# Confinement versus Chiral Symmetry

Ágnes Mócsy,* Francesco Sannino,† and Kimmo Tuominen‡
*The Niels Bohr Institute & NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark*
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We construct an effective Lagrangian which illustrates why color deconfines when chiral symmetry is restored in hot gauge theories with quarks in the fundamental representation. For quarks in the adjoint representation we show that, while deconfinement and the chiral transition do not need to coincide, entanglement between them is still present. Extension to the chemical potential driven transition is discussed.

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In the absence of quarks, the SU($N$) Yang-Mills theory has a global $Z_N$ symmetry [1]. There exists a gauge invariant operator charged under $Z_N$, the Polyakov loop, which can be identified as the order parameter of the theory, and thus be used to characterize the deconfinement phase transition [2]. One can directly study this phase transition via numerical lattice simulations. Such studies have revealed that the deconfinement phase transition is second order when the number of colors is $N_c = 2$ [3], weakly though [4], but first order for $N_c = 3$ [5], and presumably first order for $N_c \geq 4$ [6].

The picture changes considerably when quarks are added to the theory. If fermions are in the fundamental and pseudoreal representations for $N_c = 3$ and $N_c = 2$, respectively, the corresponding $Z_3$ or $Z_2$ center of the group is never a good symmetry. The order parameter is the chiral condensate which characterizes the chiral phase transition. For $N_c = 3$ and two massless quark flavors at finite temperature and zero baryon density, the chiral phase transition is in the same universality class as the three-dimensional O(4) spin model [7], becoming a smooth crossover as small quark masses are accounted for [8]. For $N_c = 2$, the relevant universality class is that of O(6) both for the fundamental and adjoint representations [9]. Even if the discrete symmetry is broken, one can still construct the Polyakov loop and study the temperature dependence of its properties on the lattice. One still observes a rise of the Polyakov loop from low to high temperatures and naturally, although improperly, one speaks of deconfining phase transition [10]. For fermions in the adjoint representation, the center of the group remains a symmetry of the theory, and thus, besides the chiral condensate, also the Polyakov loop is an order parameter.

Interestingly, lattice results [10] indicate that for ordinary QCD with quarks in the fundamental representation, chiral symmetry breaking and confinement (i.e., a decrease of the Polyakov loop) occur at the same critical temperature. Lattice simulations also indicate that these two transitions do not happen simultaneously when the quarks are in the adjoint representation. Despite the attempts to explain these behaviors [11], the underlying reasons are still unknown.

In this Letter, we propose a solution to this puzzle based on the approach presented in [12,13], envisioned first in [14], concerning the transfer of critical properties from true order parameters to noncritical fields. The order parameter field is a field whose expectation value is a true order parameter, i.e., is zero in the symmetric phase and nonzero in the spontaneously broken one. The nonorder parameter (or noncritical) fields are the ones whose expectation values do not have such a behavior.

Two general features introduced in [12,13] are essential: There exists a relevant trilinear interaction between the light order parameter and the heavy nonorder parameter field, singlet under the symmetries of the order parameter field. This allows for an efficient transfer of information from the order parameter to the fields that are singlets with respect to the symmetry of the theory. As a result, the noncritical fields have infrared dominated spatial correlators. The second feature, also due to the existence of such an interaction, is that the finite expectation value of the order parameter field in the symmetry broken phase induces a variation in the expectation value for the singlet field, whose value generally is nonvanishing in the unbroken phase.

*Fundamental representation.*—Here we study the behavior of the Polyakov loop by treating it as a heavy field that is a singlet under chiral symmetry transformations. We take the underlying theory to be two colors and two flavors in the fundamental representation. The degrees of freedom in the chiral sector of the effective theory are $2N_f^2 - N_f - 1$ Goldstone fields $\pi^a$ and a scalar field $\sigma$. For $N_f = 2$, the potential is [15,16]

$$V_{\rm ch}[\sigma, \pi^a] = \frac{m^2}{2}{\rm Tr}[M^\dagger M] + \lambda_1 {\rm Tr}[M^\dagger M]^2 + \frac{\lambda_2}{4}{\rm Tr}[M^\dagger M M^\dagger M], \tag{1}$$

with $2M = \sigma + i2\sqrt{2}\pi^a X^a$, $a = 1, \ldots, 5$, and $X^a \in \mathcal{A}(\mathrm{SU}(4)) - \mathcal{A}(Sp(4))$. $X^a$ are the generators provided explicitly in Eqs. (A.5) and (A.6) of [16]. The Polyakov loop potential in the absence of the $Z_2$ symmetry is

$$V_\chi[\chi] = g_0\chi + \frac{m_\chi^2}{2}\chi^2 + \frac{g_3}{3}\chi^3 + \frac{g_4}{4}\chi^4. \tag{2}$$

  

The field $\chi$ represents the Polyakov loop itself, while $m_\chi$ is the mass above the chiral phase transition. To complete the effective theory, we introduce interaction terms allowed by the chiral symmetry

$$V_{\rm int}[\chi, \sigma, \pi^a] = (g_1\chi + g_2\chi^2)\mathrm{Tr}[M^\dagger M] = (g_1\chi + g_2\chi^2)(\sigma^2 + \pi^a\pi^a). \qquad (3)$$

In the phase with $T < T_{c\sigma}$, where chiral symmetry is spontaneously broken, $\sigma$ acquires a nonzero expectation value, which in turn induces a modification also for $\langle\chi\rangle$. The usual choice for vacuum alignment is in the $\sigma$ direction, i.e., $\langle\pi\rangle = 0$. The extremum of the linearized potential is at

$$\langle\sigma\rangle^2 \simeq -\frac{m_\sigma^2}{\lambda}, \qquad m_\sigma^2 \simeq m^2 + 2g_1\langle\chi\rangle, \qquad (4)$$

$$\langle\chi\rangle \simeq \chi_0 - \frac{g_1}{m_\chi^2}\langle\sigma\rangle^2, \qquad \chi_0 \simeq -\frac{g_0}{m_\chi^2}, \qquad (5)$$

where $\lambda = \lambda_1 + \lambda_2$. Here $m_\sigma^2$ is the full coefficient of the $\sigma^2$ term in the tree-level Lagrangian which, due to the coupling between $\chi$ and $\sigma$, also depends on $\langle\chi\rangle$. Spontaneous chiral symmetry breaking appears for $m_\sigma^2 < 0$. In this regime, the positive mass squared of the $\sigma$ is $M_\sigma^2 = 2\lambda\langle\sigma^2\rangle$. The formulas (4) and (5) hold near the phase transition where $\langle\sigma\rangle$ is small. We have ordered the couplings such that $g_0/m_\chi^3$ and $g_1/m_\chi$ are both much greater than $g_2$ and $g_3/m_\chi$. This previous ordering does not affect our general conclusions. No such ordering will be considered for quarks in the adjoint representation of the gauge group. When computing the expectation values for the relevant fields, we will keep the full potential.

Near the critical temperature, the mass of the order parameter field is assumed to possess the generic behavior $m_\sigma^2 \sim (T - T_c)^\nu$. Equation (5) shows that for $g_1 > 0$ and $g_0 < 0$ the expectation value of $\chi$ behaves oppositely to that of $\sigma$: As the chiral condensate starts to decrease towards chiral symmetry restoration, the expectation value of the Polyakov loop starts to increase, signaling the onset of deconfinement. This is illustrated in the left panel of Fig. 1. Positivity of the expectation values implies $2g_1^2 - \lambda m_\chi^2 < 0$, which also makes the extremum a minimum. At the one-loop level, one can show [13] that also $\chi_0$ acquires a temperature dependence.

When applying the analysis presented in [12,13], the general behavior of the spatial two-point correlator of the Polyakov loop can be obtained. Near the transition point, in the broken phase, the $\chi$ two-point function is dominated by the infrared divergent $\sigma$ loop. This is so, because the $\pi^a$ Goldstone fields couple only derivatively to $\chi$, and thus decouple. We find a drop in the screening mass of the Polyakov loop at the phase transition. When approaching the transition from the unbroken phase, the Goldstone fields do not decouple, but follow the $\sigma$, resulting again in the drop of the screening mass of the Polyakov loop close to the phase transition. We consider the variation

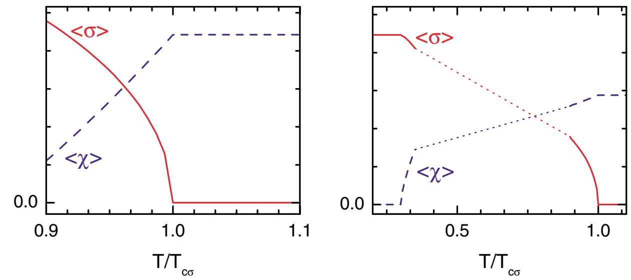


FIG. 1 (color online). Left panel: Behavior of the expectation values of the Polyakov loop and chiral condensate close to the chiral phase transition as a function of the temperature, with quarks in the fundamental representation. Right panel: Same as in left panel, for quarks in the adjoint representation and $T_{c\chi} \ll T_{c\sigma}$ (see discussion in the text).

$\Delta m_\chi^2(T) = m_\chi^2(T) - m_\chi^2$ of the $\chi$ mass near the phase transition with respect to the tree-level mass $m_\chi$. The one-loop analysis predicts

$$\Delta m_\chi^2(T) \sim -\frac{g_1^2}{|m_\sigma|} \sim t^{-(\nu/2)}, \qquad (6)$$

with $t = |T/T_c - 1|$. This result shows the strong infrared sensitivity of the two-point correlator of the field $\chi$ at the onset of chiral symmetry restoration. The detailed behavior of the screening mass of the Polyakov loop near the phase transition depends on the resummation procedure used to deal with the infrared divergences.

The large $N$ framework motivated resummation [13] leads to

$$\Delta m_\chi^2(T) = -\frac{2g_1^2(1 + N_\pi)}{8\pi m_\sigma + (1 + N_\pi)3\lambda}, \qquad T > T_{c\sigma}, \quad (7)$$

$$\Delta m_\chi^2(T) = -\frac{2g_1^2}{8\pi M_\sigma + 3\lambda}, \qquad T < T_{c\sigma}. \qquad (8)$$

This provides a qualitative improvement, since one expects that the mass of the nonorder parameter field remains finite at the phase transition. From the above equations, one finds that the screening mass of the Polyakov loop is continuous and finite at $T_{c\sigma}$, and $\Delta m_\chi^2(T_{c\sigma}) = -2g_1^2/(3\lambda)$, independent of $N_\pi$, the number of pions. Even if the mass is not critical, some associated quantities do display critical behavior. We define the slope parameters for the singlet field as

$$\mathcal{D}_\chi^{\ \pm} \equiv \lim_{T\to T_{c\sigma}^\pm} \frac{1}{\Delta m_\chi^2(T_{c\sigma})}\frac{d\Delta m_\chi^2(T)}{dT}. \qquad (9)$$

These have the critical behavior $\mathcal{D}_\chi^{\ \pm} \sim t^{\nu/2-1}$. However, as shown in [13], different critical exponents might emerge when one departs from the large $N$ limit.

This analysis is not restricted to the chiral/deconfining phase transition. The entanglement between the order parameter (the chiral condensate) and the nonorder parameter field (the Polyakov loop) is universal.

*Adjoint representation.*—As a second application, consider two color QCD with two massless Dirac quark flavors in the adjoint representation. Here the global symmetry is $SU(2N_f)$ which breaks via a bilinear quark condensate to $O(2N_f)$. The number of Goldstone bosons is $2N_f^2 + N_f - 1$. We take $N_f = 2$. There are two exact order parameter fields: the chiral $\sigma$ field and the Polyakov loop $\chi$. Since the relevant interaction term $g_1\chi\sigma^2$ is now forbidden, one might expect no efficient information transfer between them. This naive statement is partially supported by lattice data [10]. While respecting general expectations, the following analysis suggests the presence of a new and more elaborated structure which lattice data can clarify in the near future.

The chiral part of the potential is given by (1) with $2M = \sigma + i2\sqrt{2}\pi^a X^a$, $a = 1, \ldots, 9$ and $X^a \in \mathcal{A}(SU(4)) - \mathcal{A}(O(4))$. $X^a$ are the generators provided explicitly in Eqs. (A.3) and (A.5) of [16]. While the chiral part of the potential takes the same form as for the fundamental representation, there are differences when expressing the potential in terms of the component fields. These do not affect the following analysis. The $Z_2$ symmetric potential for the Polyakov loop is

$$V_\chi[\chi] = \frac{m_{0\chi}^2}{2}\chi^2 + \frac{g_4}{4}\chi^4, \tag{10}$$

and the only interaction term allowed by symmetries is

$$V_{\rm int}[\chi, \sigma, \pi] = g_2\chi^2\,\mathrm{Tr}[M^\dagger M] = g_2\chi^2(\sigma^2 + \pi^a\pi^a). \tag{11}$$

The effective Lagrangian has no knowledge of which transition, the chiral or confinement, happens first. Although lattice data already provides such information, we find it instructive to analyze separately all the possibilities.

When chiral symmetry is restored before deconfinement $T_{c\sigma} \ll T_{c\chi}$, we consider three regimes: For $T < T_{c\sigma}$, the $Z_2$ symmetry is intact, while the chiral symmetry is broken. Here $\langle\sigma\rangle^2 = -m^2/\lambda$. For $T > T_{c\chi}$, the $Z_2$ is broken, $\langle\chi\rangle^2 = -m_{0\chi}^2/g_4$, and chiral symmetry is restored. In both cases, the coefficient of the relevant quadratic term yielding condensation is not influenced by the expectation values of the other field since the latter vanishes. In the intermediate regime between the two critical temperatures both symmetries are unbroken and $\langle\sigma\rangle = \langle\chi\rangle = 0$. In this intermediate regime no trilinear interaction term between the fields is induced. For $T < T_{c\sigma}$, the interaction $\langle\sigma\rangle\sigma\chi^2$, and for $T > T_{c\chi}$ a term $\langle\chi\rangle\chi\sigma^2$ in the Lagrangian exists. These interactions are innocuous for two reasons: (i) They vanish close to their respective phase transition, and (ii) they cannot induce any infrared divergent loops [12]. Thus, for $T_{c\sigma} \ll T_{c\chi}$ the two transitions are fully separated, and neither of the two fields feels, even weakly, the transition of the other.

The situation drastically changes when $T_{c\chi} \ll T_{c\sigma}$. For $T_{c\chi} < T < T_{c\sigma}$ both symmetries are broken, and the expectation values of the two order parameter fields are linked to each other:

$$\begin{aligned}\langle\sigma\rangle^2 &= -\frac{1}{\lambda}(m^2 + 2g_2\langle\chi\rangle^2) \equiv -\frac{m_\sigma^2}{\lambda},\\ \langle\chi\rangle^2 &= -\frac{1}{g_4}(m_{0\chi}^2 + 2g_2\langle\sigma\rangle^2) \equiv -\frac{m_\chi^2}{g_4}.\end{aligned} \tag{12}$$

The coupling $g_2$ is taken to be positive. One can show that positivity of the square of the expectation values implies $\lambda g_4 - 4g_2^2 > 0$. The latter is sufficient to make the extremum of the potential a minimum. The expected behavior of $m_\chi^2 \sim (T - T_{c\chi})^{\nu_\chi}$ and $m_\sigma^2 \sim (T - T_{c\sigma})^{\nu_\sigma}$ near $T_{c\chi}$ and $T_{c\sigma}$, respectively, combined with the result of Eq. (12), yields in the neighborhood of these two transitions the qualitative situation, illustrated in the right panel of Fig. 1. On both sides of $T_{c\chi}$ the relevant interaction term $g_2\langle\sigma\rangle\sigma\chi^2$ emerges, leading to a one-loop contribution to the static two-point function of the $\sigma$ field $\propto\langle\sigma\rangle^2/m_\chi$. Near the deconfinement transition $m_\chi \to 0$, yielding an infrared sensitive screening mass for $\sigma$. Similarly, on both sides of $T_{c\sigma}$ the interaction term $\langle\chi\rangle\chi\sigma^2$ is generated, leading to the infrared sensitive contribution $\propto\langle\chi\rangle^2/m_\sigma$ to the $\chi$ two-point function. We conclude that, when $T_{c\chi} \ll T_{c\sigma}$, the two order parameter fields, *a priori* unrelated, do feel each other near the respective phase transitions. It is important to emphasize that the effective theory works only in the vicinity of the two phase transitions. Interpolation through the intermediate temperature range is shown by dotted lines in the right panel of Fig. 1. Possible structures here must be determined via first principle lattice calculations.

The infrared sensitivity leads to a drop in the screening masses of each field in the neighborhood of the transition of the other, which becomes critical, namely, of the $\sigma$ field close to $T_{c\chi}$, and of the $\chi$ field close to $T_{c\sigma}$. These drops at the transition points are expected, at the one-loop level, to behave as

$$\Delta m_\chi^2(T) \sim -\frac{(g_2\langle\chi\rangle)^2}{|m_\sigma|} \sim t^{-(\nu_\sigma/2)}, \tag{13}$$

and, similarly, we have $\Delta m_\sigma^2(T) \sim t^{-\nu_\chi/2}$ near the $Z_2$ phase transition. In the derivation of the above results, we considered the expectation values of the fields in the broken phases to be close to their asymptotic values. The resummation procedure outlined in the previous section predicts again a finite drop:

$$\Delta m_\chi^2(T_{c\sigma}) = -\frac{8g_2^2\langle\chi\rangle^2}{3\lambda}, \qquad \Delta m_\sigma^2(T_{c\chi}) = -\frac{8g_2^2\langle\sigma\rangle^2}{3g_4}. \tag{14}$$

We thus predict the existence of substructures near these transitions, when considering fermions in the adjoint representation. Searching for such hidden behaviors in lattice simulations would help to further understand the nature of phase transitions in QCD.

*Discussion.*—Via an effective Lagrangian approach, we have seen how deconfinement (i.e., a rise in the Polyakov loop) is a consequence of chiral symmetry restoration in the presence of fermions in the fundamental presentation. In nature, quarks have small but nonzero masses, which makes chiral symmetry only approximate. Nevertheless, the picture presented in this Letter still holds: Confinement is driven by the dynamics of the chiral transition. The argument can be extended even further: If quark masses were very large then chiral symmetry would be badly broken, and could not be used to characterize the phase transition. However, in such a case the $Z_2$ symmetry becomes more exact, and by reversing the roles of the protagonists in the previous discussion, we would find that the $Z_2$ breaking drives the (approximate) restoration of chiral symmetry. Which of the underlying symmetries demands and which amends can be determined directly from the critical behavior of the spatial correlators of hadrons or of the Polyakov loop [12,13].

With quarks in the adjoint representation we investigated two scenarios. In a world in which chiral symmetry is restored first, and then at some higher temperature deconfinement sets in, $T_{c\sigma} \ll T_{c\chi}$, the two phase transitions happen completely independent of each other. We know from [10], however, that $T_{c\chi} \ll T_{c\sigma}$. In this case, we have pointed to the existence of an interesting structure, which was hidden until now: There are still two distinct phase transitions, but since the fields are now entangled, the transitions are not independent. This entanglement is shown at the level of expectation values and spatial correlators of the fields. More specifically, the spatial correlator of the field which is not at its critical temperature will in any case feel the phase transition measured by the other field. Lattice simulations will play an important role in checking these predictions.

The analysis can be extended for phase transitions driven by a chemical potential. In fact, for two color QCD this is straightforward to show. When considering fermions in the pseudoreal representation, there is a phase transition from a quark-antiquark condensate to a diquark condensate [17]. We, hence, predict that, in two color QCD, when diquarks form for $\mu = m_\pi$, the Polyakov loop also feels the presence of the phase transition exactly in the same manner as it feels when considering the temperature driven phase transition. Such a situation is supported by recent lattice simulations [18]. The results presented here are not limited to describing the chiral/deconfining phase transition and can readily be used to understand phase transitions sharing similar features.

Even if the effective Lagrangian approach *à la* Ginzburg-Landau is an oversimplification, it allows, on one hand, to illuminate the relevant physics involved and, on the other hand, permits a systematic study of different effects, such as a nonzero chemical potential, quark masses, quark flavors, and axial anomaly.

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*Electronic address: agnes.mocsy@nbi.dk
†Electronic address: francesco.sannino@nbi.dk
‡Electronic address: tuominen@nordita.dk

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