Dynamical Control of Macroscopic Quantum Tunneling

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We show that the quantum Zeno and anti-Zeno effects are realizable for macroscopic quantum tunneling by current-bias modulation in Josephson junctions (and their analogs in atomic condensates).

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Frequent measurements of quantum states decaying into an energy continuum can cause either slowdown of the decay [the quantum Zeno effect (QZE)] [1,2] or, conversely, its speedup [the anti-Zeno effect (AZE)] [1,3–5]. Thus far, these effects have mainly been investigated in the context of *microscopic* systems (e.g., particles or atoms) [1–5]. Here we show that decay modifications reminiscent of the QZE and AZE are realizable for *macroscopic* quantum states that couple to the continuum via macroscopic quantum tunneling (MQT) in a superconducting current-biased Josephson junction (JJ) [6–9] or in its analogs in ultracold atomic condensates [10]. To this end, we develop a generalized, comprehensive theory of MQT through temporally modulated barriers, which includes previous treatments [9,11] as special cases. In contrast to previously discussed realizations of the QZE and AZE by frequent measurements [2,3], the effects predicted here are based upon *small-amplitude modulation* of the bias current, which is the only viable means of dynamical MQT control in a JJ.

To set our problem in its general context, consider a system ruled by the Hamiltonian

$$H = H_0 + \tilde{V}(t), \qquad \tilde{V}(t) = \epsilon(t)\sum_f V_{fn}|f\rangle\langle n| + \text{H.c.} \quad (1)$$

Here $\tilde{V}(t)$ is a perturbation that couples the initial state $|n\rangle$ only to eigenstates $|f\rangle$ of H_0 with energies in the continuous spectrum, and $\epsilon(t)$ expresses the time-dependent modulation of the perturbation. Under these assumptions, the probability amplitude $\alpha_n(t)$ of the initial state $|n\rangle$ obeys (in the interaction representation) the following *exact* integrodifferential equation [1]:

$$\dot{\alpha}_n = -\int_0^t dt' \epsilon^*(t)\epsilon(t')\mathcal{G}_n(t-t')e^{i\omega_n(t-t')}\alpha_n(t'). \quad (2)$$

Here the energy eigenvalue of $|n\rangle$ is $\hbar\omega_n$ and $\mathcal{G}_n(t) = \hbar^{-2}\int_0^\infty d\omega_f \rho(\omega_f)|V_{fn}|^2 e^{-i\omega_f t}$ is the memory (correlation) function of the coupling to the continuum, which involves the density of continuum states $\rho(\omega_f)$. For sufficiently short times one may set $\alpha_n(t') \approx 1$ in the integral in (2), resulting in the time-dependent decay rate of $|\alpha_n(t)|^2$:

$$\begin{aligned}\frac{1-|\alpha_n(t)|^2}{Q(t)} &\simeq R_n(t)\\ &= \frac{2}{Q(t)}\text{Re}\int_0^t dt' \int_0^{t'} dt'' \epsilon^*(t')\epsilon(t'')\\ &\quad\times \mathcal{G}_n(t'-t'')e^{i\omega_n(t'-t'')}\\ &= 2\pi\int_{-\infty}^{\infty} d\omega F_t(\omega-\omega_n)G_n(\omega), \end{aligned} \quad (3)$$

where $Q(t) = \int_0^t d\tau|\epsilon(\tau)|^2$ is the effective time. In Eq. (3), $R_n(t)$ is the convolution of two spectral functions: (i) $G_n(\omega) = \hbar^{-2}\rho(\omega)|V_{fn}(\omega)|^2$, the Fourier transform of the continuum memory function $\mathcal{G}_n(t)$, and (ii) $F_t(\omega) \sim |\int_0^t \epsilon(t')e^{i\omega t'}dt'|^2$, the spectral density of the modulation function (normalized to 1). The *universal* Eq. (3) [1] has much broader applicability than its golden rule counterpart, $R_{GR} = 2\pi G_n(\omega_n)$, obtained by standard perturbation theory [12]: $R_n(t)$ may vary with the modulation period τ, obeying either the QZE or the AZE, depending on whether τ is shorter than the memory (correlation) time τ_c [the time over which $\mathcal{G}(t)$ changes].

The foregoing results purport to be *universal,* but their applicability to time-modulated tunneling to the continuum is far from obvious, as detailed below. We shall consider a *low-temperature, nondissipative* JJ (with negligible thermal effects) driven by time-dependent bias-current $I_b(t)$. This system is adequately described by the following Hamiltonian, in terms of the magnetic-flux variable Φ [8]:

$$H(t) = -\frac{\hbar^2}{2C}\frac{\partial^2}{\partial\Phi^2} - I_b(t)\Phi - E_J\cos\frac{2\pi\Phi}{\Phi_0}, \quad (4)$$

where C is the junction capacitance, $E_J = \Phi_0 I_c/2\pi$ is the Josephson energy, $\Phi_0 = \hbar/2e$ is the flux quantum, and I_c is the critical current. This Hamiltonian is equivalent to that of a fictitious particle of "mass" $m = C$ and "momentum" $p = -i\hbar\partial/\partial\Phi$ moving along the coordinate $x = \Phi$ in a tilted (washboard) potential, consisting of a time-dependent tilt term and a sinusoidal "lattice" potential:

$$H(t) = \frac{p^2}{2m} - ma(t)x + U_0 \cos 2k_L x;$$
$$a(t) = \bar{a} + b\epsilon(t). \tag{5}$$

Here the time-dependent "acceleration" is $a(t) = I_b(t)/C$, $U_0 = E_J$, and $2k_L = 2\pi/\Phi_0$. We shall assume abrupt (steplike) changes of the tilt, causing the acceleration to periodically alternate between $\bar{a} + b$ ($\epsilon = 1$), within time intervals of length τ_1, and $\bar{a}$ ($\epsilon = 0$), within time intervals of length $\tau_0 - \tau_1$. This time dependence is realizable in a JJ by rapid (on time scales $\lesssim 0.1$ ns) up-down ramping of the bias current. For atomic Bose condensates trapped in optical lattices [10] we can turn the tilt up and down by fast ($\lesssim 10$ ps) modulation of the laser intensity.

The Hamiltonian in Eq. (5) is analogous to that describing cold atoms in a repeatedly accelerated (tilted) optical washboard potential, resulting in periodically interrupted tunneling from a quasibound state to the continuum [5]. The experiment in Ref. [5] has demonstrated good agreement with Eq. (3) and has provided the only convincing proof to date of both the QZE and AZE in decay to a continuum.

We first address the basic query: can we cast the driven-JJ Hamiltonian $H(t)$ in Eq. (5) into the universal form (1), namely, separate it into H_0, whose eigenstates are either bound (i.e., have discrete energies) or unbound (with energies in the continuum), and a time-dependent perturbation $\epsilon(t)V$ that couples bound and unbound states? In this respect, we note the important differences between the system in Ref. [5], characterized by a slightly tilted, shallow sinusoidal potential supporting a *single* quasibound band, and a biased JJ, describable by a *strongly tilted* potential, characterized by a near-critical acceleration ($a \lesssim a_c = 2k_L U_0/m$) and supporting many bound levels [6–9]. The sinusoidal potential in Eq. (5) can be effectively replaced in a biased JJ by the cubic form (Fig. 1, upper inset)

$$U(q) = (U_0/6)q^2(q_0 - q), \tag{6}$$

whose maximum $U_m = (2/81)U_0 q_m^3$ is at $q_m = 2q_0/3 = \sqrt{8(1 - a/a_c)}$ and $q = 2k_L x - \pi/2 + q_m/2$. We shall consider a quasibound level n localized in the well on the left (around the minimum $U = 0$ at $q = 0$). In order to cast the effective Hamiltonian (5) for such a quasibound level [13] into the required form, we rewrite it as follows:

$$H = \tilde{H}_0 + \epsilon(t)V,$$
$$\tilde{H}_0 = \frac{p^2}{2M} + U(q)\theta(q_m - q) + U_m\theta(q - q_m), \tag{7}$$
$$V = [U(q) - U_m]\theta(q - q_m).$$

Here $\theta()$ is the Heaviside step function and the effective mass is $M = m/4k_L^2$. In (7) we have *artificially divided* $U(q)$ between $\tilde{H}_0$, whose eigenstates are strictly bound if their energy $\hbar\omega_n < U_m$, and a potential $V < 0$ in the time-modulated perturbation $\epsilon(t)V$. This perturbation allows the particle to tunnel periodically from the bound state to the unbound (continuum-energy) eigenstates of H whenever $\epsilon(t) = 1$ in Eq. (5). The suggested form (7) conforms to Eq. (1) and thus answers the query above (Fig. 1, insets).

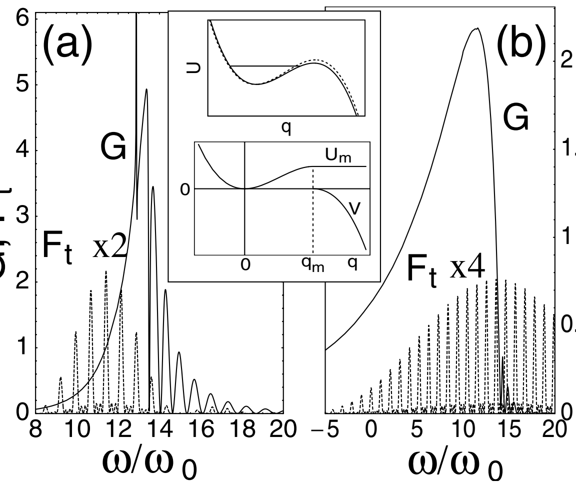

FIG. 1. Upper inset: tilted potential $U(q)$ [Eq. (6)]; solid line: maximal tilt, $\epsilon = 1$; dashed line: minimal tilt, $\epsilon = 0$ [Eq. (5)]. Lower inset: U ($\epsilon = 0$) is approximated by a binding potential for energies below U_m, since tunneling is negligible; U ($\epsilon = 1$) is divided into the same binding potential and a perturbation $V < 0$, allowing tunneling. Main figure: (a) The coupling spectrum $G_{n=12}(\omega)$ (in units of $R_{\rm GR}/2\pi$) and the modulation function $F_{t=4\tau_0}(\omega)$ (in units of ω_0^{-1}) multiplied by 2 with $\tau_1 = 1/\omega_0$ and $\tau_0 = 5\tau_1$, where ω_0 is the fundamental (harmonic) oscillation frequency in the well. (b) Same, with $G_{n=15}(\omega)$, $\tau_1 = 0.3/\omega_0$, $\tau_0 = 20\tau_1$, and $F_{t=4\tau_0}(\omega)$ ($\times 4$).

Let us next raise another important query: are abrupt changes of the tilt compatible with the assumption that $|\alpha_n(t)|^2$ evolves slowly enough, so as to warrant the use of Eq. (3), or do they conform to the impulse (shock) approximation [11,12]? Under the constraint $R_n\tau_c \ll 1$, which implies that the (modified) decay rate R_n of level n is slow on the memory (correlation) time scale of the continuum, the approximation Eq. (3) should hold, at least whenever the tilt is constant (either $\bar{a}$ or $\bar{a} + b$). We denote the corresponding survival probability by [1] $P_n(t)|_{\rm modul} \simeq e^{-R_n T}$, where $T = Q(t)$ [cf. (3)] is the total time in the interval $(0, t)$, during which the tunneling is switched on ($\epsilon = 1$). By contrast, the impulse (shock) approximation should apply during the much shorter ramping times $\tau_r \ll \tau_1$ if the sudden-change condition holds [12]: The abrupt ramping of the tilt down and up causes the higher-tilt ("preshock") wave function $|\psi_n^{(+)}(t)\rangle$ to be projected onto its lower-tilt ("postshock") counterpart $|\psi_n^{(-)}(t)\rangle$ at time τ_1, then back again after time $\tau_0 - \tau_1$. This yields the following estimate for the survival probability of the nth level after time τ_0:

$$P_n|_{\rm impulse} = |\langle\psi_n^{(-)}|\psi_n^{(+)}\rangle|^2 |\langle\psi_n^{(+)}|\psi_n^{(-)}\rangle|^2 = |\langle\psi_n^{(-)}|\psi_n^{(+)}\rangle|^4. \quad (8)$$

The answer to the foregoing query is therefore that Eqs. (8) and (3) apply at *different* time intervals, and therefore are mutually compatible.

Having established the validity of Eq. (3) in the context of abruptly modulated MQT, we proceed to calculate $G_n(\omega)$, $F_t(\omega)$, and finally their convolution, in order to obtain the dynamically modulated decay rate $R_n(t)$.

(1) *The nth-level coupling to the continuum* corresponding to Eq. (7) with $\epsilon(t) = 1$ must be evaluated for the entire energy spectrum $-\infty < \hbar\omega < \infty$. It has the form $G_n(\omega) = |\int_{-\infty}^{\infty} dq \psi_\omega^*(q) V(q) \psi_n(q)|^2$, where $\psi_n(q)$ and $\psi_\omega(q)$ are the initial (bound) and final (continuum) state wave functions, normalized to 1 and $\delta(\omega - \omega')$, respectively. Its evaluation has employed the semiclassical approximation for $\psi_\omega(q)$ well above or below the barrier top, and confluent hypergeometric functions near the top. The total width of $G_n(\omega)$, Γ_R, defines the *shortest correlation time* of the continuum $\tau_c \sim 1/\Gamma_R$, and is related to the energy distance from the top $\hbar\Gamma_R \sim \hbar\omega_m = U_m - E_n$. Figure 1 shows $G_n(\omega)$, calculated for two distinct representative cases: the states $n = 12$ and 15 (for the parameters given below). Several peaks can be discerned in $G_n(\omega)$ of Fig. 1(a) ($n = 12$). The narrow peaks below the barrier height represent enhanced coupling to the continuum (*tunneling resonances*) via quasidiscrete levels. The broader, progressively diminishing peaks at $\hbar\omega > U_m$, whose separation scales as $\omega^{1/2}$ at high ω, are above-the-barrier resonances, indicating constructive interference of waves transmitted to the right and those reflected from the "wall" on the left (Fig. 1, inset), with phase differences of 2π, 4π, etc. By contrast, the peak heights in $G_n(\omega)$ of Fig. 1(b) ($n = 15$) are relatively low, because the $n = 15$ quasibound state allows much larger overlap with continuum states near the barrier top than with the above-barrier resonances.

(2) *The steplike periodic modulation function* in Eq. (7): $\epsilon(t) = 1$ for $j\tau_0 < t < j\tau_0 + \tau_1$, $\epsilon(t) = 0$ for $j\tau_0 + \tau_1 < t < (j+1)\tau_0$ ($j = 0, 1, \ldots$), has the spectral density

$$F_{t=N\tau_0}(\omega) = \frac{2\sin^2(\omega\tau_1/2)\sin^2(N\omega\tau_0/2)}{\pi N \tau_1 \omega^2 \sin^2(\omega\tau_0/2)}. \quad (9)$$

The function (9) [plotted in Figs. 1(a) and 1(b)] consists of a "comb" of bell-shaped spectral peaks centered at $\omega_k = 2k\pi/\tau_0 (k = 0, \pm 1, \ldots)$ with the width $1/t$, whose weights diminish with $|\omega_k|$ [1]. When the alternating-tilt intervals satisfy $\tau_1 \lesssim \tau_c$ and $\tau_1 \sim \tau_0 - \tau_1$, the modulation $\epsilon(t)$ causes successive tunneling events to be strongly correlated (within the memory time τ_c). The spectral peaks of $F_t(\omega)$ are then sparse and may coincide with the peaks of $G_n(\omega)$ (tunneling resonances), as discussed below. By contrast, when the low-tilt interval $\tau_0 - \tau_1 \gg \tau_c$, and the comb is spectrally dense [Fig. 1(b)], the system effectively loses its "memory" between consecutive tunneling events, which then resemble irreversible measurements that "interrupt" the evolution (although the evolution remains unitary, in reality). We may then approximate $F_t(\omega)$ in (9) by smoothing out the comb peaks separated by $2\pi/\tau_0$ to become $F_t(\omega) \approx (\tau_1/2\pi)\mathrm{sinc}^2[(\omega - \omega_n)\tau_1/2]$, where $\mathrm{sinc}(x) = \sin x/x$. This form of $F_t(\omega)$ effectively amounts to spectral spread (broadening) of $G_n(\omega)$ over a frequency range $\sim 1/\tau_1$ in the convolution integral (3). In this case R_n is insensitive to narrow resonant peaks of $G_n(\omega)$.

(3) *The convolution of* $F_t(\omega - \omega_n)$ and $G_n(\omega)$ yields, at large enough t, the effective decay rate:

$$R_n \approx \frac{2\pi\tau_1}{\tau_0} \sum_{k=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{k\pi\tau_1}{\tau_0}\right) G\left(\omega_n + \frac{2k\pi}{\tau_0}\right). \quad (10)$$

This result provides the general, *unified* framework for the study of dynamically modulated MQT (decay) rate. The control parameters τ_0 and τ_1 in $F_t(\omega)$ [Eq. (9)] are seen to determine the relative weights and spacings of the k-dependent peaks in R_n.

We may discern three limiting regimes in the dependence of the decay rate R_n on τ_0 and τ_1.

(i) The QZE (i.e., *reduction of the decay rate* R_n *with the modulation rate*) is obtained when $1/\tau_1 \gtrsim \omega_m, \Gamma_R$. We may graphically infer from Fig. 1 that the large width of the modulation spectrum $F_t(\omega)$ [compared to the spectral width of $G_n(\omega)$] in the convolution (3) is then the origin of the QZE. The physical sense is that when the system is perturbed (interrupted) frequently enough, the QZE arises since the energy uncertainty incurred by the perturbations causes the effective decay rate to be averaged over the entire spectrum of continuum states, most of which do not contribute to the decay [$G_n(\omega) \simeq 0$]. The corresponding time-domain behavior (Fig. 2) is one of *repeatedly interrupted oscillations* of the initial-state population $P_n(t)$. The oscillatory character of the evolution attests to *short-time reversibility* of the tunneling at $\tau_1 \ll \tau_c$, i.e., well within the correlation time of the continuum. This evolution is in sharp contrast to the effectively irreversible steplike population loss associated with $P_n|_{\rm impulse}$ [Eq. (8), and Fig. 2(b), upper inset]. The loss due to $P_n|_{\rm impulse}$ should be used to calibrate the oscillatory $P_n(t)|_{\rm modul} \simeq e^{-R_n T}$ (solid curves), thereby allowing an experimentally distinct signature of the QZE.

(ii) The AZE (i.e., *decay speedup as the modulation rate increases*) is seen from Fig. 2(a) to arise (for $n = 12$) when the unperturbed energy is strongly detuned from the maximum of the coupling spectrum $G_n(\omega)$, i.e., $G_{12}(\omega_{12}) \ll G_{12}(\omega_m)$, and the modulation rate satisfies $1/\tau_1 < \omega_m$. This implies that the decay rate R_n *grows* with $1/\tau_1$, since the modulation function $F_t(\omega)$ is then probing more of the *rising* part of $G_n(\omega)$ in

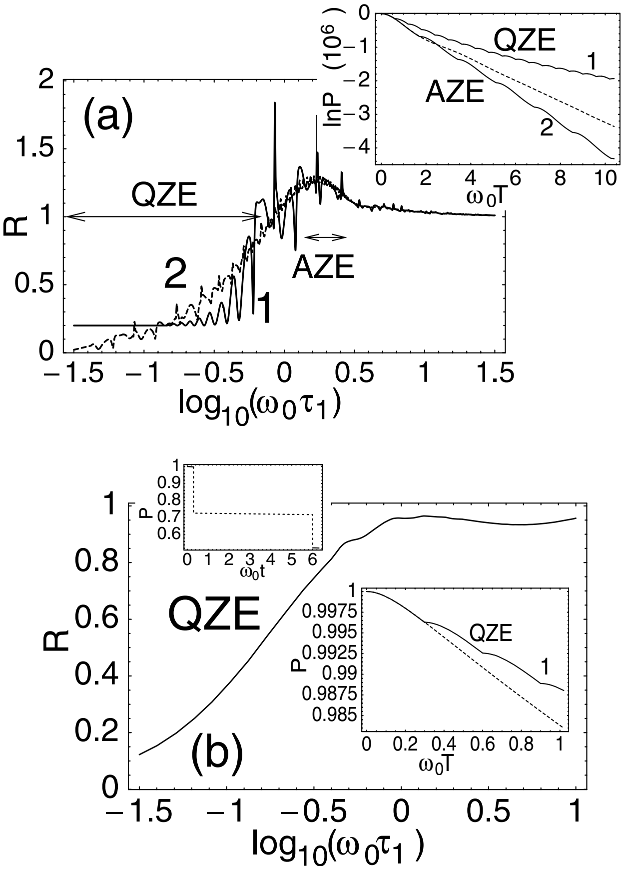

FIG. 2. (a) The decay rate R (in units of the unperturbed, golden rule rate R_{GR}), as obtained from the convolution of $F_t(\omega)$ and $G_n(\omega)$ in Fig. 1(a), for $n = 12$. The rate is plotted as a function of the interruption time τ_1 (in units of $1/\omega_0$) on a log scale for $\tau_0 = 5\tau_1$ (curve 1) and $\tau_0 = 50\tau_1$ (curve 2). The domains of QZE, QZE scaling, and AZE are marked. Inset: the evolution of $\ln P(t)$ for level $n = 12$; solid line 1: QZE-like decay; solid line 2: AZE-like decay; dashed line: unperturbed decay. (b) Same, for level $n = 15$, corresponding to Fig. 1(b). Lower inset: decay rate R shows only QZE behavior. Upper inset: P vs total time t (including the lower-tilt period), showing impulsive jumps.

the convolution (3). Physically, this means that, as the energy uncertainty grows with the modulation rate $1/\tau_1$, the state decays into more and more channels, whose weight $G_n(\omega)$ is progressively larger.

(iii) For low lying nearly harmonic levels (here $0 \leq n \leq 12$) $G_n(\omega)$ has distinct tunneling resonances if $2\pi k/\tau_0 \simeq n\omega_0$. A periodic modulation corresponding to narrow spectral peaks $F_t(\omega) \sim \delta(\omega - 2k\pi/\tau_0)$ would excite such a resonance, and thus give rise to *AZE-like resonantly enhanced* tunneling [see spikes in Fig. 2(a)]. In this regime, our theory qualitatively reproduces previous treatments [9,11] of periodically modulated barriers [with sinusoidal $\epsilon(t)$], which have predicted tunneling-rate enhancement. It should be mentioned that enhancement of the decay rate to the continuum for high-frequency bias-current modulation has been observed previously [14]. However, this observation was made under rather high-temperature conditions, where many levels are populated in the JJ potential and the present picture does not hold.

For numerical examples we chose $C = 58$ pF and either $I_c = 15\ \mu$A and $n = 12$ [Figs. 1(a) and 2(a)] or $I_c = 16\ \mu$A and $n = 15$ [Figs. 1(b) and 2(b)], yielding $\omega_0 = 3.85 \times 10^9\ \mathrm{s}^{-1}$, $\omega_m = 7.8 \times 10^9\ \mathrm{s}^{-1}$, and $R_{GR} = 1.2 \times 10^3\ \mathrm{s}^{-1}$ for $n = 12$, whereas $\omega_0 = 3.97 \times 10^9\ \mathrm{s}^{-1}$, $\omega_m = 1.1 \times 10^9\ \mathrm{s}^{-1}$, and $R_{GR} = 6.8 \times 10^7\ \mathrm{s}^{-1}$ for $n = 15$. The QZE for $n = 12$ and 15 then requires $\tau_1 \lesssim 0.1$ and 1 ns, respectively, while the AZE requires $\tau_1 > 0.1$ ns for $n = 12$ and is practically absent for $n = 15$ when I_b is modulated between $0.992\,65 I_c$ and $0.993 I_c$.

To summarize, our comprehensive treatment of dynamically controlled MQT has elucidated its hitherto unknown *short-time* evolution at low temperatures. We find a surprisingly high sensitivity of this dynamics and the resulting modification of the decay rate to *moderate* changes of the bias-current modulation and to the energy of the initial state relative to the barrier. Depending on the chosen values, the bias-current modulation has been shown to *imitate either frequent measurements or correlated perturbations* of a decaying state, between successive impulses (shocks) [11,12]. Such modulation has been demonstrated to either enhance or suppress the MQT rate. These modifications are similar to (but more complex than) the AZE or QZE. The present analysis provides useful handles on decoherence control in quantum gates based on JJ qubits [15] or their atomic-condensate counterparts [10].

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