Proposed Search for $a_0^0(980) - f_0(980)$ Mixing in Polarization Phenomena

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The K^+ and K^0 meson mass difference induces the mixing of the $a_0^0(980)$ and $f_0(980)$ resonances, the amplitude of which, between the K^+K^- and $K^0\bar{K}^0$ thresholds, is large in magnitude, of the order of $m_K\sqrt{m_{K^0}^2 - m_{K^+}^2} \approx \sqrt{\alpha}\, m_K^2$, and possesses the phase sharply varying by about 90°. We suggest performing the polarized target experiments on the reaction $\pi^- p \to \eta\pi^0 n$ at high energy in which the fact of the existence of $a_0^0(980) - f_0(980)$ mixing can be unambiguously and very easily established through the presence of a strong jump in the azimuthal asymmetry of the $\eta\pi^0$ S wave production cross section near the $K\bar{K}$ thresholds. The presented estimates of the polarization effect to be expected in experiment are to a great extent model independent.

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Study of the nature of light scalar resonances has become a central problem of nonperturbative QCD. The point is that the elucidation of their nature is important for understanding both the confinement physics and the chiral symmetry realization way in the low-energy region, i.e., the main consequences of QCD in the hadron world. The nontrivial nature of well established lightest scalar resonances is no longer denied practically anybody. In particular, there exist numerous evidences in favor of the four-quark ($q^2\bar{q}^2$) structure of these states; see, for example, Ref. [1] and references therein. In this Letter, we propose a new method for the investigation of the $a_0(980)$ and $f_0(980)$ resonances with the use of the polarization phenomena closely related to the $a_0^0(980) - f_0(980)$ mixing effect that carries important information on their nature, in particular, concerning their coupling to the $K\bar{K}$ channels.

The mixing between the $a_0^0(980)$ and $f_0(980)$ resonances was discovered theoretically as a threshold phenomenon in the late 1970s [2]. Recently interest in the $a_0^0(980) - f_0(980)$ mixing was renewed, and its possible manifestations in various reactions are intensively discussed in the literature [3–16]. For example, in Ref. [7] it was suggested that the data on the centrally produced $a_0^0(980)$ resonance in the reaction $pp \to p_s(\eta\pi^0)p_f$, in principle, can be interpreted in favor of the existence of $a_0^0(980) - f_0(980)$ mixing. In Ref. [10] it was noted that within the experimental errors and the model uncertainty in the $f_0(980)$ production cross section the result obtained in Ref. [7] does not contradict the predictions made in Ref. [2]. However, the experimental confirmation of such a scenario requires measuring the reaction $pp \to p_s(\eta\pi^0)p_f$ at a much higher energy to exclude a possible effect of the secondary Regge trajectories, for which the $\eta\pi^0$ production is not forbidden by G parity. This Letter presents a qualitatively new proposal concerning a search for the $a_0^0(980) - f_0(980)$ mixing effect. We propose the polarized target experiments on the reaction $\pi^- p \to \eta\pi^0 n$ at high energy in which the fact of the existence of $a_0^0(980) - f_0(980)$ mixing can be unambiguously and very easily established through the presence of a strong jump in the azimuthal asymmetry of the S-wave $\eta\pi^0$ production cross section near the $K\bar{K}$ thresholds.

Owing to parity conservation, the differential cross section of the reaction $\pi^- p \to (\eta\pi^0)_S n$ on a polarized proton target at fixed incident pion laboratory momentum, $P_{\text{lab}}^{\pi^-}$, has the form

$$d^3\sigma/dtdmd\psi = [d^2\sigma/dtdm + I(t,m)P\cos\psi]/2\pi, \quad (1)$$

where $(\eta\pi^0)_S$ denotes a $\eta\pi^0$ system with the relative orbital angular momentum $L = 0$, t is the four-momentum transfer squared from the incident π^- meson to the outgoing $\eta\pi^0$ system, m is the $\eta\pi^0$ invariant mass, ψ is the angle between the normal to the reaction plain, formed by the momenta of the π^- and $\eta\pi^0$ system, and the transverse (to the π^- beam axis) polarization of the protons, P is a degree of this polarization, $d^2\sigma/dtdm = |M_{++}|^2 + |M_{+-}|^2$ is the unpolarized differential cross section, M_{+-} and M_{++} are the s-channel helicity amplitudes with and without nucleon helicity flip, $I(t,m) = 2\,\text{Im}(M_{++}M_{+-}^*)$ describes the interference contribution responsible for the azimuthal (or spin) asymmetry of the cross section. In terms of the directly measurable quantities $I(t,m)$ and $d^2\sigma/dtdm$, one can also define the dimensionless normalized asymmetry $A(t,m) = I(t,m)/[d^2\sigma/dtdm]$, $-1 \le A(t,m) \le 1$ [17]. Here we are interested in the region of $m \approx 1$ GeV. The available data from unpolarized target experiments on the reaction $\pi^- p \to \eta\pi^0 n$, performed with $P_{\text{lab}}^{\pi^-}$ of 18.3 GeV at BNL [3,18,19], 38 GeV at IHEP [20,21], 32 GeV at IHEP [21], and 100 GeV at CERN [21], show that the $(\eta\pi^0)_S$ mass spectrum in this region of m is dominated by the production of the $a_0^0(980)$ resonance, $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$.

From the G-parity conservation it follows that at high energies and small $-t$ the amplitudes M_{+-} and M_{++} are defined by the t-channel exchanges with quantum numbers of the b_1 and ρ_2 Regge poles, respectively

[4](hereinafter they are denoted by $M^{b_1}_{+-}$ and $M^{\rho_2}_{++}$). In addition, the possibility of the π Regge pole exchange in the reaction $\pi^- p \to (\eta\pi^0)_S n$ arises by virtue of the process $\pi^- p \to f_0(908)n \to a_0^0(980)n \to (\eta\pi^0)_S n$ stipulated by the G-parity violating $a_0^0(980) - f_0(980)$ mixing [2,4,22]. As is well known, the amplitude of the π exchange is large in the low $-t$ region. Moreover, the modulus and the phase of the $a_0^0(980) - f_0(980)$ transition amplitude both change dramatically as functions of m near the $K\bar{K}$ thresholds. As we shall see, all of these features lead in the reaction $\pi^- p \to (\eta\pi^0)_S n$ to rather impressive consequences, which can be easily revealed in polarized target experiments because of measuring the interference between the ρ_2 and π exchange amplitudes.

Let us now turn to the quantitative estimates of the expected polarization effect. As for the G-parity violating π exchange amplitude M^{π}_{+-}, essentially all is known, including its absolute normalization [2,4,23]. This amplitude can be written as follows:

$$M^{\pi}_{+-} = e^{-i\pi\alpha_\pi(t)/2}\frac{\sqrt{-t}}{t-m_\pi^2}\, e^{\Lambda_\pi(t-m_\pi^2)/2} a_\pi e^{i\delta_B(m)} G_{a_0 f_0}(m) \times [2m^2\Gamma_{a_0\eta\pi^0}(m)/\pi]^{1/2}, \quad (2)$$

where $\alpha_\pi(t) = \alpha_\pi(0) + \alpha'_\pi t \approx 0.8(t - m_\pi^2)/\mathrm{GeV}^2$ is the π Regge pole trajectory, $a_\pi = g_{\pi NN} g_{f_0\pi^+\pi^-}/\sqrt{8\pi}s$, $g^2_{\pi NN}/4\pi \approx 14.3$, $g_{f_0\pi^+\pi^-}$ is the $f_0(980)$ coupling constant to the $\pi^+\pi^-$ channel, $s \approx 2m_p P^{\pi^-}_{\mathrm{lab}}$, $\Lambda_\pi/2 = \Lambda^0_\pi/2 + \alpha'_\pi \ln(s/s_0)$ is the residue slope, $s_0 = 1\ \mathrm{GeV}^2$, $\delta_B(m)$ is a smooth and large phase (of about 90° for $m \approx 1$ GeV) of the elastic background accompanying the $f_0(980)$ resonance in the S-wave reaction $\pi\pi \to \pi\pi$ in the channel with isospin $I = 0$ [2,23], $G_{a_0 f_0}(m) = \Pi_{a_0 f_0}(m)/[D_{a_0}(m)D_{f_0}(m) - \Pi^2_{a_0 f_0}(m)]$, $\Pi_{a_0 f_0}(m)$ is the nondiagonal matrix element of the polarization operator describing the $a_0^0(980) - f_0(980)$ transition amplitude [2], $1/D_r(m)$ is the propagator of an unmixed resonance r with a mass m_r, $D_r(m) = m_r^2 - m^2 + \sum_{ab}[\mathrm{Re}\Pi^{ab}_r(m_{f_0}) - \Pi^{ab}_r(m)]$, $r = [a_0(980), f_0(980)]$, $ab = (\eta\pi^0, K^+K^-, K^0\bar{K}^0)$ for $r = a_0(980)$, and $ab = (\pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0\bar{K}^0)$ for $r = f_0(980)$, $\Pi^{ab}_r(m)$ is the diagonal matrix element of the polarization operator for the resonance r corresponding to the ab intermediate state contribution [23], $\Gamma_{rab}(m) = \mathrm{Im}[\Pi^{ab}_r(m)]/m = g^2_{rab}\,\rho_{ab}(m)/16\pi m$ is the width of the $r \to ab$ decay, g_{rab} is the coupling constant of r to the ab channel, $\rho_{ab}(m) = [(m^2 - m_+^2)(m^2 - m_-^2)]^{1/2}/m^2$, and $m_\pm = m_a \pm m_b$. The $a_0^0(980) - f_0(980)$ transition amplitude $\Pi_{a_0 f_0}(m)$ must be determined to a considerable extent by the K^+K^- and $K^0\bar{K}^0$ intermediate states because of the proximity of the $a_0^0(980)$ and $f_0(980)$ resonances to the $K\bar{K}$ thresholds and their strong coupling to the $K\bar{K}$ channels. The sum of the one-loop diagrams $f_0(980) \to K^+K^- \to a_0^0(980)$ and $f_0(980) \to K^0\bar{K}^0 \to a_0^0(980)$, with isotopic symmetry for coupling constants, gives [2]

$$\Pi_{a_0 f_0}(m) = \frac{g_{a_0 K^+K^-} g_{f_0 K^+K^-}}{16\pi}\left[i[\rho_{K^+K^-}(m) - \rho_{K^0\bar{K}^0}(m)] - \frac{\rho_{K^+K^-}(m)}{\pi}\ln\frac{1+\rho_{K^+K^-}(m)}{1-\rho_{K^+K^-}(m)} + \frac{\rho_{K^0\bar{K}^0}(m)}{\pi}\ln\frac{1+\rho_{K^0\bar{K}^0}(m)}{1-\rho_{K^0\bar{K}^0}(m)}\right], \quad (3)$$

where $m \geq 2m_{K^0}$; in the region $0 \leq m \leq 2m_K$, $\rho_{K\bar{K}}(m)$ should be replaced by $i|\rho_{K\bar{K}}(m)|$. The "resonancelike" behavior of the modulus and the phase of $\Pi_{a_0 f_0}(m)$ is clearly illustrated in Figs. 1(a) and 1(b). Note that in the 8-MeV-wide region between the K^+K^- and $K^0\bar{K}^0$ thresholds $|\Pi_{a_0 f_0}(m)| \approx |g_{a_0 K^+K^-} g_{f_0 K^+K^-}/16\pi| \times [(m^2_{K^0} - m^2_{K^+})/m^2_{K^0}]^{1/2} \approx 0.1265 |g_{a_0 K^+K^-} g_{f_0 K^+K^-}/16\pi|$, that is, of the order of $m_K\sqrt{m^2_{K^0} - m^2_{K^+}} \approx \sqrt{\alpha}\, m_K^2$ [2]. From Eqs. (2) and (3) it follows also that the contribution of M^π_{+-} to $d^2\sigma/dtdm$, in this mass region, is controlled mainly by the production of the ratios $R_1 = g^2_{f_0 K^+K^-}/g^2_{f_0\pi^+\pi^-}$ and $R_2 = g^2_{a_0 K^+K^-}/g^2_{a_0\eta\pi^0}$, i.e., $|M^\pi_{+-}|^2 \propto \sigma(\pi^+\pi^- \to \eta\pi^0) \propto R_1 R_2$.

Within the Regge pole model the amplitudes $M^{b_1}_{+-}$ and $M^{\rho_2}_{++}$ at fixed $P^{\pi^-}_{\mathrm{lab}}$ can be written in the following form:

$$M^{b_1}_{+-} = ie^{-i\pi\alpha_{b_1}(t)/2}\sqrt{-t}\, e^{\Lambda_{b_1}t/2}(s/s_0)^{\alpha_{b_1}(0)} a_{b_1} G_{a_0}(m) \times [2m^2\Gamma_{a_0\eta\pi^0}(m)/\pi]^{1/2}, \quad (4)$$

$$M^{\rho_2}_{++} = e^{-i\pi\alpha_{\rho_2}(t)/2} e^{\Lambda_{\rho_2}t/2}(s/s_0)^{\alpha_{\rho_2}(0)} a_{\rho_2} G_{a_0}(m) \times [2m^2\Gamma_{a_0\eta\pi^0}(m)/\pi]^{1/2}, \quad (5)$$

where $\alpha_j(t) = \alpha_j(0) + \alpha'_j t$, a_j, and $\Lambda_j/2 = \Lambda^0_j/2 + \alpha'_j \ln(s/s_0)$ are the trajectory, residue, and slope of the jth Regge pole [one can accept tentatively $\alpha_{b_1}(t) \approx -0.21 + 0.8t/\mathrm{GeV}^2$ and $\alpha_{\rho_2}(t) \approx -0.31 + 0.8t/\mathrm{GeV}^2$], $G_{a_0}(m) = D_{f_0}(m)/[D_{a_0}(m)D_{f_0}(m) - \Pi^2_{a_0 f_0}(m)]$ is the propagator of the mixed $a_0^0(980)$ resonance [2]. The real situation is rather interesting. The available data from BNL [3], IHEP [20,21], and CERN [21] show that, in general, the amplitude $M^{b_1}_{+-}$ is not required at all to describe the t distributions (dN/dt) of the $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S\, n$ reaction events in the $a_0^0(890)$ mass region. All the data for $0 \leq -t \leq (0.6$–$0.8)\ \mathrm{GeV}^2$ are excellently approximated by a simplest exponential form $C\exp(\Lambda t)$ [4,20,21] corresponding to the amplitude $M^{\rho_2}_{++}$ nonvanishing for $t \to 0$ [4]. For example, the fit to the normalized BNL data [3,24] for the differential cross section $d\sigma/dt$ of the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$, shown in Fig. 1(c) by the solid curve, gives $\chi^2/n.d.f. = 15.75/22$ and $d\sigma/dt = [(945.8 \pm 46.3)\ \mathrm{nb/GeV}^2]\exp[t(4.729 \pm 0.217)/\mathrm{GeV}^2]$. That is why we consider first of all the case with the ρ_2 and π exchanges only.

In Fig. 1(c) the dashed curve shows the differential cross section due to the π exchange,

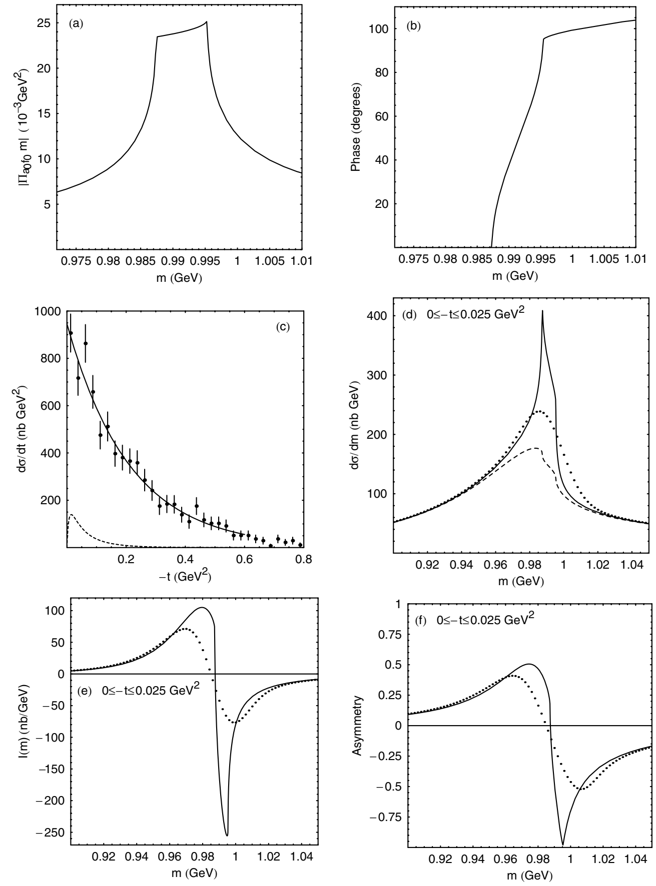

FIG. 1. (a) The modulus of the $a_0^0(980) - f_0(980)$ transition amplitude; see Eq. (3). (b) The phase of the $a_0^0(980) - f_0(980)$ transition amplitude. (c) The experimental points are the normalized BNL data for $d\sigma/dt$ of the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ [3,24]; the solid curve shows the fit to the data in the ρ_2 exchange model, and the dashed curve shows $d\sigma^\pi/dt$ for the process $\pi^- p \to f_0(980)n \to a_0^0(980)n \to (\eta\pi^0)_S n$ due to the π exchange mechanism only at $P_{\rm lab}^{\pi^-} = 18.3$ GeV. (d),(e),(f) The manifestation of the $a_0^0(980) - f_0(980)$ mixing in the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ on a polarized target at $P_{\rm lab}^{\pi^-} = 18.3$ GeV in the ρ_2 and π exchange model. The solid curves in (d),(e),(f) show $d\sigma/dm$, $I(m)$, for the region $0 \leq -t \leq 0.025$ GeV^2, and the corresponding asymmetry $A(0 \leq -t \leq 0.025\ \text{GeV}^2, m)$ [17], respectively; the common sign of the $I(m)$ and asymmetry was chosen arbitrarily. The dashed curve in (d) shows the ρ_2 exchange contribution to $d\sigma/dm$. The dotted curves in (d),(e),(f) show $d\sigma/dm$, $I(m)$, smoothed with the Gaussian mass distribution with the dispersion of 10 MeV, and the corresponding asymmetry, respectively.

$d\sigma^\pi/dt = \int |M_{+-}^\pi|^2 dm$, corresponding to the region of integration over m from 0.8 to 1.2 GeV at $P_{\rm lab}^{\pi^-} = 18.3$ GeV and $\Lambda_\pi/2 \approx 4.5$ GeV^{-2} [25,26]. When constructing this curve for $d\sigma^\pi/dt$, the curve for $|\Pi_{a_0 f_0}(m)|$ in Fig. 1(a), as well as the curves in Figs. 1(d)–1(f) illustrating the expected polarization effect, we used the following tentative values of the $f_0(980)$ and $a_0(980)$ resonance parameters: $m_{f_0} \approx 0.980$ GeV, $g_{f_0\pi^+\pi^-}^2/16\pi \approx \frac{2}{3}\, 0.1$ GeV^2, $g_{f_0 K^+K^-}^2/16\pi \approx \frac{1}{2} 0.4$ GeV^2, $\delta_B(m) \approx 35.5° + 47° m/$GeV, $m_{a_0} \approx 0.9847$ GeV, $g_{a_0 K^+K^-}^2/16\pi \approx g_{f_0 K^+K^-}^2/16\pi \approx \frac{1}{2} 0.4$ GeV^2, and $g_{a_0\eta\pi^0}^2/16\pi \approx 0.25$ GeV^2, in addition, see Refs. [2,23,27–31]. Note that a strong variation (by about 90°) of the phase of the amplitude $\Pi_{a_0 f_0}(m)$ between the K^+K^- and $K^0\bar{K}^0$ thresholds [see Fig. 1(b) and Eq. (3)], being crucial for polarization phenomena, is independent of the $f_0(980)$ and $a_0^0(980)$ resonance parameters. On integrating $d\sigma^\pi/dt$ over t, we obtain $\sigma^\pi \approx 10.9$ nb, which makes up about 5.5% of the total $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ reaction cross section, which is ≈ 200 nb at 18.3 GeV [4,24]. Let us emphasize that the indicated value of σ^π should be considered as its rather reliable lower bound [2,4]. At the maximum, located near $t \approx -0.0149$ GeV^2, $d\sigma^\pi/dt \approx 139$ nb/ GeV^2, which accounts for approximately 14.7% of $(d\sigma/dt)|_{t\approx 0}$, see Fig. 1(c). However, the main point is that the whole value of $d\sigma^\pi/dt$ at given t, in fact, comes from the narrow region of m near the $K\bar{K}$ thresholds, see Fig. 1(a), whereas the values of the total differential cross section $d\sigma/dt$ are assembled over the m region which is at least by an order of magnitude wider. Thus, at low $-t$ and m near the $K\bar{K}$ thresholds, the π exchange contribution can be quite comparable with that of the G-parity conserving ρ_2 exchange. In Figs. 1(d)–1(f) are shown $d\sigma/dm = \int [|M_{++}^{\rho_2}|^2 + |M_{+-}^\pi|^2] dt$, $d\sigma^{\rho_2}/dm = \int |M_{++}^{\rho_2}|^2 dt$, $I(m) = \int I(t,m) dt = \int 2\,\mathrm{Im}[M_{++}^{\rho_2}(M_{+-}^\pi)^*] dt$, pertaining to the $-t$ region from 0 to 0.025 GeV^2 at $P_{\rm lab}^{\pi^-} = 18.3$ GeV, and the corresponding asymmetry $A(0 \leq -t \leq 0.025\ \text{GeV}^2, m)$. In so doing, the parameters of the ρ_2 exchange, which we substitute in Eq. (5), correspond to the above-mentioned fit to the BNL data; see Fig. 1(c). Notice that the $I(m)$ and asymmetry are determined only up to the sign because the relative sign of the ρ_2 and π exchanges is unknown. Figures 1(d)–1(f) show that the polarization effect caused by the interference between the amplitudes $M_{++}^{\rho_2}$ and M_{+-}^π is highly significant. A natural measure of the effect is the magnitude of a distinctive jump of the asymmetry, which takes place in the m region from 0.965 to 1.01 GeV. As is seen from Fig. 1(f) the corresponding difference between the maximal and minimal values of the asymmetry smoothed at the expense of the finite $\eta\pi^0$ mass resolution turns out to be approximately equal to 0.95 in this mass region, see the dotted curve in Fig. 1(f) and the figure caption. Note that any noticeable variation of the interference pattern does not arise if one refits the BNL data in Fig. 1(c) by adding the π exchange contribution, indicated in the same figure, to the ρ_2 exchange one. To demonstrate the polarization effect, we have restricted ourselves to a single interval of $-t$ from 0 to 0.025 GeV^2 only from brevity considerations. We constructed the figures, analogous to Figs. 1(d)–1(f), for the intervals $0 \leq -t \leq 0.05, 0.1, 0.2$ GeV^2 and made sure that the relative magnitude of the polarization effect is practically unchanged. Furthermore, we examined the effects of the b_1 exchange contribution in detail. We enabled the b_1

exchange contribution to reach 40% of the total cross section. But even so, the influence of the b_1 exchange is inessential in the low $-t$ region. The resulting conclusion is that in this case the smoothed asymmetry pertaining to any interval of $0 \leq -t \leq 0.025, \ldots, 0.1$ GeV^2 must also undergo a jump of order one in the region $0.965 \leq m \leq 1.01$ GeV owing to the π exchange admixture.

Finally, we emphasize that observing the asymmetry jump does not require at all any very high-quality $\eta\pi^0$ mass resolution that would be absolutely necessary to recognize the $a_0^0(980) - f_0(980)$ mixing manifestation in the $\eta\pi^0$ mass spectrum in the unpolarized experiment.

Currently, experimental investigations utilizing the polarized beams and targets are on the rise. Therefore, this proposal seems to be quite opportune. The indicated polarization effect can be investigated at any high energy, for example, in the range from 8 to 100 GeV because of nearness of the π, ρ_2, and b_1 Regge trajectories. The relevant experiments on the reaction $\pi^- p \to \eta\pi^0 n$ on a polarized proton target, in principle, can be realized at KEK, BNL, IHEP, CERN (COMPASS), FNAL, ITEP, and Institut für Kernphysik in Jülich. Discovery of the $a_0^0(980) - f_0(980)$ mixing would open one more interesting page in investigation of the nature of the puzzling $a_0^0(980)$ and $f_0(980)$ states. Of course, the general idea using polarization phenomena as an effective tool for the observation of the $a_0^0(980) - f_0(980)$ mixing connected with the great variation (by about 90°) of the phase of the $a_0^0(980) - f_0(980)$ mixing amplitude in the narrow energy region (8 MeV) between the K^+K^- and $K^0\bar{K}^0$ thresholds is also applicable to other reactions. A more extended discussion of the questions touched on here will be presented elsewhere.

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[24] The experimental points in Fig. 1(c) correspond to the BNL data for dN/dt at $P_{\rm lab}^{\pi^-} = 18.3$ GeV [3] normalized on the $a_2^0(1320)$ formation cross section in the reaction $\pi^- p \to a_2^0(1320)n$ in such a way as it was done in Ref. [4]. The total cross section σ for the reaction $\pi^- p \to a_0^0(980)n \to (\eta\pi^0)_S n$ at 18.3 GeV is approximately equal to 200 nb [4]. This number refers to the m region from 0.8 to 1.2 GeV and to the whole region of $t \leq 0$. Note that the above value of σ is in close agreement with the estimate presented in Ref. [20].
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