Comment on "Exact Self-Similar Solutions of the Generalized Nonlinear Schrödinger Equation with Distributed Coefficients"

The nonlinear Schrödinger equation (NLSE) is one of the most important nonlinear evolution equations of modern science. The generalized NLSE, with distributed dispersion, nonlinearity, and gain or loss, arises frequently in the mathematical formulation of quasisolitons and soliton dispersion management concepts [1]. How can we determine whether a given nonlinear evolution equation is integrable or not? The ingenious method to answer this question—the inverse scattering transform (IST) method—was discovered by Gardner, Green, Kruskal, and Miura; Lax; Zakharov and Shabat; Ablowitz, Kaup, Newell, and Segur [2]. In particular, this method has been extended to the nonisospectral generalization of the Zakharov-Shabat eigenvalue problem, and fundamental soliton management regimes have been discovered in Ref. [3].

In a recent Letter [4], Kruglov, Peacock, and Harvey claimed that (i) they "present the discovery of a broad class of exact self-similar solutions to the nonlinear Schrödinger equation with gain or loss" and (ii) "these equations are not integrable by the inverse scattering method, and, therefore, they do not have soliton solutions; however, they do have solitary wave solutions which have often been called solitons." We wish to point out that many of the solutions found in [4] have been presented recently [3], and that these solutions are in fact soliton solutions.

Let us start by mentioning that the following generalized NLSE arises in the framework of the extended IST method with variable spectral parameter $\Lambda(T)$

$$i\frac{\partial q}{\partial T}+\frac{1}{2}D(T)P^2(T)\frac{\partial^2 q}{\partial S^2}+R(T)|q|^2q+i\left[V(T)P+\frac{S}{P}\left(\frac{\partial P}{\partial T}+DP^2\right)\right]\frac{\partial q}{\partial S}+\left[2\gamma_0-V(T)S-\frac{2}{P(T)}\int_0^S\frac{\partial\Lambda}{\partial T}dS\right]q$$
$$-\frac{S^2}{2P^2}\left(\frac{\partial P}{\partial T}+DP^2\right)q=\frac{i}{2}\left\{\frac{W[R(T),D(T)]}{R(T)D(T)}-D(T)P(T)\right\}q, \tag{1}$$

which has the Lax representation (see Ref. [3] for details). The gain (or absorption) coefficient in Eq. (1) is determined by the Wronskian $W[R(T),D(T)]=RD'_T-DR'_T$ of two arbitrary functions, dispersion $D(T)$ and nonlinearity $R(T)$, and is dependent on the "moving in time" focus length $f(T)=P(T)$ as well (see Ref. [3]). It was found [3] that soliton solutions exist only under certain conditions and the parameter functions describing dispersion, nonlinearity, and gain inhomogeneities cannot be chosen independently. Consequently, a key result [Eq. (8)] of the Letter [4] is consistent with the condition for the existence of solitons published previously in Physical Review Letters [3]. Additionally, the exact N-soliton solutions for Eq. (1) have been found recently in [5]; this fact, in contradiction with [4], also emphasizes the ideal soliton features of solutions discussed. Notice also, in contradiction with [4] that, in the case when the dispersion and nonlinearity are constants, the exactly integrable generalized NLSE model represents the cylindrical NLSE with the phase modulated soliton solutions discovered in [6].

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