Skewness as a Test of the Equivalence Principle

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The skewness of the large-scale distribution of matter has long been known to be a probe of gravitational clustering. Here we show that the skewness is also a probe of violation of the equivalence principle between dark matter and baryons. The predicted level of violation can be tested with the forthcoming data from the Sloan Digital Sky Survey.

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The normalized third order moment of the galaxy distribution, or skewness, is defined as

$$S_3 = \frac{\langle \delta(x)^3 \rangle}{(\langle \delta(x)^2 \rangle)^2}, \tag{1}$$

where $\delta(x)$ is the density contrast at the point x. Its value at large (weakly nonlinear) scales can be calculated assuming that structure forms only via gravitational instability: In a flat universe dominated by matter with Gaussian initial conditions, the well-known result is [1] $S_3 = 34/7$. If the density contrast is smoothed through a window function of size R, the skewness becomes [2,3]

$$\hat{S}_3 = S_3 + \frac{d \log \sigma^2(R)}{d \log R}, \tag{2}$$

where σ^2 is the variance of the density field smoothed through the same window function. In a series of papers it has been shown that S_3 remains extremely close to $34/7$ in dark energy models [4,5], in curved spaces [6], in brane-induced gravity [7], and in Brans-Dicke models [8], with deviations that hardly exceed 1% in the observationally acceptable range of cosmological parameters. These results have shown that S_3 can be considered one of the best probes of the gravitational instability picture at large scales [9]. Only scenarios with radically different features predict values of S_3 that deviate sensitively from the standard results: non-Gaussian initial conditions [10], cosmic strings [11], Cardassian cosmologies [7,12], and modified gravity models based on Birkhoff's law [13].

Since the skewness is such a good test of gravity, it seems interesting to ask whether it is also a good test of the universality of gravity, that is of the equivalence principle. In this Letter, we focus on the class of violations of the equivalence principle in which the violating force is mediated by a scalar field. In other words, we investigate the effects on S_3 of a scalar field coupled to matter in a species-dependent way. These models, first studied in [14], have been revived in the context of coupled dark energy (CDE), in which the same scalar field couples to matter and drives the accelerated expansion [15,16].

Let us recapitulate the calculation of the skewness as detailed in [2] and recently reviewed in [9]. We start by writing down the Newtonian equations for a pressureless fluid with density ρ, density contrast δ, and dimensionless peculiar velocity $v_i = v_{\mathrm{pec},i}/\mathcal{H}$, where $\mathcal{H} = aH$ is the conformal time Hubble function and a is the scale factor. Defining the gravitational potential Φ and the auxiliary variable $\Delta = \Phi/(4\pi\rho a^2)$, the Newtonian equations are

$$\delta' + \nabla_i (1 + \delta) v_i = 0, \tag{3}$$

$$v_i' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right) v_i + v_j \nabla_j v_i = -\frac{3}{2} \Omega_m \nabla_i \Delta, \tag{4}$$

and the Poisson equation is $\nabla_i \nabla_i \Delta = \delta$, where ∇_i derivates with respect to comoving coordinates and the prime denotes derivation with respect to $\alpha = \log a$. These equations have to be complemented by the Friedmann equation for $\mathcal{H}'$ and by the matter conservation equations.

We generalize now the equations introducing a scalar coupling to dark matter. Such a coupling is realized in any theory which admits in the Lagrangian a Brans-Dicke term of the form $f(\tilde{\phi}, \tilde{R})$; the low-energy limit of superstring theory is the most interesting example [17]. Upon a conformal transformation, this theory can be written as Einsteinian gravity in which matter (subscript c) and scalar field (ϕ) interact through an exchange term in their conservation equations (see, e.g., [15,18,19]):

$$\begin{aligned} T^{\mu}_{(c)\nu;\mu} &= -\sqrt{2/3}\kappa^2 \beta(\phi) T_{(c)} \phi_{;\nu}, \\ T^{\mu}_{(\phi)\nu;\mu} &= \sqrt{2/3}\kappa^2 \beta(\phi) T_{(c)} \phi_{;\nu}, \end{aligned} \tag{5}$$

where $\kappa^2 = 8\pi G$ and the dimensionless coupling $\beta(\phi)$ depends on the function $f(\tilde{\phi}, \tilde{R})$. The coupling introduces two distinct effects on the Newtonian equations: First, due to the interaction an additional force appears as a source; second, the matter energy density is no longer conserved. The first effect implies that matter feels an extra force due to its interaction with the scalar field that will add to the right-hand side in Eq. (4). We can write this term in all generality as $-\frac{3}{2}\Omega_m \eta(a, r) \nabla_i \Delta$, where $\eta(a, r)$ is a function that in general will depend on time and distance. For a potential $V(\phi)$ the field effective mass is $m^2 = d^2V/d\phi^2$ and the interaction scale is $\lambda = 1/m$. Then the new force introduces a Yukawa correction in the gravitational potential which becomes $\eta(r)/r$, where $\eta(r) = 1 + (4/3)\beta^2 e^{-r/\lambda}$. Here, however, we will assume for simplicity that λ is infinite, or at least much larger than the observed scales: On one hand, a very small λ would be unobservable, since the interaction would be effectively damped; on the other, if we interpret the scalar field as dark energy then its interaction scale λ is expected to be of the order of the Hubble size. It is possible, however, to generalize the calculations to a finite λ.

The second effect arises because of the nonconservation of the matter energy density. Equation (5) implies in fact in a homogeneous and isotropic metric an equation of the form

$$\rho' + 3\rho = -\sqrt{2/3}\kappa^2\beta(\phi)\phi'\rho, \tag{6}$$

whose solution $\rho = n_0 m_0 a^{-3} e^{-\sqrt{2/3}\kappa^2\int\beta(\phi)\,d\phi}$ can be interpreted as a varying dark matter mass, $\rho = n_0 a^{-3} m(\phi)$ with $m(\phi) = m_0 e^{-\sqrt{2/3}\kappa^2\int\beta(\phi)\,d\phi}$, and n_0 is the numerical density of particles at present. This time-dependent mass introduces an extra friction term in the Euler equations. Therefore, the scalar-Newtonian Euler equation for a coupled fluid can be written in the form

$$\mathbf{v}_c' + \tfrac{1}{2}F_c(\alpha)\mathbf{v}_c + (\mathbf{v}_c\cdot\vec{\nabla})\mathbf{v}_c = -\tfrac{3}{2}S_c(\alpha)\vec{\nabla}\Delta_c, \tag{7}$$

where the two functions, the friction $F_c(\alpha)$ and the source $S_c(\alpha)$, are in general time dependent. In [19], we have shown that the full relativistic perturbation treatment of CDE reduces to Eqs. (3) and (7) in the Newtonian limit, with

$$F_c(\alpha) = 2\left[1 + \frac{\mathcal{H}'}{\mathcal{H}} - 2\beta\frac{\phi'}{\sqrt{6}}\right],$$

$$S_c(\alpha) = \Omega_c\left(1 + \frac{4}{3}\beta^2\right),$$

where the β terms in F_c, S_c quantify the two effects due to the scalar interaction. Let us stress again that the function β is in general field dependent.

The upper bounds on a scalar interaction with baryons (subscript b) are very strong, of the order of $\beta_b < 0.01$ [20]: In the following, we assume that the interaction to baryons is effectively zero but will generalize to $\beta_b \neq 0$ at the end (in Ref. [21] it has been proposed a model in which such constraints can be escaped but only for suitably chosen potentials). The bounds on a coupling to dark matter (subscript c) are, however, much weaker. In [22] astrophysical observations were employed to derive $\beta_c < 1.5$ roughly; N-body simulations [23] have shown that the dark matter halo profile depends sensitively on β_c in a class of dark energy models but, due to the controversial status of the observations, it is difficult to derive firm upper limits. Finally, in [24], we found that cosmic microwave background (CMB) requires $\beta_c < 0.13$; however, this result assumes that the coupling remains constant throughout the universe lifetime and it is actually most sensitive to the value of β_c at early times. Moreover, the limits obviously depend on the assumed priors on the cosmological parameters, especially on the Hubble constant. Therefore, there are no strong upper bounds to the *present* value of a scalar field coupling to dark matter; if β_c varies with time, then even a value of order unity at present is not definitively excluded. We assume therefore the coupling to dark matter as a free parameter and drop the subscript c in β_c. Since the baryons are practically uncoupled, the scalar interaction violates the equivalence principle. The value of β is therefore also a measure of the equivalence principle violation.

For an uncoupled and subdominant (i.e., $\Omega_b \ll \Omega_c$) component such as the baryons, Eq. (7) becomes

$$\mathbf{v}_b' + \tfrac{1}{2}F_b(\alpha)\mathbf{v}_b + (\mathbf{v}_b\cdot\vec{\nabla})\mathbf{v}_b = -\tfrac{3}{2}S_b(\alpha)\vec{\nabla}\Delta_c, \tag{8}$$

where $F_b = 2 + 2\mathcal{H}'/\mathcal{H}$, $S_b = \Omega_c + \Omega_b \approx \Omega_c$. In the standard pure matter case $\mathcal{H}'/\mathcal{H} = -1/2$ and $F_{c,b} = S_{c,b} = 1$, while in a flat universe with a scalar field component with equation of state $w_\phi = p_\phi/\rho_\phi$, one has $\mathcal{H}'/\mathcal{H} = -[1 + 3w_\phi(1-\Omega_m)]/2$ (here and in the following $\Omega_m = \Omega_c + \Omega_b$).

Following the notation of Refs. [2,5], we expand the scalar-Newtonian equations in a perturbation series, $\delta = \sum_i \delta^{(i)}$, and $\Delta = \sum_i \Delta^{(i)}$. For each component we define the growth function $D_1(\alpha)$, $\delta^{(1)} = D_1(\alpha)\delta_0^{(1)}$, where $\delta_0^{(1)}$ is the density contrast at the initial time (assumed Gaussian distributed) and the growth exponent $m(\alpha) = D_1{}'/D_1$. At first order, we derive therefore the equations

$$\delta_b^{(1)\prime\prime} + \frac{F_b}{2}\delta_b^{(1)\prime} - \frac{3}{2}S_b\delta_c^{(1)} = 0, \tag{10}$$

$$\delta_c^{(1)\prime\prime} + \frac{F_c}{2}\delta_c^{(1)\prime} - \frac{3}{2}S_c\delta_c^{(1)} = 0. \tag{11}$$

Asymptotically, the dark matter drives the evolution of the baryons, so that the two components grow with the same exponent $m(\alpha)$ but with a biased amplitude, $b = \delta_b^{(1)}/\delta_c^{(1)}$. Subtracting the two equations, we see that $b = S_b/[S_c + m(F_b - F_c)]$ [19]. In the limit of small ϕ', which is a typical occurrence for a slowly rolling dark energy field, $F_c = F_b$ and the (anti)bias is simply

$$b = (1 + 4\beta^2/3)^{-1}, \tag{12}$$

constant in time and space. In the same limit, it appears that both the background evolution and the perturbation equations depend on β^2 so that the sign of β is irrelevant. It is remarkable that in the opposite limit in which the field kinetic energy is much larger than the potential energy (for instance, in the original Brans-Dicke model in which $V = 0$) so that the field does not drive the acceleration, it turns out that $\phi' \propto \beta$ [24] and the product $\beta\phi'$ in F_c is proportional to β^2. Then even in this case the sign of β does not matter. In the following, we put $\beta > 0$.

Here and below, we will assume for the numerical integrations an inverse power law potential $V \sim \phi^{-n}$.

The potential appears only in the background equations and, indirectly, in the assumption $\lambda \to \infty$. For this potential, the present equation of state is approximated by $w_{\phi 0} = -2/(n+2)$ [25] during the tracking regime (which may or may not extend to the present epoch; in the latter case $w_{\phi 0} \to -1$). Integrating numerically Eqs. (10) and (11), we find a fit for m:

$$m \approx \Omega_m^{0.56(1-1.73\beta^2)}, \tag{13}$$

almost independent of n [in the range $n \in (0, 2)$].

We proceed now to second order. We define for each component b, c the second-order Fourier amplitude $\delta_{b,c}^{(2)}(k, \alpha) = D_{2b,c}(\alpha)\delta_{b,c}^{(2)}(k)$ and, following the standard technique of Fourier convolution (see, e.g., [2,5,9]), we obtain for $D_{2b,c}$ the equations

$$D_{2b}'' + \frac{F_b}{2}D_{2b}' - \frac{3}{2}S_b D_{2c} = \left(\frac{3}{2}S_b b + \frac{4}{3}m^2 b^2\right)D_1^2, \tag{14}$$

$$D_{2c}'' + \frac{F_c}{2}D_{2c}' - \frac{3}{2}S_c D_{2c} = \left(\frac{3}{2}S_c + \frac{4}{3}m^2\right)D_1^2, \tag{15}$$

with the initial conditions $D_{2b,c}(\alpha_{in}) = D_{2b,c}'(\alpha_{in}) = 0$. It is interesting to note that equations similar to (15) and (11) have been derived in [7,13] for a general single-component density expansion of Friedmann equations. However, the mass nonconservation induced by scalar gravity introduces nonstandard friction terms $F_{b,c}$ that cannot be accounted for within the class explored in Refs. [7,13]. As it has been shown in [5,6], the dominant term in the skewness is $S_{3c} = 6D_{2c}/D_1^2$ and $S_{3b} = 6D_{2b}/(bD_1)^2$. To derive an approximate analytical solution, we can assume $D_{2b} = b^{(2)}D_{2c}$ with a constant second-order bias $b^{(2)}$ since, as in the linear equations, the baryon evolution is driven by the dark matter one. The bias $b^{(2)}$ is not to be confused with the coefficient b_2 of the Taylor expansion of a nonlinear biasing (see, e.g., [9]). It turns out that for small β^2 the leading nontrivial term is

$$\frac{S_{3b}}{S_{3c}} = \frac{b^{(2)}}{b^2} \approx 1 + \beta^2\left(\frac{34\Omega_m}{28m^2 + 57\Omega_m}\right). \tag{16}$$

(here we employed the approximation $S_{3c} \approx 34/7$; see below). It appears that S_{3b} is almost independent of the potential slope n and, since $m^2 \approx \Omega_m$, also almost independent of Ω_m.

The system (10), (11), (14), and (15), along with the background equations, constitutes a complete set of differential equations for the unknowns b, m, S_{3b}, S_{3c} as a function of the cosmological parameters w_ϕ, Ω_m, β. Each of the functions b, m, S_{3b}, S_{3c} depends on β and is therefore in principle a test of the equivalence principle and, more in general, of coupled dark energy. In the long term, this redundancy can be exploited to set more stringent limits to the coupling and to break degeneracies with other cosmological parameters. However, b and S_{3c} require the detection of the large-scale clustering of the dark matter component, while m, the growth rate, requires accurate observations over an extended range of redshifts and, consequently, the problematic removal of redshift-dependent selection effects. Moreover, Eq. (13) implies a strong level of degeneracy between β and the parameters that enter $\Omega_m(z)$. On the other hand, S_{3b} is an efficient probe of the scalar interaction since it requires only observations of the baryon distribution at a fixed redshift.

We plot in Fig. 1 the functions $S_{3b,c}(\Omega_m, \beta)$ obtained through numerical integration. As anticipated, we find that S_{3c} is close to the standard value 34/7 in the whole parameter range while S_{3b} deviates from it by more than 1% for $\beta > 0.1$, following the approximate fit

$$S_{3b} = \frac{34}{7}(1 + 0.6\beta^2)\Omega_m^{-0.0005(\beta^2)^{0.025}}. \tag{17}$$

This result is almost independent of Ω_m (in fact the last factor can be omitted) and n and also independent of time (if β is constant) and of scale: It shows therefore that S_{3b} is a direct test of the equivalence principle.

The analytical behavior (16) is relatively accurate (error on $S_{3b} < 1\%$) only for $\beta < 0.2$. Thus far, we assumed

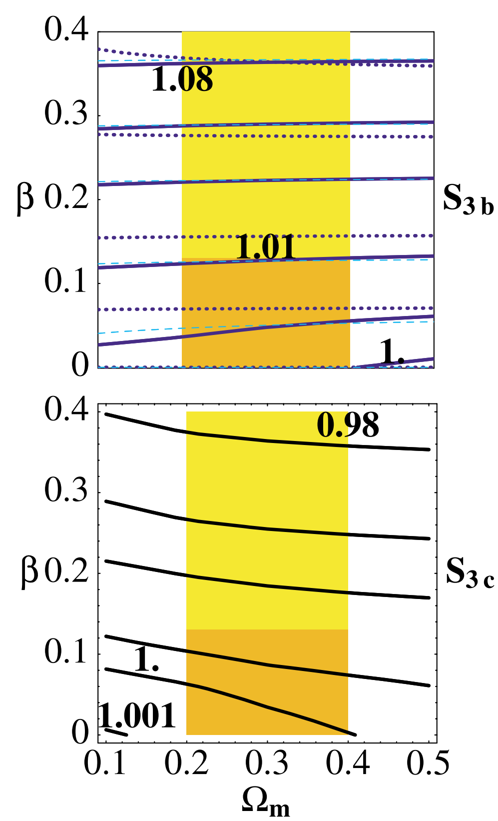

FIG. 1 (color online). Contour plot of the observable quantities $S_{3b} \cdot (7/34)$, $S_{3c} \cdot (7/34)$ calculated numerically as a function of Ω_m, β. For S_{3b}, the lines correspond to the contour values 1.0, 1.002, 1.01, 1.03, 1.05, and 1.08, while for S_{3c} they are 1, 1.001, 0.999, 0.995, 0.99, and 0.98, both from bottom to top. The short-dashed curves in the plot S_{3b} are the fit (17); the dotted curves the approximation (16). The light rectangle marks the astrophysical bounds on Ω_m, the darker one adds the CMB constraint assuming constant β.

$\beta_b = 0$ but it is not difficult to see that, in the limit $\Omega_b \ll \Omega_c$, Eqs. (13), (16), and (17) generalize to a finite β_b by simply replacing β^2 with $\beta_c(\beta_c - \beta_b)$. Let us remark also that, although we performed the numerical integration with a dark energy potential, the scalar field need not be the field responsible of the accelerated expansion. The only condition on the potential concerning the validity of our numerical calculations is that the interaction scale λ be much larger than the astrophysical scale at which the observations are carried out. For instance, the original Brans-Dicke model, which does not give acceleration and where $V = 0$, fulfills this condition.

In [9], the authors compiled an extensive list of present constraints on the smoothed skewness $\hat{S}_3$ from angular and redshift galaxy catalogs. Although several experiments quote values of $\hat{S}_3$ with errors of 5%–10%, many results are clearly not compatible with each other. This points to the presence of systematic errors, likely to reside in sampling and finite volume effects and redshift distortions, so it is premature to perform a direct comparison with data. However, analyses from larger redshift surveys such as the Sloan Digital Sky Survey (SDSS) promise to measure $\hat{S}_3$ at large scales with a precision of less than 10% and perhaps down to 1% (see, e.g., preliminary results in [26] and, for the 2dF survey, in [27]). At this level, SDSS might detect the scalar interaction or put a stringent upper limit to its present value.

It is, however, to be stressed that our calculations refer to the properties of baryons, while observations deal with light, i.e., with the fraction of baryons that collapsed in sufficiently bright galaxies. The relation between the two populations is not well known, although at large scales, where hydrodynamical effects and strong nonlinearities are smeared out, one does not expect significant segregation. To ascertain this relation, it will be necessary to perform N-body simulations with broken equivalence, as in [23] or study objects that seem to trace with more accuracy (or in a simpler way) the underlying baryon component, such as Lyman-α clouds [28]. Errors less than 10% in the bispectrum at large scales are predicted in [29] using a Lyman-α forest that simulate SDSS data.

Although models with non-Gaussian initial conditions, nongravitational effects, or nonstandard Friedmann equations predict $S_3 \neq 34/7$ [7,9,13], they also predict a specific time and/or scale dependence that makes them distinguishable, at least in principle, from a scalar interaction. Further information can be gained by the full bispectrum $B(\mathbf{k}_1, \mathbf{k}_2) = \langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{-\mathbf{k}_1 - \mathbf{k}_2} \rangle$, rather than by the integrated skewness. In [30], it has been shown that the scale dependence of the bispectrum may be of great help in constraining primordial non-Gaussianity. The behavior of the bispectrum for the present model will be reported in subsequent work. Forthcoming large-scale skewness data offer therefore the exciting opportunity to test the equivalence principle in a realm inaccessible to laboratory experiments.

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