

Spontaneous Ratchet Effect in a Granular Gas

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The spontaneous clustering of a vibrofluidized granular gas is employed to generate directed transport in two different compartmentalized systems: a *granular fountain* in which the transport takes the form of convection rolls, and a *granular ratchet* with a spontaneous particle current perpendicular to the direction of energy input. In both instances, transport is not due to any system-intrinsic anisotropy, but arises as a spontaneous collective symmetry breaking effect of many interacting granular particles. The experimental and numerical results are quantitatively accounted for within a flux model.

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The conversion of random fluctuations into directed motion in a periodic system — termed ratchet effect — is of interest in a wide variety of physical, biological, and technological contexts [1]. Overall, two necessary conditions for the emergence of the effect have been identified [2], namely, (i) the absence of thermal equilibrium and (ii) the breaking of the inversion symmetry. The most common realization of the latter condition is by means of an intrinsic anisotropy in the system, e.g., a spatially periodic but asymmetric potential, or a so-called dynamical asymmetry due to a time-dependent driving force with zero mean but nonvanishing higher moments [2]. In this Letter, we are concerned with *perfectly symmetric systems* where the symmetry breaking arises spontaneously as a collective effect of many interacting particles via an ergodicity breaking nonequilibrium phase transition [3]. While a proof of principle for this so-called spontaneous ratchet effect has been provided theoretically with several different “minimal models” [2,3], we explore here for the first time a *real* system, namely, a vibrofluidized granular gas, experimentally, numerically, and analytically.

Transport in granular systems due to a ratchet effect has been previously demonstrated both for asymmetric potentials [4] and for dynamical asymmetries [5]. While in both these cases even a single granular particle moves in a preferential direction, the spontaneous ratchet effect we consider here is a genuine *collective phenomenon*. The underlying mechanism is the enhanced energy loss in already dense regions of a granular gas due to the inelastic collisions, resulting in a spontaneous separation into dense and dilute regions [6]. This clustering effect can be controlled by confining the particles to a series of compartments that are connected by small apertures (slits) and subjected to gravity [7,8]. When the granular system is fluidized by shaking it vertically, the particles cluster into the compartments which are initially, or due to random fluctuations, slightly denser, until a dynamical equilibrium is established in which the particle flux out of

a compartment is balanced by the influx coming from its neighbors.

Hence, all ingredients for a spontaneous ratchet effect are present: First, random fluctuations in the form of deterministic noise are created intrinsically by the chaotic dynamics. Second, the system is kept away from thermal equilibrium by the vertical vibrations. Third, in a perfectly symmetric, periodic compartmentalization, stable steady states with a spontaneously broken symmetry yet still respecting the periodicity seem possible, but are not at all obvious: their actual existence is the main finding of our present work.

We start with a granular gas in a container that is divided into $K = 2$ equal compartments by a wall with a slit at a certain height h (cf. Fig. 1). At vigorous shaking, this system has a unique steady state with an equal population of both compartments [7]. Upon reducing the driving, this symmetric state loses stability via a continuous phase transition, resulting in a pair of stable solutions with spontaneously broken symmetry: They are

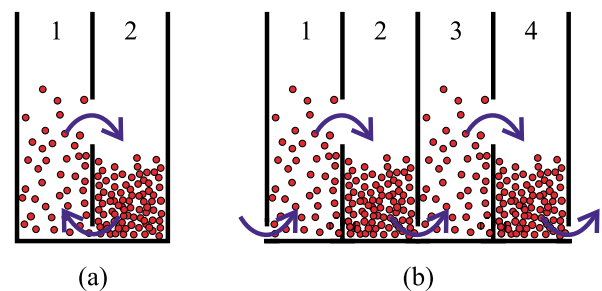


FIG. 1 (color online). (a) Schematic cross section of a granular fountain exhibiting spontaneous symmetry breaking into a “cold” and a “hot” compartment and a concomitant spontaneous circular particle flow. (b) By folding out the geometry of several adjacent fountains, and adding cyclical boundary conditions, a granular ratchet is obtained with almost unchanged populations of the respective compartments and the particle currents between them (here shown for $K = 4$ compartments).

clustered states with one diluted and one crowded compartment [7]. If we now add a sufficiently small hole at the bottom of the wall separating the compartments, the clustered states are expected to subsist [see Fig. 1(a)], resulting in a particle flow through the hole, balanced by a flow through the slit in the opposite direction. We thus expect a *granular fountain*, i.e., a spontaneous convective flow, imposed upon the system by the clustering phenomenon.

To verify these predictions, we conducted both experiments and molecular dynamics simulations. In the experiments we used $N = 407$ beads of stainless steel (radius $r = 1.18$ mm, normal restitution coefficient $\nu \approx 0.9$) in a high Perspex container with a ground surface of 41.5×25.0 mm². When the particles are at rest, this corresponds to a filling level of 2.0 layers. The box was divided into $K = 2$ equal compartments by a wall, with a horizontal slit of 25.0 mm \times 5.0 mm, starting at height $h = 25.0$ mm, and a hole of dimensions 3.0 mm \times 3.0 mm at the bottom. The entire device was subjected to vertical, sinusoidal vibrations with variable frequency f (ranging from 47 to 150 Hz) and fixed amplitude $a = 1.0$ mm. The results are given in Fig. 2 in terms of the dimensionless control parameter

$$B = 4\pi \frac{gh}{a^2 f^2} (1 - \nu^2)^2 \left(\frac{r^2 N}{\Omega K} \right)^2, \quad (1)$$

which is motivated [7] from the kinetic theory for dilute granular gases [see also Eq. (2)]. Here Ω ($= 19.7 \times 25.0$ mm²) is the surface area of each compartment and $g = 9.81$ m/s². As anticipated, *vigorous shaking* (small B) leads to a symmetric steady state, giving way to a

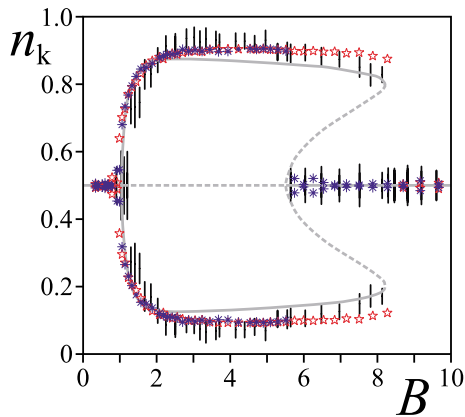


FIG. 2 (color online). Bifurcation diagram for the granular fountain. Depicted are the steady state fractions of particles n_k , $k \in \{1, 2\}$, in the two compartments from Fig. 1(a) for different values of the driving frequency f in units of $B \propto f^{-2}$ from Eq. (1). Experimental results are indicated by the black error bars; the other symbols represent molecular dynamics simulations using uniform (asterisks) and clustered (stars) initial conditions. The gray lines represent the theoretical prediction from the flux model (1)–(3) for $\lambda = 0.018$: solid lines correspond to stable configurations; dashed lines to unstable ones.

clustered state with the predicted fountain effect for *intermediate driving*. For very *weak driving* (large B) the system returns to equipartition via the hole at the bottom of the container, since the flux through the slit [cf. Fig. 1(a)] becomes negligibly small. Furthermore, a pronounced hysteresis is observed in Fig. 2 for B values between 5.5 and 8.2.

Figure 2 also contains the results of molecular dynamics *simulations* in a setup comparable to the experiment: $K = 2$ rectangular compartments with ground area 19.4×25.0 mm² and filled with $N = 400$ particles of radius $r = 1.18$ mm and normal restitution coefficient $\nu = 0.9$ [9]. While the slit at height h was as in the experiment, the hole at the bottom was chosen slightly larger (4.2×4.2 mm²) to compensate for the vanishing thickness of the apertures and the neglected dissipation from particle-wall collisions in the simulations. Experiments and simulations are seen to agree well.

These findings can be explained in the spirit of a previously established *flux model* [7] with the help of two flux functions F and G describing the number of particles which escape out of a compartment per time unit through the slit and the hole, respectively:

$$F(n_k) = An_k^2 e^{-BK^2 n_k^2}, \quad G(n_k) = A\lambda n_k^2. \quad (2)$$

Here, B is defined in (1), n_k is the fraction of particles in compartment $k \in \{1, \dots, K\}$ (hence $n_k \in [0, 1]$, $\sum_1^K n_k = 1$), and A essentially accounts for the size of the slit, $[A] = \text{s}^{-1}$. The flux $F(n_k)$ grows from zero for $n_k = 0$ to a maximum at $n_k = (BK^2)^{-1/2}$, and then decreases asymptotically towards zero as a result of the inelastic collisions between the particles. The flux through the hole, $G(n_k)$, is obtained by taking the limit $h \rightarrow 0$ of $F(n_k)$. The ratio λ of $G(n_k)$ and $F(n_k)$ in the low density limit is determined by the ratio of the surface areas of the hole and the slit, and also accounts for the anisotropy of the velocity distribution just above the bottom. Obviously, λ should be chosen considerably smaller than unity, otherwise the flux through the hole will completely overpower that through the slit, and hence the clustering effect. For our experimental setup one finds as a rough estimate $\lambda \approx 0.02$ under the assumption that the finite particle radius effectively reduces the dimensions of slit and hole by 1.8 mm and neglecting any velocity anisotropies.

In the fountain geometry ($K = 2$) from Fig. 1(a) the time evolution of the relative populations $n_1 = n_1(t)$ and $n_2 = 1 - n_1$ follow by adding up all the fluxes through slit and hole:

$$\dot{n}_1 = F(1 - n_1) + G(1 - n_1) - F(n_1) - G(n_1). \quad (3)$$

The uniform distribution $n_1 = n_2 = 1/2$ is a stationary solution for all B , but is linearly stable only if the Jacobian $-2F'(1/2) - 2G'(1/2) = -2A[(1 - B)\exp(-B) + \lambda]$ is negative. This is the case when B is either small or large. If $\lambda < e^{-2}$ there exists a B interval for which the uniform distribution is unstable, and one can show that now the only stable solutions of (3) are a symmetric pair of

clustered, fountain states. When fitting with $\lambda = 0.018$, the comparison of the predictions of the flux model with the experiment in Fig. 2 is fairly good; especially the bifurcation and the hysteresis are well reproduced.

Convection rolls somewhat similar to our present granular fountain but governed by different physical mechanisms are well known for ordinary liquids and gases out of equilibrium and also for granular systems at very high densities. For a dilute granular gas, as we consider it here, convection in a single container has been reported recently [10], but neither a compartmentalized setup nor a theoretical modeling have been addressed before.

We now proceed towards the *granular ratchet* setup in Fig. 1(b): Starting with any number of copies of the granular fountain placed next to each other and closed cyclically, we then move each hole to its adjacent wall. Within the flux model (1),(2) this amounts to replacing (3) by

$$\begin{aligned} \dot{n}_k &= G(n_{k+1}) + F(n_{k-1}) - F(n_k) - G(n_k), & \text{even } k, \\ \dot{n}_k &= F(n_{k+1}) + G(n_{k-1}) - F(n_k) - G(n_k), & \text{odd } k, \end{aligned} \quad (4)$$

with an even number of compartments K and periodic boundary conditions $n_{K+k} = n_k$. Because of (2) it follows that any periodically continued steady state solution of the original fountain geometry translates into a steady state solution of the extended granular ratchet geometry; the particle currents through the slits remain unchanged, and those through the holes are simply inverted. Less obvious is the stability of these solutions and the possible coexistence of further stable solutions.

In the simplest case of a granular ratchet consisting of just $K = 2$ periodically closed compartments, one obtains, as expected, a bifurcation diagram (not shown) which practically coincides with that in Fig. 2, and similarly for the particle currents. Turning to $K = 4$ compartments as depicted in Fig. 1(b), we have performed molecular dynamics simulations with the same dimensions of compartment, hole, and slit and the same average particle number $N/K = 200$ per compartment as in the $K = 2$ fountain case. The resulting bifurcation diagram in Fig. 3 displays as its most prominent new feature *two different types of coexisting stable states* with spontaneously broken symmetry for not too large B values: (i) *Ratchet states* with alternating dense and dilute compartments and a resulting finite directed particle flux, as quantitatively exemplified in Fig. 4: As the system evolves towards its stable steady state, both fluxes in Fig. 4 converge to the same constant mean. Its finite value is the most immediate signature of a spontaneous ratchet effect. (ii) *Fluxless clustered states* with alternating pairs of dense and dilute compartments and no resulting net particle flux; see Fig. 4. Our experimental data (not shown) on the $K = 4$ ratchet are in agreement with these findings.

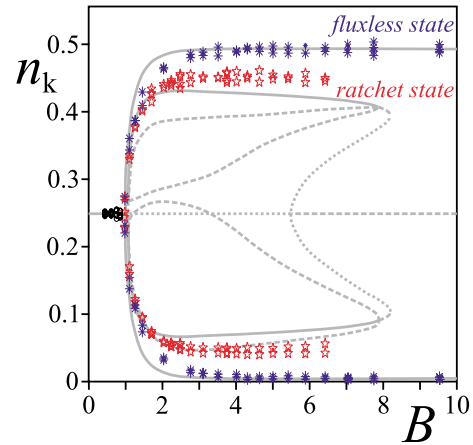


FIG. 3 (color online). Bifurcation diagram of the $K = 4$ granular ratchet. Symbols: results from molecular dynamics simulations. Gray lines: prediction from the flux model (1), (2), and (4) for $\lambda = 0.018$. Solid lines correspond to stable configurations; dashed lines to unstable ones. For further details regarding parameters and methods, see main text. For small B the uniform distribution, $n_1 = n_2 = n_3 = n_4 = 1/4$, is the unique steady state. At $B = 1$ a pair of symmetry broken stable states takes over, namely, $n_2 = n_3 \neq n_4 = n_1$, indicated by (blue) asterisks. They exhibit no net particle flow and are called *fluxless clustered states*. For slightly larger B another pair of stable states emerges, both of the form $n_1 = n_3 \neq n_2 = n_4$ [see Fig. 1(b)], indicated by (red) stars. They do carry a net flow (cf. Fig. 4) and are called *ratchet states*. For $B > 6.8$ only the fluxless stable states survive.

The intricate bifurcation diagrams for even larger K will be presented elsewhere. The main (and quite plausible) finding is that current carrying ratchet states with alternating dense and dilute compartments are always stable solutions within an entire interval of B values. Moreover, one finds an increasing number of coexisting states with spontaneously broken symmetry. These are

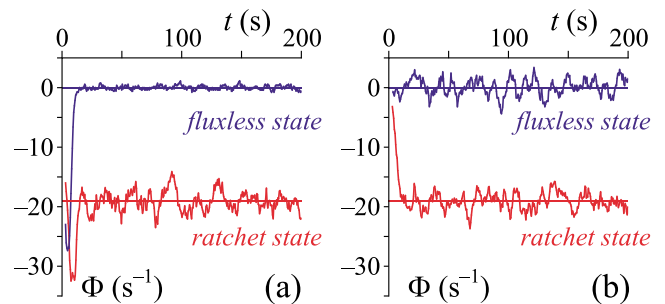


FIG. 4 (color online). Running time average (over 5 s) of the net particle flux $\Phi(t)$ for the same molecular dynamics simulations as in Fig. 3 with $B = 3.0$, measured through the rightmost slit (a), and through the rightmost hole (b) in Fig. 1(b). The upper (blue) curves in each plot correspond to the clustered state with densely populated leftmost and rightmost compartments in Fig. 1(b). The lower (red) curves belong to the ratchet state with densely populated compartments 1 and 3 in Fig. 1(b). For both states the straight line corresponds to the long-time asymptote of the particle current.

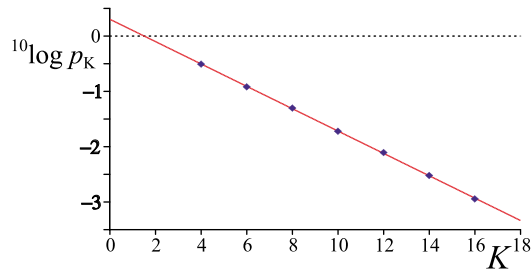


FIG. 5 (color online). Fraction p_K of ratchet solutions versus number $K(\geq 4)$ of compartments in a granular ratchet system with a constant initial particle number N/K per compartment. Symbols represent numerical solutions of the (deterministic) flux model (1), (2), and (4) for $B = 3$ and $\lambda = 0.018$, averaged over initial distributions with a superimposed randomization, mimicking fluctuations due to the finite particle number within each compartment. The straight line is a linear best fit.

long-lived hybrid states, containing both ratchetlike regions (with alternating dense and dilute compartments) and fluxless clustered regions.

Which specific steady state solution the system will settle in strongly depends on the initial condition, and at phase boundaries also on random fluctuations. An interesting example of the latter type is a uniform initial distribution. The corresponding probability p_K of ending up in a ratchet state is depicted in Fig. 5. For an intuitive explanation we view the system as $K/2$ identical “dimers,” each consisting of two compartments with a common slit at height h , and connected with each other by holes. Since the holes which connect neighboring dimers are much smaller than the slits, inside each dimer a quasisteady distribution is approached much faster than the full equilibration between all dimers. Hence, an initially uniform distribution evolves inside each dimer towards either of two clustered states with probability $q = 1/2$, independently of the other dimers. This implies that a granular ratchet system with K compartments will end in one of the two ratchet states with alternating dense and dilute compartments with probability

$$p_K = 2q^{K/2}. \quad (5)$$

This prediction is in excellent agreement with the linear best fit $p_K = 2.02(0.394)^{K/2}$ in Fig. 5. The reduced q value can be attributed to the fact that the dimers are actually not entirely independent due to the finite hole size.

In practice, a ratchet state spontaneously arises only with a small probability for large K , as far as uniform initial conditions are concerned. This situation is comparable to other systems with spontaneous symmetry breaking, e.g., a ferromagnet [11]: Just like a piece of iron picked from the shelf normally appears to be “unmagnetized” and an external magnetic field is needed to produce a magnetization on a global scale, a ratchet state only emerges with appreciable probability for large K if there is some initial bias towards this state. Without breaking

the symmetry of the setup, this could be realized (in a reproducible direction, if so desired) by applying a periodically modulated initial particle distribution or a small external force in the horizontal direction in Fig. 1(b), acting during a certain preparatory time span.

In conclusion, we have exploited the clustering effect to create spontaneous directed transport in symmetrically compartmentalized granular gases. In the granular fountain it takes the shape of a convection roll, and in the granular ratchet it appears as a current perpendicular to the vertical driving. In both cases, the directed transport arises as a collective effect of the stochastically colliding particles.

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