Measuring a Photonic Qubit without Destroying It

G. J. Pryde, J. L. O'Brien, A. G. White, S. D. Bartlett, and T. C. Ralph
Centre for Quantum Computer Technology, Department of Physics, University of Queensland, Brisbane 4072, Australia
(Received 5 December 2003; published 12 May 2004)

Measuring the polarization of a single photon typically results in its destruction. We propose, demonstrate, and completely characterize a *quantum nondemolition* (QND) scheme for realizing such a measurement nondestructively. This scheme uses only linear optics and photodetection of ancillary modes to induce a strong nonlinearity at the single-photon level, nondeterministically. We vary this QND measurement continuously into the weak regime and use it to perform a nondestructive test of complementarity in quantum mechanics. Our scheme realizes the most advanced general measurement of a qubit to date: it is nondestructive, can be made in any basis, and with arbitrary strength.

DOI: 10.1103/PhysRevLett.92.190402 PACS numbers: 03.65.Ta, 03.67.–a, 42.50.Xa

At the heart of quantum mechanics is the principle that the very act of measuring a system disturbs it. A quantum nondemolition (QND) scheme seeks to make a measurement such that this inherent *backaction* feeds only into unwanted observables [1,2]. Such a measurement should satisfy the following criteria [3]: (i) The measurement outcome is correlated with the input, (ii) the measurement does not alter the value of the measured observable, and (iii) repeated measurement yields the same result—*quantum state preparation* (QSP). Originally proposed for gravity wave detectors, most progress in QND has been in the continuous variable (CV) regime, involving measurement of the field quadrature of bright optical beams [3]. Demonstrations at the single-photon level have been limited to intracavity photons due to the requirement of a strong nonlinearity [4,5]. In addition, there has been no complete characterization of a QND measurement due to a limited capacity to prepare input states and thus inability to observe all the required correlations.

The ability to measure properties of a single photon is critical for optical quantum computing [6] and quantum cryptography [7]. Such measurements are traditionally strong and destructive, employing direct photodetection. However, quantum mechanics allows general measurements [8] that range from strong to arbitrarily weak (one obtains maximum to negligible information) and can be nondestructive. Such general measurements may find application in a range of quantum information protocols [9–11]. General measurements of a single photon's polarization, a common qubit encoding in many protocols, will be useful for quantum cryptography protocols employing weak measurements [12], testing the security of quantum cryptography protocols against an eavesdropper using weak QND measurements [13,14], and feedback onto a quantum system [15] in the context of quantum control of a single-photon state. They are also important [16] for tests of *wave-particle duality* [17] and other fundamental tests of quantum mechanics [17,18].

Here we propose, demonstrate, and completely characterize a scheme for the QND measurement of the polarization of a free-propagating single-photon qubit—a flying qubit. This is achieved nondeterministically by using a measurement induced nonlinearity. The scheme is heralded but requires photon number resolving detectors. Our demonstration uses standard detectors and therefore requires simultaneous detection of two photons. The measurement can be performed on all possible input states. Eigenstate inputs result in strong correlation with the measurement outcome; coherent superpositions exhibit "collapse" and a corresponding loss of coherence as a result of the measurement. Direct observation of all correlations demonstrates that the criteria (i)–(iii) have been satisfied. To quantify the performance against these criteria, we introduce measures that are applicable to *all* QND measurements. Finally, we show how our measurement scheme can be varied continuously from a strong measurement into the regime of a nondestructive weak measurement of polarization. Using these weak measurements we perform a fundamental test of complementarity using "which-path" information without destroying the photon. Our scheme implements a measurement that is nondestructive, can be made in any basis, with arbitrary strength, and is therefore the most advanced general measurement to date.

Our scheme for QND measurement of the polarization of a single photon in the horizontal (H)/vertical (V) basis is illustrated in Fig. 1(a). After interaction, a destructive measurement of the polarization (H or V) of an ancilliary *meter* photon realizes a QND measurement of the free-propagating *signal* photon. The required strong optical nonlinearity, which couples the signal and meter, is realized using only linear optics and photodetection following the principles developed for optical quantum computing [6,10]. As with those schemes, our QND measurement is nondeterministic: it succeeds with nonunit probability, but whenever precisely one photon is detected in the meter output, it is known to have succeeded. The

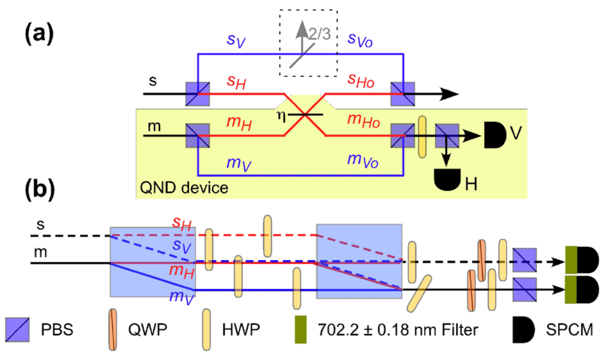

FIG. 1 (color online). A QND measurement of a polarization encoded single photon qubit. (a) A linear optics scheme, as explained in the text. The $\frac{2}{3}$ BS is used only for weak operation. (b) A schematic of the experimental setup. Pairs of photons were generated through spontaneous parametric down-conversion in a β-barium-borate crystal cut for beamlike [19,20] type-II phase matching and coupled [21] into single mode optical fibers (details are given in [10]). The output of each fiber was collimated and wave plates in each input beam allow preparation of the required signal and meter states. The tilted HWP corrects a systematic phase shift between the meter modes. A standard polarization analyzer—a half wave plate (HWP), quarter wave plate (QWP), and polarizing beam splitter (PBS)—is used in each output before a SPCM. A coincidence window of 5 ns was used with coincident counts of $\sim 100\ \mathrm{s}^{-1}$, and we do not subtract accidental counts.

key to the operation of this circuit is that the QND device makes a photon number measurement $|n = 0, 1\rangle$ in the s_H arm of the signal interferometer: the H component of the meter experiences a π phase shift *conditional* on the signal being in the state $|H\rangle_s$ (i.e., in the mode s_H). This conditional phase shift is realized by a nonclassical interference between the two photons at the η reflectivity beam splitter (BS) and conditioned on the detection of a single meter photon.

When the signal photon is in a polarization eigenstate, we require that the meter and signal outputs be the same state (i.e., $|H\rangle_s|H\rangle_m$ or $|V\rangle_s|V\rangle_m$). Consider the modes labeled in Fig. 1(a): the m_V and s_V modes are simply transmitted, while $s_{H_o} \rightarrow -\sqrt{\eta}s_H + \sqrt{1-\eta}m_H$ and $m_{H_o} \rightarrow \sqrt{1-\eta}s_H + \sqrt{\eta}m_H$. For the signal in the eigenstate $|V\rangle_s$, the signal and meter do not interact. We require the meter output to be $(|H\rangle_m + |V\rangle_m)/\sqrt{2}$, which is rotated by 45° to $|V\rangle_m$ by the HWP set at 22.5°. This is realized by preparing the meter state:

$$|D(\eta)\rangle_m = \sqrt{1/(1+\eta)}|H\rangle_m + \sqrt{\eta/(1+\eta)}|V\rangle_m. \quad (1)$$

The $\sqrt{1-\eta}$ loss experienced by the H component makes the H and V components equal and the signal-meter output state is

$$|\phi_{\mathrm{out}}^{V_s}\rangle = \sqrt{\eta/(1+\eta)}|V\rangle_s(|H\rangle_m + |V\rangle_m) + \sqrt{(1-\eta)/(1+\eta)}|H\rangle_s|V\rangle_s, \quad (2)$$

where the first term represents successful operation, and the second a failure mechanism corresponding to two photons in the signal output and no photons in the meter output. After rotation of the meter state by 45° the successful output state is $|V\rangle_s|V\rangle_m$.

When the signal is in the other eigenstate input $|H\rangle_s$ the output state is

$$|\phi_{\mathrm{out}}^{H_s}\rangle = \sqrt{1/(1+\eta)}[(1-2\eta)|H\rangle_s|H\rangle_m - \eta|H\rangle_s|V\rangle_m] + \ldots, \quad (3)$$

where the terms not shown are ones with two photons in one of the outputs and zero in the other. We require that the coefficients be equal so that the meter output state is $(|H\rangle_m - |V\rangle_m)/\sqrt{2}$ (which is rotated to $|H\rangle_m$ by the HWP). This is satisfied only for $\eta = \frac{1}{3}$, and thus we prepare the meter in the state $|D'\rangle_m \equiv |D(\frac{1}{3})\rangle_m = (\sqrt{3}/2)|H\rangle_m + \frac{1}{2}|V\rangle_m$.

The probability of success for an arbitrary input state $\gamma|H\rangle_s + \delta|V\rangle_s$ is $P = (\gamma^2 + 3\delta^2)/6$. The fact that P is dependent on the input state must be taken into account when inferring populations from repeated measurements of identically prepared input states. It is possible to introduce the $\frac{2}{3}$ loss shown in the s_V mode in Fig. 1(b) to make $P = \frac{1}{6}$ independent of δ and γ, as done below for weak measurement operation. Successful QND would then be signaled by the detection of a single photon in the meter output and no photon in this extra loss mode.

Figure 1(b) outlines our experimental design for realizing the schematic circuit of Fig. 1(a). Polarization beam displacers (PBDs) separate the horizontal and vertical components of the inputs into parallel spatial modes and are used to form stable signal and meter Mach-Zehnder polarization interferometers. Note that to realize a nonclassical interference at an ordinary BS the two photons must have the same polarization [Fig. 1(a)]. In contrast, a nonclassical interference at a PBD (or PBS) requires the two photons to be orthogonally polarized. Therefore, in Fig. 1(b) the s_V and m_H modes interfere nonclassically. In the center region each mode passes through two HWPs. The combined effect is to rotate the polarization of s_H and m_V by 90° (so that the polarization modes of the signal and meter each recombine correctly), and s_V and m_H by 125° (which in conjunction with the second PBD implements a $\frac{1}{3}$ BS). Note that wave plates can be used in front of the circuit of Fig. 1(b) to measure the signal in any basis.

The data collected are coincident counts—simultaneous detection of a single photon at each of the detectors—for two reasons. First, in order to characterize the QND measurement we need to measure the polarization of both the signal and meter photons to determine the correlation between them. By adjusting our analyzers we can directly measure the probability P_{HH} of the two photons being horizontally polarized, etc. Second, although in principle measurement of a *single* meter photon would indicate that the measurement worked, currently available single-photon counting modules (SPCMs) cannot distinguish between one and many photons, and only operate with moderate quantum efficiency.

We prepared the signal in the eigenstates $|H\rangle_s$ and $|V\rangle_s$, as well as the superposition states $|D^{\pm}\rangle_s \equiv (\sqrt{3}/2)|H\rangle_s \pm \frac{1}{2}|V\rangle_s$ and $|R^{\pm}\rangle_s \equiv i(\sqrt{3}/2)|H\rangle_s \pm \frac{1}{2}|V\rangle_s$ (states which give equal probability of measuring $|H\rangle_s$ and $|V\rangle_s$), and measured the probabilities $P_{sm} = P_{HH}$, P_{HV}, P_{VH}, and P_{VV} (Table I). The QND measurement works most successfully in the case of a $|H\rangle_s$ signal, because it requires only the splitting and recombining of the meter components. In contrast, all other measurements require both classical and nonclassical interference.

To quantify the performance of a QND measurement relative to the criteria (i)–(iii), we define new measures that can be applied to all input states. These measures each compare two probability distributions p and q over the measurement outcomes i, using the (classical) fidelity $F(p,q) = (\sum_i \sqrt{p_i q_i})^2$. For polarization qubits, $i \in \{H, V\}$; also, $F = 1$ for identical distributions, $F = \frac{1}{2}$ for uncorrelated distributions (for example, the distributions $p = \{1, 0\}$ and $q = \{\frac{1}{2}, \frac{1}{2}\}$), and $F = 0$ for anticorrelated distributions (for example, $p = \{1, 0\}$ and $q = \{0, 1\}$). For a QND measurement there are three relevant probability distributions: p^{in} of the signal input, p^{out} of the signal output, and p^m of the measurement. These distributions, and hence fidelities, are functions of the signal input state. The requirements (i)–(iii) demand *correlations* between these distributions as follows:

(i) The success of the measurement is quantified by the *measurement fidelity* $F_M = F(p^{\text{in}}, p^m)$, which measures the overlap between the signal input and measurement distributions. For signal eigenstates, we measure $F_M(|H\rangle_s) = P_{HH} + P_{VH} = 0.97 \pm 0.03$ and $F_M(|V\rangle_s) = P_{VV} + P_{HV} = 0.81 \pm 0.04$. For all superposition states, $|D^{\pm}\rangle_s$ and $|R^{\pm}\rangle_s$, $F_M > 0.99$.

(ii) For the measurement to be *nondemolition,* the signal output probabilities should be identical to those of the input. This is characterized by the QND fidelity $F_{\text{QND}} = F(p^{\text{in}}, p^{\text{out}})$. For all signal inputs measured (eigenstates and superpositions), $F_{\text{QND}} > 0.99$.

(iii) When the measurement outcome is i, a good QSP device gives the signal output state $|i\rangle_s$ with high probability. We denote this conditional probability $p_i^{\text{out}}|i$ and define the QSP fidelity $F_{\text{QSP}} = \sum_i p_i^m p_i^{\text{out}}|i$, which is an average fidelity between the expected and observed conditional probability distributions. For our scheme $F_{\text{QSP}} = P_{HH} + P_{VV}$. The average for the six inputs quantifies the performance as a QSP device for *any* unknown input and is 0.88 ± 0.05. This quantity is also known as the *likelihood* L [22] of measuring the signal to be H or V given the meter outcome H or V, respectively.

TABLE I. Experimental values for the joint probabilities for the signal and meter polarization P_{sm}. For the inputs $|D^-\rangle$ and $|R^-\rangle$ (not shown) results are almost identical to those for $|D^+\rangle$ and $|R^+\rangle$.

Signal input	$\|H\rangle_s$	$\|V\rangle_s$	$\|D^+\rangle$	$\|R^+\rangle$
P_{HH}	0.97(3)	0.012(3)	0.44(3)	0.46(3)
P_{HV}	0.024(3)	0.000 13(7)	0.016(3)	0.022(3)
P_{VH}	0.007(1)	0.18(1)	0.10(1)	0.104(8)
P_{VV}	0.0005(3)	0.81(4)	0.44(3)	0.41(2)

In the CV regime, $L = 0$ due to the continuous spectrum of the measurement outcome. To compare directly with CV experiments, the QSP performance could also be quantified by the *correlation function* [23] between the signal and meter: $P_{HH} + P_{VV} - P_{HV} - P_{VH}$. This correlation is also referred to as the *knowledge* K; for qubits, $K = 2L - 1$. Both K and L are useful for characterizing the weak measurements, which we now describe.

Along with performing strong measurements of polarization, our device also allows for nondestructive weak measurements. By varying the input state of the meter $|\Psi\rangle_m = \alpha|H\rangle_m + \beta|V\rangle_m$, we vary the strength of the measurement: for $\alpha = \sqrt{3}/2$ a strong measurement is realized (as above); for $\alpha = 0$ the signal and meter do not interact, and no measurement is realized; while for $0 < |\alpha| < \sqrt{3}/2$ a weak measurement is realized. We also introduce a $\frac{2}{3}$ loss in the s_V mode [Fig. 1(a)] so that the H and V components of the signal *both* experience a $\frac{2}{3}$ loss. In the case of a strong measurement this loss is not present, making the probabilities of projecting onto either eigenstate (H/V) asymmetric. In contrast, for a weak measurement, projection onto an eigenstate does not occur, and in the absence of the $\frac{2}{3}$ loss the signal state would be effectively rotated. The most interesting behavior can be seen for an equal superposition signal input, e.g., $(|H\rangle_s + |V\rangle_s)/\sqrt{2}$. The output state for an *arbitrary* meter input is then

$$|\phi_{\text{out}}\rangle = (1/2\sqrt{3})[|H\rangle_s(\sqrt{2/3}\alpha|H\rangle_m + \sqrt{2}\beta|V\rangle_m) + |V\rangle_s(\sqrt{2/3}\alpha|H\rangle_m + \sqrt{2}\beta|V\rangle_m)] + \ldots, \quad (4)$$

where again the terms not shown are failure mechanisms. We vary α by adjusting the angle θ of a HWP at the meter input: nominally $\alpha = \sin(2\theta)$.

We can characterize this weak measurement by measuring the one-qubit reduced density matrix ρ_s of the signal output [Figs. 2(a) and 2(b)]. The H and V populations do not change, regardless of the meter input state. However, for a HWP setting of $\theta = 0°$ ($\alpha \simeq 0$) we observe a coherent superposition, while for the strong measurement setting $\theta = 33°$ ($\alpha \simeq \sqrt{3}/2$), we observe an incoherent mixture as expected. In the intermediate region the signal output is partially mixed. The degree of coherence is determined by measuring the visibility V of the linear polarization fringes as shown in Fig. 2(c). These results explicitly demonstrate the *decoherence* that would appear in quantum cryptography due to an eavesdropper using weak QND measurement of polarization, as simulated in [14].

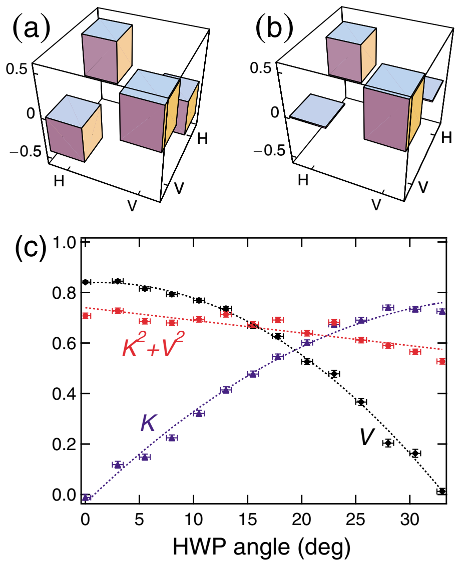

FIG. 2 (color online). Nondestructive weak measurement of a polarization qubit in an equal superposition state, demonstrating the principle of complementarity. (a) When $\theta = 0°$, the meter makes *no measurement* and the signal output state is quite pure, as seen in the real part of the reduced density matrix ρ_s: the one-qubit linear entropy $S_L = 2(1 - \mathrm{Tr}[\rho_s^2]) = 0.22$. (b) When $\theta = 33°$ the meter makes a *strong measurement* and the signal output state ρ_s is highly mixed: $S_L = 0.98$. (Note that all the imaginary components (not shown) are <0.08.) (c) Weak measurement and complementarity: Plots of K, V, and $K^2 + V^2$ for a number of different values of θ. The error bars take into account the count statistics and the input wave plate angle setting, respectively.

The fundamental principle of complementarity [17], in particular, wave-particle duality, can be tested with our general measurement in the fashion originally proposed in Ref. [16]. In the spatial interferometer of Fig. 1(b), K quantifies the degree of which-path information, and V the quantum indistinguishability. They must satisfy $V^2 + K^2 \leqslant 1$ [24]. In Fig. 2(c) we plot K, V, and $K^2 + V^2$ for a range of values of α. As K increases, V decreases. Ideally our weak QND scheme is optimal: $K^2 + V^2 = 1$ for all meter polarizations. In our experiment $K^2 + V^2 < 1$ due to nonideal mode matching. The decline of $K^2 + V^2$ with increasing K can be attributed to the increasing requirement for nonclassical (as well as classical) interference as the strength of the QND measurement is increased. In contradistinction to the nondestructive scheme presented here, a previous photonic test of complementarity relied on encoding which-path information onto a different degree of freedom of the photon [25], so that which-path information is obtained only destructively, when the photon is measured.

In summary, we have proposed, demonstrated, and characterized a nondeterministic scheme for general measurement of a flying qubit. We have introduced the first set of fidelity measures to characterize the quality of any QND measurement. Because we are able to measure these fidelities directly and prepare all input states with high fidelity, we have demonstrated the most comprehensive characterization of a QND measurement to date. When the measurement is successful we find that our device performs well against all three requirements of a QND measurement, with fidelities greater than 80% for all measures and all input states. Finally, operating in the weak regime, we have performed a nondestructive test of complementarity.

We thank J. S. Lundeen and G. J. Milburn for helpful discussions. This work was supported by the ARC and by the NSA and ARDA under ARO Contract No. DAAD 19-01-1-0651.

[1] C. M. Caves *et al.*, Rev. Mod. Phys. **52**, 341 (1980).
[2] M. F. Bocko and R. Onofrio, Rev. Mod. Phys. **68**, 755 (1996).
[3] P. Grangier, J. A. Levenson, and J.-P. Poizat, Nature (London) **396**, 537 (1998).
[4] Q. A. Turchette *et al.*, Phys. Rev. Lett. **75**, 4710 (1995).
[5] G. Nogues *et al.*, Nature (London) **400**, 239 (1999).
[6] E. Knill, R. Laflamme, and G. J. Milburn, Nature (London) **409**, 46 (2001).
[7] N. Gisin *et al.*, Rev. Mod. Phys. **74**, 145 (2002).
[8] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000), p. 91.
[9] P. Kok, H. Lee, and J. P. Dowling, Phys. Rev. A **66**, 063814 (2002).
[10] J. L. O'Brien *et al.*, Nature (London) **426**, 264 (2003).
[11] W. Dür and H. Briegel, Phys. Rev. Lett. **90**, 067901 (2003).
[12] A. Botero and B. Reznik, Phys. Rev. A **61**, 050301 (2000).
[13] M. J. Werner and G. J. Milburn, Phys. Rev. A **47**, 639 (1993).
[14] D. S. Naik *et al.*, Phys. Rev. Lett. **84**, 4733 (2000).
[15] S. Lloyd and J.-J. E. Slotine, Phys. Rev. A **65**, 012307 (2000).
[16] B. C. Sanders and G. J. Milburn, Phys. Rev. A **39**, 694 (1989).
[17] D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, NJ, 1951).
[18] K. J. Resch and A. M. Steinberg, Phys. Rev. Lett. **92**, 130402 (2004); K. J. Resch, J. S. Lundeen, and A. M. Steinberg, quant-ph/031009.
[19] S. Takeuchi, Opt. Lett. **26**, 843 (2001).
[20] C. Kurtsiefer, M. Oberparleiter, and H. Weinfurter, J. Mod. Opt. **48**, 1997 (2000).
[21] C. Kurtsiefer, M. Oberparleiter, and H. Weinfurter, Phys. Rev. A **64**, 023802 (2001).
[22] W. K. Wootters and W. H. Zurek, Phys. Rev. D **19**, 473 (1979).
[23] The correlation function is defined $C_{MS} = \langle \hat{O}_M \hat{O}_S \rangle / (\langle \hat{O}_m^2 \rangle \langle \hat{O}_s^2 \rangle)^{1/2}$, where $\hat{O}$ is the QND observable: the Stokes operator $\hat{S}1$ for our measurement; the quadrature amplitude $\hat{X}$ for CV systems.
[24] B.-G. Englert, Phys. Rev. Lett. **77**, 2154 (1996).
[25] P. D. D. Schwindt, P. G. Kwiat, and B.-G. Englert, Phys. Rev. A **60**, 4285 (1999).