description, the inflaton may be identified with a four-dimensional scalar field parametrizing the angular position ϕ of the anti-D3-branes in the internal directions. The mass squared of the inflaton at the false ground state is $m_\phi^2 \sim (1/\alpha')(r_0/R)^2$, where α' is the string length squared and (r_0/R) is the warp factor (R is the Anti–de Sitter radius and r_0 is the location of the stack on antibranes along the fifth dimension). The mass squared is much larger than

$$H_*^2 \simeq 2p\frac{T_3}{3M_p^2}\left(\frac{r_0}{R}\right)^4, \tag{14}$$

where T_3 is the tension of a single anti-D3-brane. The false vacuum for the potential $V(\phi)$ exists only if the number of anti-D3-branes is smaller than a critical number [29]. If a sufficient number of anti-D3-branes travels from the UV towards the IR region, thus increasing the value of p, inflation stops as soon as p becomes larger than the critical value $p_{\rm cr}$. Once an anti-D3-brane appears in the UV region, it rapidly flows towards the IR region, and it starts oscillating with a frequency $\sim\sqrt{2}H_*$ around the minimum where the p anti-D3-branes sit. Since the Universe is in a de Sitter phase, the amplitude of the oscillations decreases as rapidly as $\sim e^{-3N/2}$. Once the energy density stored in the oscillations becomes smaller than $\sim 1/\alpha'^2$, the anti-D3-brane stops its motion and gets glued with the p anti-D3-branes in the IR, increasing their number by one unity. The oscillating phase lasts for a number of e-folds of the order of $\sim\frac{2}{3}\ln[p^3(r_0/R)^4/M_p^2\alpha']$ [28]. Once the number of anti-D3-branes becomes equal to $p_{\rm cr}$, inflation ends since the curvature around the false vacuum becomes negative. The system rolls down the supersymmetric vacuum, and the graceful exit from inflation is attained. In such a stringy version of the old inflation, it is possible to find scalar fields which have all the necessary properties to play the role of the curvaton. For instance, the imaginary part of the volume modulus behaves like an axion field, and its mass is exponentially suppressed upon volume stabilization. Because of this exponential suppression, the condition $m_\sigma^2 \ll H_*^2$ during inflation does not require any particular fine tuning [28]. Furthermore, since the nonperturbative superpotentials necessary for volume stabilization arise in the UV region, no warping suppression is expected and the curvaton scale σ_* will be naturally of the order of M_p in the four-dimensional effective theory. Furthermore, since p has to be larger than about $(R/r_0)^4$ in order for the p anti-D3-branes to dominate the energy density of the Universe, the Hubble rate during inflation is naturally large:

$$H_*^2 \gtrsim \frac{T_3}{M_p^2} \sim 10^{-3}\frac{1}{g_s\alpha'^2 M_p^2}, \tag{15}$$

where g_s is the string coupling constant. For $g_s \sim 0.1$, requiring that H_* is larger than $\sim 2\times 10^{12}$ GeV to ensure gravity-wave detection [23], implies the fundamental scale $1/\sqrt{\alpha'}$ to be of the order of 10^{16} GeV, a perfectly natural value.

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