Coulomb Blockade of a Noisy Metallic Box: A Realization of Bose-Fermi Kondo Models

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We focus on a metallic quantum dot coupled to a reservoir of electrons through a single-mode point contact and capacitively connected to a back gate, by including that the gate voltage can exhibit *noise;* this will occur when connecting the gate lead to a transmission line with a finite impedance. The voltage fluctuations at the back gate can be described through a Caldeira-Leggett model of harmonic oscillators. For *weak* tunneling between the lead and the dot, exploiting the anisotropic Bose-Fermi spin model, we show that zero-point fluctuations of the environment can markedly alter the Matveev Kondo fixed point leading to an amplification of the charge quantization phenomenon.

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The single-electron box, the simplest single-electron circuit, is probably the ideal system in which to test the theory of quantum charge fluctuations. The box has been vividly investigated theoretically because this is a model system for understanding electron-electron interactions and because the quantum fluctuations in the box are analogous to the Kondo effect [1]. In practice, charge fluctuation measurements necessitate a (large) dot at the micron scale with a very dense spectrum [2]. This is referred to as a *metallic dot*. In earlier theoretical treatments of the metallic dot coupled to a reservoir lead [1,3], the gate voltage was treated as a fixed parameter of the Hamiltonian. As depicted in Fig. 1, in this Letter we consider the more general situation where the source of the gate voltage is placed in series with an impedance $Z(\omega)$ of the gate lead showing resistive behavior at low frequency. From the fluctuation-dissipation theorem this resistance will introduce noise in the gate voltage even at zero temperature. Below, dot and lead are coupled through a single-mode quantum point contact (QPC).

Since the charge on a quantum box can now be measured up to few thousandths of an electron [4], it is then important to ask to what extent zero-point fluctuations of the electrical environment affects the (Kondo) ground-state energy, i.e., the capacitance, of a metallic dot already coupled to a reservoir of electrons through a single-mode QPC [1]. While the transport through a single ultrasmall tunnel junction coupled to an electromagnetic (dissipative) environment has been thoroughly investigated [5], to our knowledge a discussion for the Coulomb blockade oscillations of a metallic box subject to a fluctuating gate voltage seems not to be available. According to conventional wisdom, tunneling rates of quasiparticles become strongly suppressed in the presence of dissipative environments because an additional energy is needed to excite the environment [6]. Hence, this should certainly modify the Kondo picture ascribed by Matveev for electron tunneling between the island and the lead [1].

The back gate here is embodied by a voltage fluctuating around its average value V_g. We treat the full dynamics of the fluctuations in a self-consistent manner. $\delta V_g(t)$ representing the voltage fluctuations (or the charge displacement up to a factor) on the back gate is assumed to be given by the sum over the normal coordinates of a Caldeira-Leggett–type bath of harmonic oscillators [7]:

$$H_B = \sum_{j=1}^{+\infty}\left(\frac{P_j^2}{2M} + \frac{M\omega_j^2}{2}X_j^2\right), \tag{1}$$

and

$$\delta V_g(t) = \gamma \sum_j \lambda_j X_j(t). \tag{2}$$

This mapping is entirely justified in the context of a long dissipative transmission line. The precise correspondence [8] between mechanical and electrical quantities can be found, e.g., in the recent Ref. [9]. In the case of a linear circuit element with Ohmic resistance R, one expects Johnson-Nyquist correlation functions $\langle \delta V_g \delta V_g\rangle_\omega \approx \hbar R\omega \coth(\hbar\omega/2k_BT)$. This is usually referred to as an Ohmic dissipative environment. For convenience, we

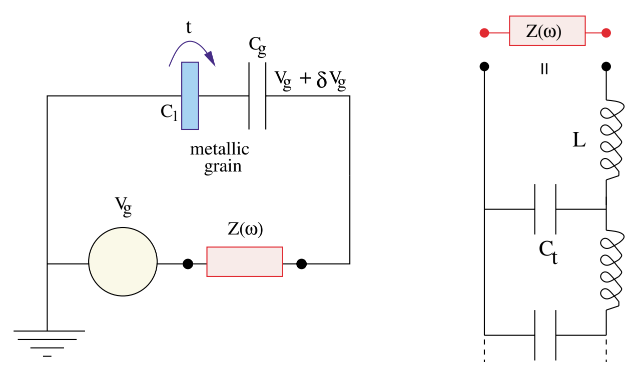

FIG. 1 (color online). Circuit diagram of the metallic dot coupled to a bulk lead and subject to a fluctuating back-gate voltage $V_g + \delta V_g(t)$. The circuit is supposed to exhibit an Ohmic resistance $R = Z(\omega = 0)$. We incorporate the finite resistance in a Hamiltonian fashion, through a long dissipative transmission line with capacitance C_t and inductance L: $R = (L/C_t)^{1/2}$.

have extracted the *dimensionless* parameter $\gamma = \sqrt{R/R_K}$, which naturally characterizes the coupling between the bosonic bath and electrons of the box, $R_K = h/e^2 \approx 25.8\ \mathrm{k}\Omega$ being the quantum of resistance. Having in mind a long dissipative transmission line with capacitance C_t and inductance L, the Ohmic resistance is $R = \sqrt{L/C_t}$. Furthermore, the couplings λ_j, the mass, and the frequencies of the oscillators enter only through the bath's spectral density $J(\omega) = \gamma^2 \sum_j \lambda_j^2 \pi\delta(\omega - \omega_j)/(2M\omega_j) = \hbar R\omega$ in the quantum limit; $\omega_c = \sqrt{1/(LC_t)}$ is a natural low-frequency cutoff for the transmission line. Using the circuit of Fig. 1, the Coulomb term for the quantum box takes the form

$$H_c = E_c(Q - N)^2 - E_c N^2 - eQ\delta V_g. \tag{3}$$

Here, $eN = V_g C_g$ is proportional to the mean gate voltage and eQ depicts the charge on the dot. Here, the charging energy of the granule takes the form $E_c = e^2/(2C_\Sigma)$ where $C_\Sigma = C_g + C_l$, C_g denotes the classical capacitance between the back gate and the grain, and C_l is the capacitance between the lead and the island.

Let us first study the case of a low-transparency tunnel barrier and focus on the point $N = 1/2$ where the states with $Q = 0$ and 1 are energy degenerate, and charge fluctuations in principle can be large. We introduce the small parameter $h = eV_g - e^2/(2C_g) \propto (N - 1/2)$ to measure deviations of N from the degeneracy point. Our treatment of electron tunneling is based on the assumption that the tunneling Hamiltonian takes the form [1]

$$H_t = \sum_{k,q} |t| c_{dk}^\dagger c_{lq} S^+ + \mathrm{H.c.}, \tag{4}$$

where $|t| c_{dk}^\dagger c_{lq}$ is the tunneling term transferring an electron from the lead (l) to the (large) dot (d), and where S^+ is an operator raising the charge Q on the dot from 0 to 1. The tunneling matrix element t is assumed not to depend on the momenta k and q. Below, we will discuss in detail the case of *spinless* electrons, assuming that a strong in-plane magnetic field has been applied, and as mentioned previously the case of a point contact most probably with *one* conducting transverse mode [1,10]. We are led to the obvious identification $Q = S^z + 1/2$, and finally to

$$H_c = -hS^z - \gamma S^z \Phi; \tag{5}$$

$\Phi(t) = e\sum_j \lambda_j X_j(t)$ now stands for the bosonic variable. It is also appropriate to visualize H_t as a transverse Kondo Hamiltonian $H_t = (J_\perp/2)(s^+ S^- + \mathrm{H.c.})$ where $J_\perp = 2|t|$ and $s^+ = \sum_{k,q} c_{lq}^\dagger c_{dk}$. Bear in mind that here a Kondo flip process means physically the transfer of an electron from lead to dot or vice versa, the effective spin index $\alpha = l, d$ is, in fact, the position of an electron in the structure, and then $s^z = \sum_{k,q}(c_{lk}^\dagger c_{lq} - c_{dk}^\dagger c_{dq})$.

Hence, in the limit of weak tunneling ($|t|\rho) \ll 1$, our system is embodied by the Bose-Fermi spin Hamiltonian

$$H = \left(\sum_{\alpha=l,d} H_{\mathrm{kin}}^\alpha - hS^z\right) + H_B + \frac{J_\perp}{2}(s^+ S^- + \mathrm{H.c.}) - \gamma\Phi S^z. \tag{6}$$

H_{kin}^α stand for the usual kinetic terms in the lead and dot. The densities of states in the dot and lead have been assumed to be equal for simplicity and are denoted by ρ. It is suitable to recall that the Bose-Fermi Kondo model has been analyzed in detail in the context of quantum critical points of certain heavy fermion materials [11–13], but to our knowledge Eq. (6) would constitute its first realization in mesoscopic structures. We get the following renormalization group (RG) equations akin to those of Zhu and Si [12] and Zarand and Demler [13] (for $\epsilon = 0$)

$$\frac{d\lambda_\perp}{dl} = \lambda_\perp \lambda_z - \frac{\nu}{2}\lambda_\perp \gamma^2, \qquad \frac{d\lambda_z}{dl} = \lambda_\perp^2, \qquad \frac{d\gamma}{dl} = -\frac{1}{2}\gamma\lambda_\perp^2, \tag{7}$$

$l = \ln(\Lambda_0/\Lambda)$ denotes the scaling variable with $\Lambda_0 \approx \min\{E_c, \hbar\omega_c\}$ the initial value of the energy cutoff and $\Lambda_0 = E_c$ when $R = 0$ ($L = 0$). Our bare values obey $\lambda_\perp = J_\perp\rho = 2|t|\rho$, $\lambda_z = J_z\rho = 0$, $\gamma^2 = R/R_K < 1$; $J_z s^z S^z$ refers to the induced Ising part of the Fermi Kondo coupling [1], and ν which is of the order of 1 refers to the normalization constant occurring in the boson time propagator. Note that here there is no generated transverse Kondo part for the bosonic coupling. It is obvious that since the bosonic heat bath adjusts itself to a given (pseudo)spin configuration this tends to suppress the renormalization of the Kondo process, i.e., the coherent tunneling of electrons from lead to dot. The competition between the bosonic and fermionic fields then will give rise to *two* stable quantum phases corresponding to a *Kondo Fermi-liquid phase* (where the Kondo energy scale will depend on R) and to a *merely bosonic Ising* regime.

It is advantageous to rewrite the RG equations above in terms of the two variables $\lambda_\perp$ and $\hat{\lambda}_z = [\lambda_z - (\nu/2)\gamma^2]$. As long as $\gamma^2 \ll 1$, this enables us to recover the renowned Kosterlitz-Thouless equation flow: $d\lambda_\perp/dl = \lambda_\perp\hat{\lambda}_z$ and $d\hat{\lambda}_z/dl = \lambda_\perp^2$. Hence, this strongly validates the idea of a critical value for the external resistance, namely $R_c[|t|\rho] \approx 4|t|\rho R_K/\nu$, at which the Kondo physics will be washed out. Let us underline that this formula is strictly applicable only in the case of a low-transparency tunnel barrier where $R_c[|t|\rho] \ll R_K$, i.e., in the scope of validity of the perturbative RG. For $R < 4|t|\rho R_K/\nu$, since $\lambda_\perp$ and $\hat{\lambda}_z$ will both grow under renormalization, a Kondo phase with restoration of the SU(2) symmetry emerges. The correction to the classical capacitance $C = \partial\langle Q\rangle/\partial V_g$ is proportional to the impurity susceptibility $\chi = \partial\langle S^z\rangle/\partial h$. An important point is that when applying a strong magnetic field the fixed point corresponds to the usual one-channel Kondo Fermi liquid where $\chi = (T_K[R])^{-1}$, and the Kondo energy scale can be approximated by the one of the completely symmetrical model $T_K[R] \approx \Lambda_0 \exp[-1/(\lambda_\perp - \nu\gamma^2/2)] \ll E_c$.

Close to the degeneracy point $N = 1/2$, the average dot's charge $\langle Q\rangle$ then varies linearly with the applied gate voltage V_g at zero temperature and the slope $C = \partial\langle Q\rangle/\partial V_g$ progressively increases with R (see Fig. 2). When $R \geq 4|t|\rho R_K/\nu$, the Kondo temperature vanishes, which means that $\lambda_\perp(l) \to 0$ and that γ remains almost unrenormalized. Since $\langle\Phi\rangle = 0$, we expect a perfect jump in $\langle Q\rangle$ or $\langle S^z\rangle$ similar to a free spin in magnetic field: $\langle Q\rangle = 1$ if $h > 0$ ($N > 1/2$) and $\langle Q\rangle = 0$ if $h < 0$ ($N < 1/2$). This shows that a reasonably large resistance in the electrical circuit can markedly restore a perfect Coulomb staircase behavior by suppressing the virtual tunneling of electrons between the lead and the metallic dot. Note that an analogous phenomenon has been reported in the different context of a *small* "noisy" dot (embodied by the lowest unoccupied electron level as opposed to our many-body grain) embedded in a *small* mesoscopic ring (described by the topmost occupied electron level as opposed to our semi-infinite lead) [9]. In the case of spinful fermions, i.e., without magnetic field, the RG flow would be quite similar to that of Eq. (7). However, the effect of the external impedance should be much less spectacular because already in the Kondo phase there is a blatant divergence in the capacitance for $N \approx 1/2$ [1],

$$C \propto -\frac{2E_c}{|t|\rho}\exp\left(\frac{1}{2|t|\rho}\right)\ln\left(\frac{1}{|2N-1|}\right)\cos(2\pi N), \quad (8)$$

emerging clearly from the two-channel Kondo model (the two channels being the spin polarizations of an electron).

A maybe more physical way of understanding the effect of the electrical environment is to absorb δV_g in the tunneling term using a unitary transformation [15] following Refs. [5,6]. In our situation, H_t naturally

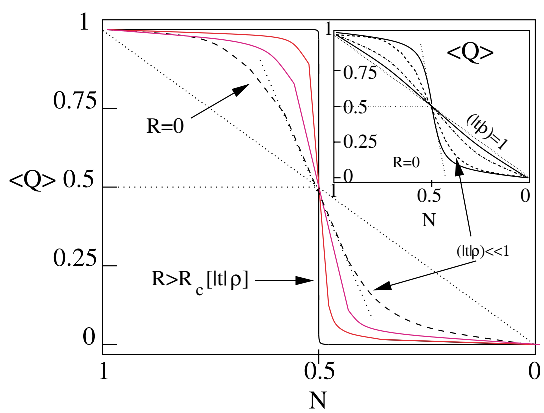

FIG. 2 (color online). Average dot's charge $\langle Q\rangle$ versus the rescaled voltage $N = V_g C_g/e$ for *weak* tunneling ($|t|\rho) \ll 1$ and in the case of a single-mode spin-polarized point contact. When R will exceed $R_c[|t|\rho] \approx 4|t|\rho R_K/\nu$, we predict the restoration of perfect Coulomb steps [14]. Inset: Evolution of the Coulomb blockade oscillations for $R = 0$ and various dot-lead couplings.

turns into

$$\hat{H}_t = J_\perp e^{i\varphi} s^+ S^- + \text{H.c.}, \quad (9)$$

where the phase $\varphi(t) = (e/\hbar)\int_{-\infty}^{t} \delta V_g(t')\,dt'$ at the time t represents the integral over the voltage fluctuations felt by the dot [8]. It becomes (more) transparent that a transferred charge onto the dot must be accompanied by zero-point fluctuations (excitations) of the environment. In this formulation of the tunneling Hamiltonian, the Kondo coupling J_z will be renormalized by the contribution

$$J_\perp^2 \int d\tau\, \mathrm{sgn}(\tau) G(\tau) e^{K(\tau)}, \quad (10)$$

$G(\tau) = 1/\tau$ corresponds to the usual fermionic imaginary time propagator. The main difference with the original Kondo problem is the occurrence of the phase correlator $K(\tau) = \langle\varphi(\tau)\varphi(0) - \varphi(0)^2\rangle$. An exact calculation of $K(\tau)$ can be performed at zero temperature for the transmission line, i.e., for the bath of harmonic oscillators, and for large $\tau = it$ we find $K(\tau) = -2(R/R_K) \times \ln(\omega_c|\tau|)$ [6,9]. When R is negligible, $K(\tau) \approx 0$, one easily recovers the second line of Eq. (7) and then the expected one-channel Kondo fixed point. By increasing R, the integral $\int_{\hbar/\Lambda_o}^{\hbar/\Lambda} d\tau\, \tau^{-2\alpha-1}$ involved in Eq. (10) will remain very small whatever the energy Λ. Hence, this leads to the same conclusion as previously: $J_z(l)$ ($J_\perp(l)$) will persist to be insignificant up to zero temperature and then the (pseudo)spin S^z will satisfy $\langle S^z\rangle = \pm 1/2$. This can be entirely attributed to the fact that, for large R, the probability $P(\tau) = e^{K(\tau)}$ to observe zero-point fluctuations in the environment rapidly decreases for a long time τ [5,6] and as a result long time (low-energy) Kondo (tunneling) processes will be eliminated by the noise.

Now, we shall concentrate on the opposite limit, i.e., close to perfect transmission. Here, electronic wave packets are clearly spread out between the bulk lead and the granule, and hence one could anticipate that the zero-point fluctuations of the environment will have more difficulty to affect the charge fluctuations of the quantum box. To make it more quantitative, we can resort to bosonization and tackle the one-dimensional problem emerging at the QPC along the lines of Refs. [1,10]. Consequently, the Coulomb term becomes

$$H_c = \frac{E_c}{\pi}[\phi(0) - \sqrt{\pi}N]^2 - E_c N^2 - \frac{e}{\sqrt{\pi}}\phi(0)\delta V_g, \quad (11)$$

where the boson $e\phi(0)/\sqrt{\pi}$ describes the charge on the granule and the coordinate $x = 0$ refers to the entrance of the dot (or QPC). First, we already note that exactly at perfect transmission ($\rho|t| = 1$), as long as $\langle\delta V_g\rangle = 0$, the capacitive coupling with the back gate will definitely impose $\langle\phi(0)\rangle \approx \sqrt{\pi}N$, and then the grain's charge will vary linearly with the (mean) back-gate voltage whatever the external resistance (inset of Fig. 2). Obviously, the previous qubit basis $S^z = \pm 1/2$ would not be appropriate in that limit. The weak backscattering contribution at the

QPC then can be manipulated as [1]

$$H_{bs} = -\frac{v_F}{\pi a}|r|\cos(\sqrt{4\pi}\delta\phi(0))\cos(2\pi N), \tag{12}$$

where a is a short-distance cutoff, v_F denotes the Fermi velocity, the small reflection amplitude obeys $|r| = [1 - (\rho|t|)^2]^{1/2} \ll 1$, and the charge fluctuation field is $\delta\phi(0) = \phi(0) - \sqrt{\pi}N$. For spinless fermions, the (first-order) correction to the ground-state energy naturally takes the form $\delta E_1 = -v_F/(\pi a)|r|e^{-2\pi\langle\delta\phi(0)^2\rangle}\cos(2\pi N)$.

Let us first recall that when $R/R_K \to 0$, $\langle\delta\phi(0)^2\rangle$ is large but finite, and a simple calculation provides $\langle\delta\phi(0)^2\rangle = -1/(2\pi)\ln(a\mu E_c/v_F)$ where $\mu = e^C$ and $C \approx 0.5772$ is Euler's constant; the phonons with energies below E_c are pinned down by the interaction term. The periodic correction to the ground-state energy then reads

$$\delta E_1 = -\frac{|r|}{\pi}\mu E_c\cos(2\pi N). \tag{13}$$

The average charge in the dot is given by $e\langle Q\rangle = eN - e/(2E_c)\partial\delta E_1/\partial N = eN - \mu e|r|\sin(2\pi N)$. One recovers a curve similar to the dashed line in Fig. 2 even though the amplitude of the charge oscillations is much smaller (i.e., this is much closer to the dotted straight line).

Now, we revisit the quantum fluctuations of the charge displacement for larger external impedances. The part of the Coulomb term containing $\delta\phi(0)$ can be turned into

$$H_c = \frac{E_c}{\pi}\delta\phi(0)^2 - e\delta\phi(0)\sum_j \beta_j X_j, \tag{14}$$

with $\beta_j = \gamma\lambda_j/\sqrt{\pi}$. It is suitable to note the resemblance with a Brownian particle in a harmonic potential. By exploiting this analogy [7,16], we can check that the following correlation function $\langle\delta\phi(0,t)\delta\phi(0,0)\rangle \propto -R/(E_c^2 t^2)$ almost vanishes for long times and hence that the noise is rather irrelevant for $|r| \ll 1$. The charge displacement $\langle\delta\phi(0)^2\rangle$ will still be dominated by the logarithmic phonon contribution above cutoff at the time $\hbar/E_c$. Zero-point fluctuations of the transmission line cannot fundamentally affect the result of Eq. (13), but for very large and probably unrealistic impedances $R/R_K \to +\infty$, we could expect that the logarithmic contribution of $\langle\delta\phi(0)^2\rangle$ will be cutoff at $\tau_c \leq \hbar/E_c$. In the absence of the (Coulomb) term $E_c\delta\phi(0)^2/\pi$, for long times one would still obtain a logarithmic growth of the $\delta\phi(0)$ correlation function [17], $\langle\delta\phi(0,t)\delta\phi(0,0)\rangle \propto -(R_K/R)\ln t$. More physically, the scattering of electrons at the QPC produces a phase shift in the wave functions $\psi_k(x) = \cos(k|x| - \delta)$ that is naturally related to the average charge entering onto the dot through the Friedel sum rule $\delta = \pi\langle Q\rangle/e$. As long as $|r| \ll 1$, this means that we can approximate $\langle Q\rangle/e \approx N$ whatever the external Ohmic resistance (again $\langle\delta N\rangle = 0$). This naturally ensures the following Friedel oscillation of the electron density $\rho(x) \approx E_c/(\hbar v_F)\cos(2k_F|x| - 2\delta)$ for $|x| \ll v_F/E_c$ with $2\delta \sim 2\pi N$. We can then infer that the shift in the ground-state energy $-\int dx\rho(x)V(x)\delta(x)$, where the potential $V = |r|\hbar v_F$, (roughly) is still equal to δE_1.

In closing, we have shown that in the context of a metallic granule and a low-transparency tunnel barrier, zero-point fluctuations of a transmission line can alter the quantum tunneling of electrons *at low energy* and then restore a perfect Coulomb staircase behavior. A clear observation of such a suppression of the quantum charge fluctuations necessitates the application of a strong in-plane magnetic field, a reasonable external impedance (a non-negligible fraction of R_K), and again a small transmission coefficient between the metal lead and the grain. When approaching the perfect transmission, quantum charge fluctuations become so prominent that the electrical environment is rather inefficient.

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