Comment on "Ferrofluids as Thermal Ratchets"

In a recent experiment [1], a plastic sphere filled with a ferrofluid (FF) was suspended on a fiber aligned with the vertical z axis and placed in a horizontal magnetic field. The uniform field included the constant component $H_x = H_0$ and the transverse alternating one $H_y = H_1 f(t)$,

$$f(t) = \cos\omega t + a\sin(2\omega t + \beta). \tag{1}$$

Engel *et al.* [1] wrote, "After switching on the fields, the ferrofluid sphere immediately starts to rotate. Switching off either the static field H_0 or the modulation amplitude a the torque disappears." This was interpreted as a "thermal ratchet behavior: by rectifying thermal fluctuations...the noise driven rotation of the microscopic ferromagnetic grains is transmitted to the carrier liquid."

Such phraseology looks superfluous and screens only a true cause of the effect. As demonstrated below, under the experimental conditions of [1] *any* magnetic sample rotates because of *nonlinearity* of its *magnetization*.

Let us replace the FF sphere by a *solid* paramagnet. Far from the area of paramagnetic resonance (in [1] the field frequency $\omega/2\pi$ was merely 200 Hz), the magnetization obeys the Debye relaxation equation

$$d\mathbf{M}/dt = -(\mathbf{M} - \mathbf{M}^{\rm eq})/\tau, \tag{2}$$

where $\mathbf{M}^{\rm eq}$ denotes the local-equilibrium magnetization always directed along the field $\mathbf{H}$. The sample rotation is owing to the *magnetic torque* $\mathbf{N} = \mathbf{M} \times \mathbf{H}$ or, as follows from Eq. (2), $\mathbf{N} = \tau\mathbf{H} \times \dot{\mathbf{M}}$. One needs to find its time-averaged component $\overline{N}_z$ in the field (1). Since $\overline{\dot{g}(t)} = 0$ for any periodic function $g(t)$, we obtain

$$\overline{N}_z = \tau H_1 \overline{M_x \dot{f}(t)}. \tag{3}$$

This is the central point. Under linear magnetization law $\mathbf{M}^{\rm eq} = \chi\mathbf{H}$, one has $M_x = \chi H_0$ and "the resulting time-averaged torque vanishes identically," as noted truly in [1]. Take now a weakly nonlinear magnetization:

$$\mathbf{M}^{\rm eq} = \chi(1 - \epsilon H^2)\mathbf{H} \quad \text{till } \epsilon H^2 \ll 1. \tag{4}$$

Then, although $H_x = H_0$, $M_x^{\rm eq}$ becomes time dependent,

$$\begin{aligned} M_x^{\rm eq} &= \chi H_0[\text{const} - \epsilon H_1^2 f^2(t)] \\ &= \text{const} - \tfrac{1}{2}\epsilon\chi H_0 H_1^2[\cos 2\omega t - 2a\sin(\omega t + \beta) + R(t)], \end{aligned} \tag{5}$$

where $R(t)$ is a linear function of $\sin(n\omega t)$ and $\cos(n\omega t)$ with $n = 3, 4$. According to Eq. (2), M_x lags behind $M_x^{\rm eq}$ because of the finite relaxation time τ:

$$M_x = e^{-t/\tau} \int M_x^{\rm eq}\, e^{t/\tau} d(t/\tau). \tag{6}$$

Substituting Eqs. (1), (5), and (6) into Eq. (3) yields

$$\overline{N}_z = \frac{3}{2}\,\epsilon a \chi H_0 H_1^3 \omega^2 \tau^2 \frac{(1 + 2\omega^2\tau^2)\sin\beta + \omega\tau\cos\beta}{(1 + \omega^2\tau^2)(1 + 4\omega^2\tau^2)}. \tag{7}$$

Note that $\overline{N}_z \propto a$ is also due to nonlinearity: $M_x(f^2(t))$ and $\dot{f}(t)$ contain the same frequencies—and hence their time-averaged product (3) differs from zero—only if $a \neq 0$.

Engel *et al.* assumed FF to be Langevin's paramagnet, $\mathbf{M}^{\rm eq} = M_s \mathcal{L}(\alpha)\boldsymbol{\alpha}/\alpha$, and limited an expansion of the Langevin function $\mathcal{L}(\alpha)$ to the first two terms:

$$\begin{aligned} \mathbf{M}^{\rm eq} &\approx M_s(1 - \alpha^2/15)\boldsymbol{\alpha}/3 = \chi(1 - \alpha^2/15)\mathbf{H}, \\ \boldsymbol{\alpha} &= m\mathbf{H}/k_BT, \qquad \chi = mM_s/3k_BT. \end{aligned} \tag{8}$$

This definition of $\mathbf{M}^{\rm eq}$ coincides with Eq. (4) at $\epsilon = \frac{1}{15}(m/k_BT)^2$. Substituting the value into Eq. (7) and introducing the dimensionless fields $\alpha_x = mH_0/k_BT$, $\alpha_y = mH_1/k_BT$, we obtain

$$\overline{N}_z = \frac{M_s^2 \alpha_x \alpha_y^3 a \omega^2[(2 + \omega^2)\sin\beta + \omega\cos\beta]}{180\chi(1 + \omega^2)(4 + \omega^2)}; \tag{9}$$

here time is scaled by 2τ just as in Ref. [1]. This expression is very similar to the result of [1]

$$\overline{N}_{E,z} = \frac{M_s^2 \alpha_x \alpha_y^3 a \omega^2(2\sin\beta + \omega\cos\beta)}{90\chi(1 + \omega^2)(4 + \omega^2)^2}. \tag{10}$$

Some difference between them appears because the authors solved them somewhat differently from Eq. (2), the magnetization equation [2]. It was derived specially for FF, and so $\overline{N}_{E,z}$ fits the experimental data [1] perhaps better than $\overline{N}_z$. In principle, however, a specific form of the equation *does not matter:* Any reasonable magnetization equation should yield the same qualitative result since the torque is only due to *nonlinearity* of the magnetization. All other FF features prove to be off the point: they make no qualitative alteration in the solid-body result (9).

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