Bloch Modes Dressed by Evanescent Waves and the Generalized Goos-Hänchen Effect in Photonic Crystals

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It is common knowledge that in an infinite periodic medium, for instance, an infinite photonic crystal, the direction of propagation of a monochromatic wave packet is given by the normal to the isofrequency diagram. We show that this is no longer true in a finite size medium, due to the existence of evanescent waves near the interfaces of the photonic crystal. We derive a renormalized isofrequency diagram giving the correct direction. We give a physical interpretation, showing that this phenomenon can be considered as a generalized Goos-Hänchen effect.

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Photonic crystals are periodically structured artificial materials [1,2]. They originate in Bragg mirrors, which have been used as filters in optics for decades. The extension of periodicity to two and three dimensions of space was first imagined to permit the realization of full band gaps and the inhibition of spontaneous emission [3,4]. However, it soon became clear that the very existence of conduction bands and their intricate structure could be exploited in optics. In particular, the superprism phenomenon and the ultrarefractive effects are widely studied [5–9]. From a theoretical point of view, it is known that, in a periodic medium, the propagation of a monochromatic wave packet is ruled by the normal to the dispersion curves. More precisely, the mean Poynting vector $\langle P\rangle$ over a basic lattice cell Y is proportional to the normal to the isofrequency diagram: $\nabla_k \omega \propto \langle P\rangle$. This result is known both in quantum mechanics for the mean probability current and in waveguide theory. The point at issue in the present work is to know to what extent this result holds for structures that are not infinitely periodic but finite in size. Let us dwell on that point: photonic crystals are to be used in scattering experiments; that is, photons are created by some source outside the crystal, they interact with it, and then they leave it and are annihilated at some other place (the detector, for instance). This means that photons interact strongly with the boundaries of the crystal, resulting in the creation of evanescent fields. The consequence of this fact is that the field inside a finite photonic crystal (the finiteness being the actual experimental situation) cannot be represented by Bloch waves only [10]. The aim of this work is to show that, in certain circumstances, the relation between the normal of the Bloch isofrequency diagram and the direction of propagation of a beam no longer holds. We show in the following that the bare Bloch propagator has to be dressed by evanescent waves produced by the interfaces and that this results in a renormalized isofrequency diagram whose normal is found to give the correct direction of propagation. We start by deriving a mathematical formulation of the problem, and then we elaborate on the physical meaning of the encountered phenomena. We consider a two-dimensional photonic crystal such as that depicted in Fig. 1 (the particular symmetry of the lattice is not relevant for the following analysis). It is made of a stack of N diffraction gratings, thus infinite in the horizontal direction (to make the theoretical analysis easier) and finite in the vertical one. The period of one grating is denoted d and its height by h. This device is illuminated by an incident beam u^i, with mean incidence angle θ_0 and wavelength λ, so that we have $u^i(x, y) = \int A(\theta)e^{ik(x\sin\theta - y\cos\theta)}d\theta$ ($k = \frac{2\pi}{\lambda}$) where $A(\theta)$ takes significative values only near θ_0 [for instance, we can choose a Gaussian beam $A(\theta) = k(W/\sqrt{\pi})e^{-k^2W^2(\sin\theta-\sin\theta_0)^2/4} \times \cos\theta$]. For the sake of simplicity, we assume that for each plane wave constituting the incident field u^i, there is only one reflected and one transmitted order through the photonic crystal (cf. Fig. 1). For instance, this situation happens for any angle if $\lambda > 2d$. That way, far enough from the crystal, the reflected field reads $u^r(x, y) = \int r_0(\theta)A(\theta)e^{ik(x\sin\theta + y\cos\theta)}d\theta$, and the transmitted field reads $u^t(x, y) = \int t_0(\theta)A(\theta)e^{ik(x\sin\theta - y\cos\theta)}d\theta$. The preceding equalities represent the propagative part of the

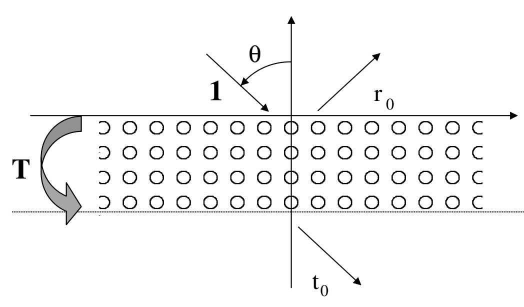

FIG. 1. Sketch of the photonic crystal.

fields only. Near the interfaces, complete expressions, i.e., including the evanescent fields, read

$$\begin{pmatrix} u^r(x, y) \\ u^t(x, y) \end{pmatrix} = \int d\theta \sum_{n=0}^{\infty} \begin{pmatrix} r_n(\theta) \\ t_n(\theta) \end{pmatrix} A(\theta) e^{i[x\alpha_n(\theta) - y\beta_n(\theta)]}, \quad (1)$$

where $\alpha_n = k\sin\theta + \frac{2n\pi}{d}$ and $\beta_n^2 = k^2 - \alpha_n^2$. However, it is quite important to note that the coefficients r_0 and t_0 are not defined by assuming that there are no evanescent waves, they are simply the propagating part of the reflected and transmitted fields. Considering the photonic crystal as a "black box," we can characterize its scattering behavior by introducing a dressed transfer matrix $\mathbb{T} = (t_{ij})$, which relates the propagating fields above and below the crystal. The main point here is that the dressed transfer matrix $\mathbb{T}$ takes into account the totality of the field inside the structure and not only Bloch waves (hence the term "dressed"). Therefore, the renormalized isofrequency diagram that is obtained from it is, in general, different from that obtained by considering Bloch waves only, that is, by considering that the medium is an infinitely periodic one.

This matrix is defined by imposing that it has real entries t_{ij} and satisfies

$$\mathbb{T}\begin{pmatrix} 1 + r_0 \\ i\beta_0(1 - r_0) \end{pmatrix} = t_0\begin{pmatrix} 1 \\ i\beta_0 \end{pmatrix}. \quad (2)$$

Provided that the system is not dissipative, and hence $|r_0|^2 + |t_0|^2 = 1$, a tedious but straightforward calculation shows that the determinant of $\mathbb{T}$ is equal to 1. From this result, we see that we have reduced the 2D scattering problem to a 1D one. We conclude that the eigenvalues of $\mathbb{T}$ are roots of the polynomial $X^2 - \mathrm{tr}(\mathbb{T})X + 1$ and are inverse one of the other; i.e., they are of the form (γ, γ^{-1}). It is well known that in 1D systems (whether they are quantum mechanical or electromagnetical ones), the Bloch diagram is obtained by considering the eigenvalues of the transfer matrix. Indeed, if $|\mathrm{tr}(\mathbb{T})| < 2$ then the eigenvalues are complex with modulus one, which corresponds to effective conduction bands, whereas if $|\mathrm{tr}(\mathbb{T})| > 2$ then the eigenvalues have modulus different from one, which corresponds to effective band gaps.

From relation (2), we can derive the expression of the reflection and transmission coefficients:

$$r_0(k, \theta) = \frac{(\gamma^2 - 1)f}{\gamma^2 - g^{-1}f}, \qquad t_0(k, \theta) = \frac{\gamma(1 - g^{-1}f)}{\gamma^2 - g^{-1}f} \quad (3)$$

with f and g defined by

$$f = \frac{i\beta_0 v_1 - v_2}{i\beta_0 v_1 + v_2}, \qquad g = \frac{i\beta_0 w_1 - w_2}{i\beta_0 w_1 + w_2}, \quad (4)$$

where $\{\mathbf{v} = (v_1, v_2), \mathbf{w} = (w_1, w_2)\}$ is a basis of eigenvectors for $\mathbb{T}$. The eigenvectors can be chosen in such a way that $|g| < |f|$. For complex eigenvalues, hence in an effective conduction band, we have $f = \overline{g}^{-1}$ and therefore $|g| < 1$.

The interest of these forms lies in the fact that they provide a natural series expansion of the coefficients. More specifically, let us consider the transmission coefficient. As $|g/f| < 1$, we have

$$t_0(k, \theta) = \left(1 - \frac{g}{f}\right)\gamma \sum_p \gamma^{2p}\left(\frac{g}{f}\right)^p. \quad (5)$$

From this expansion, we get the following expression for the propagating part of the transmitted field:

$$u^t(x, y) = \sum_p \int A(\theta)(1 - |g|^2)|g|^{2p}\gamma^{2p+1} e^{ik(x\sin\theta + y\cos\theta)} d\theta. \quad (6)$$

Although this expression might look complicated at first sight, it is rather easy to form a physical picture of this series: each term in the sum represents a beam and the overall series accounts for the multiple scattering on the upper and lower interface of the crystal.

At each multiple reflection, the intensity of the transmitted part of the beam decreases, meaning that we can consider the first transmitted beam only [corresponding to $p = 0$ in (6)]:

$$u^t(x, y) \sim \int A(\theta)(1 - |g|^2)\gamma e^{ik(x\sin\theta + y\cos\theta)} d\theta. \quad (7)$$

We are now in a position to compute the direction followed by the beam inside the medium. To do so, we consider the field on the lower interface $u^t(x, Nh)$, and we compute the barycenter of the field:

$$X_G = \frac{\int x|u^t(x, Nh)|^2 dx}{\int |u^t(x, Nh)|^2 dx}. \quad (8)$$

This gives the position of the center of the first emerging beam. Noting that γ has modulus 1, we denote $\gamma = e^{i\beta(\alpha)Nh}$ and $\alpha = k\sin\theta$. With these notations, the renormalized Bloch diagram is given by the curves $(\alpha, \beta(\alpha))$. Using the Parseval-Plancherel relation, we get, for a sufficiently spectrally narrow beam, $X_G \sim -\frac{\partial\beta}{\partial\alpha}Nh$. This value is to be compared with the corresponding quantity for the incident field, which defines the point where the beam enters the crystal. Elementary trigonometry then gives the refraction angle θ_r of the beam inside the photonic crystal:

$$\tan\theta_r = -\frac{\partial\beta}{\partial\alpha}. \quad (9)$$

This shows that the direction of the beam is given by the normal to the renormalized isofrequency diagram (α, β). However, this dispersion diagram is not the one obtained by the usual Bloch theory: it is renormalized by the evanescent waves. Of course, one can wonder about the importance of this renormalization. To exemplify its importance, let us give a numerical example. We consider

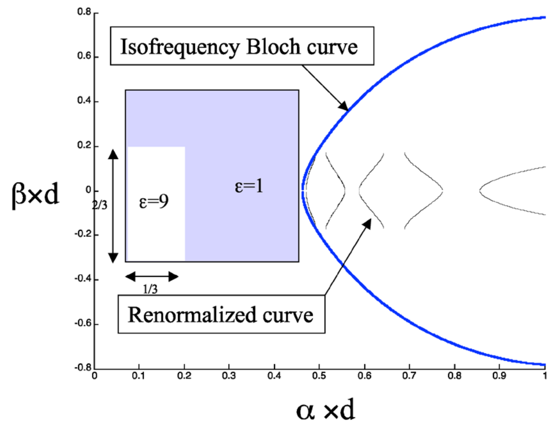

FIG. 2 (color online). Isofrequency curves obtained from Bloch theory and from the dressed transfer matrix. The basic cell of the structure is shown in insert.

the square photonic crystal (period d) whose basic cell is depicted in the inset of Fig. 2. It is a stack of six lamellar gratings with relative dielectric constant $\varepsilon = 9$. We have plotted in Fig. 2 the isofrequency curves ($\lambda/d = 2$) obtained from Bloch theory and from the dressed transfer matrix. Though there is a strong discrepancy between both curves, it should be noted that the relevant quantity here is not the curve but the normals to the curve. We note the existence of bands of wavelengths corresponding to gaps for the renormalized modes and to conduction bands for the Bloch modes. Inside these regions, the field inside the crystal is projected essentially on evanescent waves, so that these bands are seen as if they were gaps. Let us now illuminate the structure with an s-polarized Gaussian beam with waist $W/\lambda = 25$ and mean angle $\theta_0 = 57°$. The refraction angle as calculated by Bloch wave theory is $\theta_r = 28°$. A direct rigorous calculation, using grating theory, gives $\theta_n = 50°$, and the angle computed from the renormalized Bloch diagram is $\theta_d = 45°$. This shows that the difference is up to a factor of 1.8 between the correct value and that predicted by the nonrenormalized Bloch theory. Evidently, in that situation, the effect due to evanescent waves is far from being a mere correction term. At this stage, one could wonder whether this effect is compatible with a good transmission ratio. Indeed, we have stressed the fact that for this effect to be important, evanescent waves were needed. In general, the presence of a huge evanescent field prevents wave propagation, the clearest example being the phenomenon of band gaps. However, this is not the case here, where the transmission is near 1 for a plane wave with incidence θ_0. What happens is that the damping of the evanescent waves is not that huge, which allows one to propagate a noticeable part of the field by tunnel effect. Besides, due to the finite width of the device, there exist Perot-Fabry resonances in the photonic crystal, the wavelength that we used corresponds to one of these resonances. A fundamental point should be checked here: when the wavelength over ratio is

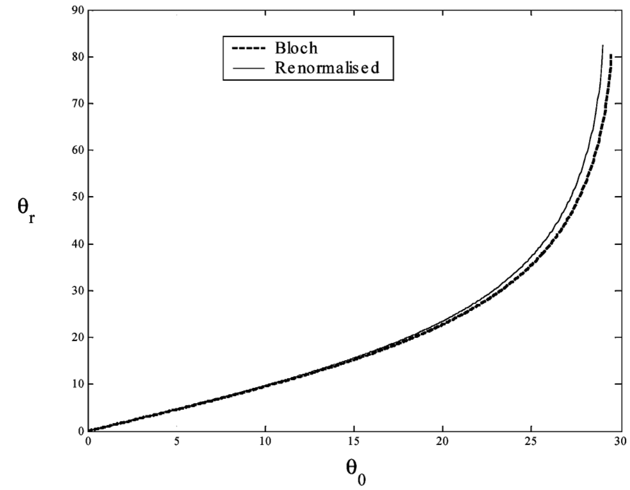

FIG. 3. Predictions for the refraction angle inside the photonic crystal obtained from Bloch theory and the dressed transfer matrix for $\lambda/d = 3$.

great, it is known by homogenization theory that the photonic crystal behaves as a homogeneous medium; therefore both predictions from the dressed matrix and Bloch theory should fit because the influence of the evanescent waves becomes negligible. Let us then choose $\lambda/d = 3$. In that case, we have plotted the predictions for the refraction angle θ_r for both theories in Fig. 3, where we see a perfect adequation, although a slight deviation is observed when one gets off the normal incidence, due to the apparition of evanescent waves. On the contrary, the effect does not disappear as the number of stacks increases because it depends on the interfaces only. To show this, we have plotted in Fig. 4 the refraction angle θ_r obtained by the rigorous grating theory versus the number of gratings constituting the photonic crystal (up to a stack of 40 gratings). It is seen that the deviation has

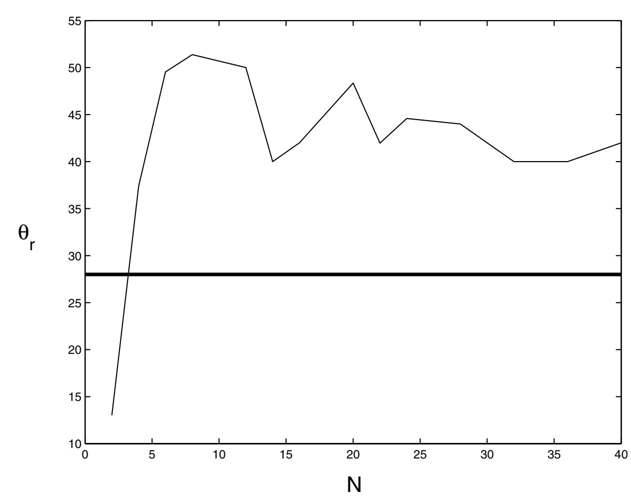

FIG. 4. Evolution of the angle of deviation with respect to the number N of periods. The bold line corresponds to the deviation obtained by Bloch theory.

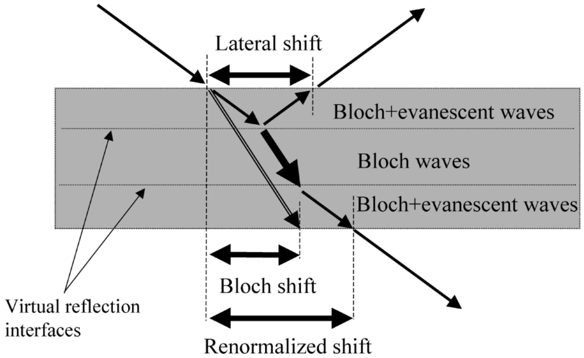

FIG. 5. A physical interpretation in terms of the Goos-Hänchen effect.

an oscillatory behavior that is certainly linked to Perot-Fabry–like oscillations, but also that the angle tends to a value near 45°, well above the value expected from Bloch theory.

How to form a physical picture of the influence of the evanescent field? The situation encountered here has much in common with the so-called Goos-Hänchen effect [11–16]: when a light beam illuminates a plane interface under total reflection, the reflected beam is laterally shifted with respect to the incident one, this shift being due to the existence of evanescent waves below the interface. This effect happens also for external reflection upon a photonic crystal inside a band gap [17]. In our situation, we are not in the case of total reflection, but we have already stressed that evanescent waves do exist in the crystal. A physical picture for the Goos-Hänchen effect amounts to saying that, due to the skin effect, all happens as if the incident beam were reflected on a plane slightly below the actual interface, where the distance between both the real interface and the virtual one is of the order of the skin depth. A similar physical interpretation can be formed in our situation. The interior of the photonic crystal can be divided into three zones: a central one, into which the field exists approximately as a superposition of Bloch waves, by which we mean that the evanescent field is negligible, and two exterior zones where the field is the sum of both propagating and nonpropagating waves. Within these zones, the beam is subjected to a lateral Goos-Hänchen–like shift. We have sketched the situation in Fig. 5. This kind of situation had already been considered in [18] for the Goos-Hänchen shifts for both the transmission and the reflection. For a semi-infinite medium, we would still have a lateral Goos-Hänchen shift for the reflected field, due to the first zone. This situation is treated in [17]. The question to know why a non-negligible part of the field travels on evanescent modes rather than on propagating ones is a very difficult one, if of fundamental importance. We have shown in a preceding paper [10] that a crucial quantity is the damping ratio of the evanescent mode of lowest order. In particular, we have shown that such a situation (i.e., transmission by tunneling in a conduction band) was more likely to happen at the band edges of certain gaps (the so-called local gaps, defined in [10]), although it was not required. However, a complete theory is still lacking.

We have shown that the direction of propagation of a spectrally narrow beam inside a photonic crystal is sometimes not given by the normal to the isofrequency dispersion diagram, contrary to common knowledge. This effect, which can be rather large, is due to the existence of the interfaces and the evanescent field that they create, and can be considered as a generalized Goos-Hänchen effect. We have developed a theory in the subwavelength regime, but similar effects should exist for multiple orders, in which case a generalized dressed transfer matrix can also be defined. This phenomenon could have an impact on the design of photonic crystal-based devices in integrated optics.

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