Coulomb Blockade of Proximity Effect at Large Conductance

P. M. Ostrovsky* and M. A. Skvortsov
Landau Institute for Theoretical Physics, Chernogolovka, Moscow region, 142432 Russia

M. V. Feigel'man
Landau Institute for Theoretical Physics, Chernogolovka, Moscow region, 142432 Russia
and Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
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We consider the proximity effect in a normal dot coupled to a bulk superconducting reservoir by the tunnel contact with large normal conductance. Coulomb interaction in the dot suppresses the proximity minigap induced in the normal part of the system. We find exact expressions for the thermodynamic and tunneling minigaps as functions of the junction's capacitance. The tunneling minigap interpolates between its proximity-induced value in the regime of weak Coulomb interaction to the Coulomb gap in the regime of strong interaction. In the intermediate case a nonuniversal two-step structure of the tunneling density of states is predicted. The charge quantization in the dot is also studied.

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When a normal metal forms a contact with a superconductor it acquires some superconductor features. One of them is the suppression of the electron density of states (DOS) at the Fermi energy. This effect is governed by the Andreev processes at the normal-metal-superconductor (NS) boundary. Cooper pairs tunnel into the normal metal preserving their phase and thus inducing the superconductive correlations. Coulomb repulsion leads to phase fluctuations and reduces this proximity effect [1]. For a disordered normal metal, a minigap of the order of the inverse escape time appears in the excitation spectrum [2]. Below we study the suppression of the minigap by Coulomb interaction.

To measure the density of states the tunneling spectroscopy technique is widely employed. The conductance as a function of the voltage between the external probe and the system under investigation is proportional to the *tunneling* density of states (TDOS). Coulomb interaction affects the TDOS suppressing tunneling conductance at small voltages. This suppression is known as the zero bias anomaly [3,4]. The interplay between the zero bias anomaly and the proximity effect was first studied in Ref. [5] for a 2D thin normal film coupled by the tunnel junction to a superconductor. The renormalization group procedure yields the power-law suppression of the minigap in the TDOS by the Coulomb interaction.

Coulomb repulsion is ultimately strong in restricted geometry. In such systems direct observation of charge quantization is made possible by the Coulomb blockade effect [6]. When a zero-dimensional (0D) normal grain is coupled to a *normal* reservoir by the tunnel junction with small dimensionless (in units of $e^2/\hbar$) conductance G, the equilibrium charge of the grain is a step function of the gate voltage at zero temperature. Contrary, if $G \gg 1$ the charge of the grain is no longer conserved and only exponentially weak modulation of charge-voltage dependence remains [7]. The situation changes dramatically if the reservoir is *superconducting* [8]. Now a single electron cannot escape into the superconductor and charge quantization is observed even at large normal conductance.

In this Letter we consider the 0D NS system described above. The parameters of the system are assumed to satisfy the following conditions:

$$E_{\mathrm{Th}} > \Delta \gg (E_g, E_C) \gg \delta. \tag{1}$$

Here E_{Th} is the Thouless energy, Δ is the superconductor gap, δ is the electron mean level spacing in the grain per one spin component, $E_g = G\delta/4$ is the proximity minigap [2] in the absence of Coulomb repulsion, and $E_C = e^2/(2C)$ is the charging energy, with C being the junction's capacitance. The dimensionless conductance G is assumed to be large. We neglect effects of quasiparticle transport assuming temperature sufficiently low and thus quasiparticle conductance $G_{\mathrm{qp}} = G\exp(-\Delta/T) \ll 1$.

Coulomb interaction modifies the thermodynamic and tunneling DOS in different ways. The minigap $\tilde{E}_g$ in the thermodynamic DOS is gradually suppressed with the increase of E_C due to enhanced phase fluctuations, whereas the dependence of the minigap E_g^{tun} in the TDOS on E_C is more complicated. Being suppressed at weak interaction by phase fluctuations, it eventually reaches the Coulomb gap E_C in the strong interaction limit. Qualitatively, these two regimes are distinguished by the relation between the charging energy E_C and the energy $E_J = (E_g^2/\delta)\log(\Delta/E_g)$, which is the Josephson energy of the fictitious system where the normal grain is replaced by the weak superconductor with the gap E_g. Below we find the exact dependence of both the DOS and TDOS on the strength of the interaction for arbitrary E_C/E_J.

We also study the charge quantization in the grain accounting for both Coulomb and proximity effects. The

result of Ref. [8] is valid only for $E_C \gg E_J$. In the opposite case proximity coupling smears the Coulomb staircase preserving however the sharp discontinuities at half-integer charge.

To investigate the electron properties of the NS system we use the nonlinear σ model [9] in the Matsubara representation. Disorder average is made with the help of a replica trick. Thus the σ model is formulated in terms of the matrix field Q operating in the Nambu-Gor'kov (Pauli matrices $\hat{\tau}_i$), energy, and replica spaces. The 0D Coulomb interaction is decoupled by the electric potential ϕ. The σ-model action reads [9]

$$S[Q,\phi] = -\frac{\pi}{\delta}\mathrm{Tr}[(\varepsilon\hat{\tau}_3 + \phi)Q] - \frac{\pi G}{4}\mathrm{Tr}[Q_S Q] + \int d\tau \frac{\phi^2}{4E_C}. \tag{2}$$

Hereafter we omit replica indices for brevity. We will use $\hat{\tau}_1$ for the Q matrix of the superconductor, Q_S, restricting our model to the subgap region $\varepsilon \ll \Delta$. The contribution from higher energies leads [10] to the renormalization of the capacitance $C \mapsto C + e^2 G/(2\Delta)$ and corresponding renormalization of the charging energy E_C.

In the action (2) the matrix Q is linearly coupled to the potential ϕ. This means that Q contains not only soft electron modes of the system with energies close to the Fermi energy but also the high energy fluctuations corresponding to the shift of the whole energy band by the electric potential. To get rid of this contribution we make a gauge transformation proposed in Ref. [11]: $Q_{\tau\tau'} = e^{i\hat{\tau}_3 K(\tau)}\tilde{Q}_{\tau\tau'}e^{-i\hat{\tau}_3 K(\tau')}$ with $K(\tau) = \int^\tau \phi(\tau)d\tau$. In terms of these new variables the action has the form:

$$S[\tilde{Q},K] = -\frac{\pi}{\delta}\mathrm{Tr}(\varepsilon\hat{\tau}_3\tilde{Q}) + \int d\tau\left[\frac{\dot{K}^2}{4E_C} - \frac{2\pi E_g}{\delta}(\tilde{Q}^{(1)}_{\tau\tau}\cos 2K + \tilde{Q}^{(2)}_{\tau\tau}\sin 2K)\right], \tag{3}$$

where the symbols $\tilde{Q}^{(i)} = \mathrm{tr}(\hat{\tau}_i\tilde{Q})/2$ denote the Nambu-Gor'kov components of the matrix $\tilde{Q}$.

The general dynamics governed by the action (3) is complicated as the variables $\tilde{Q}$ and K are strongly coupled with each other. Fortunately, in the region of the parameters specified by Eq. (1) the characteristic frequencies of the variable $\tilde{Q}$ (of the order of the renormalized proximity gap $\tilde{E}_g$) appear to be much smaller than those of the variable K. Therefore, it is possible to employ the *adiabatic* approximation, integrating first over the "fast" variable K. The resulting effective action $S[\tilde{Q}]$ for the "slow" variable $\tilde{Q}$ should then be subject to the saddle-point approximation (SPA). Fluctuations of the Q matrix describe the effects of level statistics, therefore the SPA works fine if the relevant energy scale $\tilde{E}_g$ exceeds δ. Such a condition is definitely satisfied in the regime of weak Coulomb repulsion: $E_g/\delta = G/4 \gg 1$. Below we will find the applicability range of both the adiabatic and the saddle-point approximations at arbitrary strength of the Coulomb interaction.

The action averaged over K is obviously uniform in time that allows one to assume the stationary saddle point

$$\underline{\tilde{Q}}_{\varepsilon\varepsilon'} = 2\pi\delta(\varepsilon - \varepsilon')[\hat{\tau}_3\cos\alpha(\varepsilon) + \hat{\tau}_1\sin\alpha(\varepsilon)], \tag{4}$$

parametrized by the single function $\alpha(\varepsilon)$. The possibility to choose $\underline{\tilde{Q}}$ in the form (4) without the $\hat{\tau}_2$ term is related to the zero mode with respect to $K \mapsto K + \mathrm{const}$.

Substituting the ansatz (4) into the action (3) we find the imaginary time evolution for K. It is determined by the simple Hamiltonian

$$\hat{H} = E_C[-\partial^2/\partial K^2 - 2q\cos 2K]. \tag{5}$$

The parameter $q = (E_g/2E_C\delta)\int_{-\Delta}^{\Delta} d\varepsilon \sin\alpha(\varepsilon)$ measuring the relative strength of the proximity coupling with respect to Coulomb interaction is to be determined self-consistently later on. The 2π-periodic boundary conditions for K are implied.

At zero temperature the dynamics of $K(\tau)$ is frozen at the ground state of (5). The ground state energy can be expressed in terms of the zeroth Mathieu characteristic value $a_0(q)$: $E_0 = E_C a_0(q)$. After averaging over K one obtains the effective action for $\tilde{Q}$:

$$S[\tilde{Q}] = \int d\tau\left[-\frac{1}{\delta}\int d\varepsilon\varepsilon\cos\alpha(\varepsilon) + E_0(q)\right]. \tag{6}$$

Minimizing this expression we find the BCS-like solution $\alpha(\varepsilon) = \arctan(\tilde{E}_g/\varepsilon)$. The renormalized value of the minigap $\tilde{E}_g$ satisfies the following system of equations:

$$\frac{\tilde{E}_g}{E_g} = -\frac{1}{2E_C}\frac{\partial E_0}{\partial q}, \qquad q = \frac{E_g\tilde{E}_g}{E_C\delta}\log\frac{2\Delta}{\tilde{E}_g}. \tag{7}$$

The dependence of the ground state energy E_0 on q is especially simple in two limiting cases. If the Coulomb interaction is weak the parameter q is large and we can estimate the potential energy in Eq. (5) by the oscillator potential. This gives a small correction to the bare minigap value E_g:

$$\tilde{E}_g = E_g - \frac{1}{2}\sqrt{\frac{E_C\delta}{\log(2\Delta/E_g)}}, \qquad q \gg 1. \tag{8}$$

In this regime the variable K has the frequency $E_C\sqrt{q}$ which is $\sqrt{E_C/\delta}$ times larger than E_g. In the opposite limit of strong Coulomb interaction the potential in Eq. (5) can be treated perturbatively yielding $a_0(q) = -q^2/2$. The minigap appears to be exponentially small in E_C:

$$\tilde{E}_g = 2\Delta\exp\left(-\frac{2E_C\delta}{E_g^2}\right), \qquad q \ll 1. \tag{9}$$

At the same time, the frequency of K is $E_C \gg \tilde{E}_g$, ensuring the validity of the adiabatic approximation.

The whole dependence of $\tilde{E}_g$ on E_C is shown in Fig. 1. The thermodynamic DOS itself is just the conventional BCS density of states with the gap $\tilde{E}_g$.

Now we turn to the tunneling density of states. In the σ-model formalism the TDOS is the analytic continuation of the function

$$\rho(\varepsilon) = \frac{1}{\delta}\int d\tau e^{i\varepsilon\tau}\mathrm{tr}\langle\hat{\tau}_3 e^{i\hat{\tau}_3 K}\tilde{Q}e^{-i\hat{\tau}_3 K}\rangle_{\tau,0} \qquad (10)$$

from positive Matsubara energies ε to real energies E. The angular brackets imply averaging with the action (3). In the adiabatic approximation this means averaging over K with the Hamiltonian (5) and substitution $\tilde{Q} \to \underline{\tilde{Q}}$. The result for the TDOS is the convolution of the thermodynamic DOS with the phase correlator $C(\tau) = \langle e^{i[K(\tau)-K(0)]}\rangle$ given by

$$C(\omega) = \sum_n |\langle 0|e^{iK}|n\rangle|^2 \frac{2(E_n - E_0)}{(E_n - E_0)^2 + \omega^2} \qquad (11)$$

in the frequency representation at zero temperature. Here E_n are the energy levels of the Hamiltonian (5). The matrix elements in Eq. (11) are nonzero only for $n = 4k + 1$ and $n = 4k + 2$. Evaluating the convolution of the thermodynamic DOS with $C(\omega)$ and performing analytic continuation, we obtain for the TDOS

$$\rho(E) = \frac{2}{\delta}\sum_n |\langle 0|e^{iK}|n\rangle|^2 \frac{|x_n|\,\theta(|x_n| - \tilde{E}_g)}{\sqrt{x_n^2 - \tilde{E}_g^2}}, \qquad (12)$$

where $x_n = E - (E_n - E_0)$. The tunneling minigap associated with the $n = 1$ term is given by

$$E_g^{\mathrm{tun}} = \tilde{E}_g + E_1 - E_0. \qquad (13)$$

Figure 2 shows its dependence on E_C which interpolates from the proximity gap (8) in the weak interaction regime to the Coulomb gap E_C at large E_C. Note the reentrant behavior of E_g^{tun} as a function of E_C at small E_C.

The form of the TDOS depends on the strength of the Coulomb interaction. Because of the fast decrease of the matrix element $|\langle 0|e^{iK}|n\rangle|^2$ with n only the first two terms may significantly contribute to the sum in Eq. (12). In the weak Coulomb interaction regime ($q \gg 1$), the $n = 1$ contribution dominates. Since the level splitting $E_1 - E_0$ is exponentially small the TDOS coincides with the thermodynamic DOS in this limit. When the Coulomb interaction is strong ($q \ll 1$), the matrix elements $|\langle 0|e^{iK}|1\rangle|^2 \approx |\langle 0|e^{iK}|2\rangle|^2 \approx 1/2$ and $\rho(E) \approx (2/\delta)\,\theta(|E| - E_C)$. In the intermediate regime ($q \sim 1$) the TDOS acquires a nonuniversal two-step structure, being zero for $E < E_g^{\mathrm{tun}}$, of the order of $|\langle 0|e^{iK}|1\rangle|^2$ for $E_g^{\mathrm{tun}} < E < \tilde{E}_g + E_2 - E_0$, and close to 1 for $E > \tilde{E}_g + E_2 - E_0$; see the inset of Fig. 2. The BCS-type singularities are still present at the step edges but they are relatively weak.

At nonzero temperature all the above results are still valid provided $T \ll \tilde{E}_g$. If the temperature is higher all integrations over energies should be replaced by summations over Matsubara frequencies. Moreover, the phase K is not frozen at the ground state in this case, and the temperature dependent free energy should be used instead of the ground state energy in Eq. (6) and below. These calculations show that the true tunneling minigap vanishes when the temperature is of the order of $\tilde{E}_g(T = 0)$.

Now we briefly comment on the validity of our approach. The above results were obtained using the adiabatic approximation for the integral over K and the saddle-point approximation for $\tilde{Q}$. To justify this approach one can calculate the first fluctuation correction to the TDOS. To this end we parametrize $\tilde{Q}$ in the standard way in terms of the matrix W, and average over K with the Hamiltonian (5) taking into account W

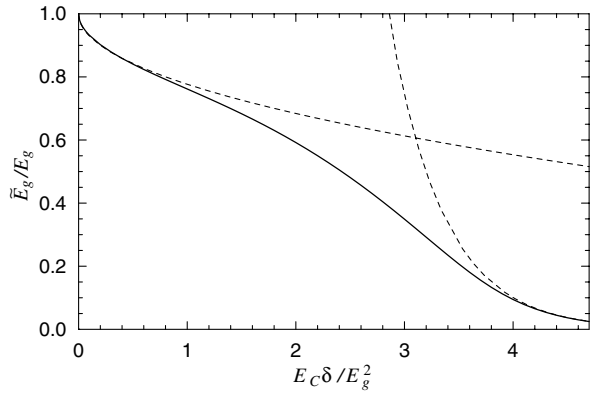

FIG. 1. The minigap in the thermodynamic DOS as a function of the Coulomb energy E_C. Dashed lines illustrate the asymptotic behaviors (8) and (9). The crossover from weak to strong fluctuations regime occurs at $E_C\delta/E_g^2 \sim \log(\Delta/E_g)/2$. This figure shows the case $\log(\Delta/E_g) = 5$.

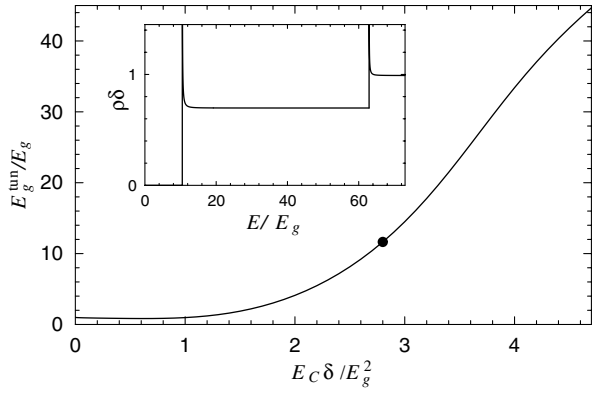

FIG. 2. The minigap in the tunneling DOS as a function of the Coulomb energy. For small E_C it follows $\tilde{E}_g$ given by Eq. (8). In the opposite limit $E_g^{\mathrm{tun}} = E_C$. This figure is plotted assuming $\log(\Delta/E_g) = 5$ and $G = 40$. Inset: two-step tunneling density of states in the intermediate regime. The charging energy is such that $E_C\delta/E_g^2 = 2.8$ (as shown by the dot on the main graph) which corresponds to $q = 0.95$.

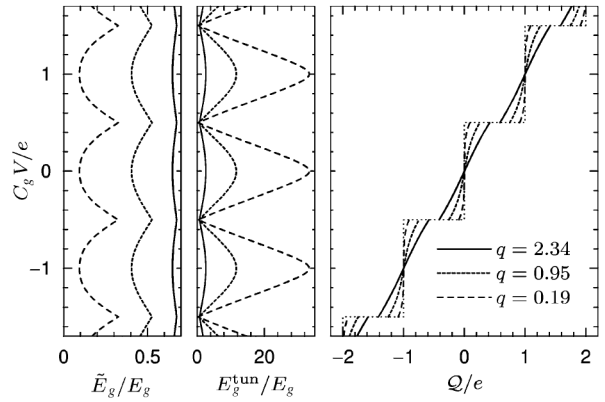

FIG. 3. The dependence of the thermodynamic minigap, the tunneling minigap, and the average charge on the gate voltage V (vertical axis) at three different values of Coulomb energy. The parameters are $G = 40$, $\log(\Delta/E_g) = 5$.

perturbatively. As a result we end up with the quadratic action for the fluctuating W. Evaluating the first order perturbative correction to the density of states we find that it is small provided that $E_C \gg \delta$ (adiabaticity condition) and $\tilde{E}_g \gg \delta$ (validity of the SPA). All the above results were obtained in the lowest-order approximation over tunneling probability $\mathcal{T}$ across the NS interface. The effect of higher-order terms can be neglected if Andreev conductance through this interface is small, $G_A \ll 1$.

Finally, we consider the case of a nonzero gate voltage V applied to the normal grain in the standard Coulomb blockade setup [8]. If the grain is coupled to the gate by the capacitance C_g the charging energy [6] becomes $E_C(\hat{\mathcal{Q}}/e - N)^2$, where $\hat{\mathcal{Q}}$ is the charge operator of the grain, $eN = C_g V$ is the equilibrium charge, and we redefined $E_C = e^2/2(C + C_g)$. The Hamiltonian for K is changed to

$$\hat{H} = E_C[(-i\partial/\partial K - N)^2 - 2q\cos 2K]. \qquad (14)$$

The energy spectrum of this Hamiltonian has the standard band structure. The boundary conditions pick out *two* energy levels from each band with the quasimomentum determined by the gate potential. When the equilibrium charge N approaches half-integer values these two energy levels cross. Since all physical quantities depend periodically on N, in what follows we assume $|N| < 1/2$.

In the weak Coulomb interaction regime, $E_C \ll E_J$, the lowest energy band is exponentially narrow. The lowest level of the Hamiltonian depends weakly on N, and all the results for the DOS and TDOS obtained above are left unchanged. For strong Coulomb interaction we calculate the ground state energy perturbatively in small q. Neglecting the q term we have $E_n^{(0)} = E_C(n - N)^2$. The second order in the q correction to the ground state energy $E_0^{(2)} = -E_C q^2/2(1 - N^2)$. From the self-consistency Eqs. (7) we find the thermodynamic minigap $\tilde{E}_g = 2\Delta \exp[-(2E_C\delta/E_g^2)(1 - N^2)]$. The minigap in the TDOS is determined by Eq. (13) yielding $E_g^{\rm tun} = \tilde{E}_g + E_C(1 - 2|N|)$. This quantity depends strongly on N decreasing to the exponentially small value $\tilde{E}_g$ at $N = 1/2$. These results are illustrated in Fig. 3.

The average charge of the grain is $\mathcal{Q} = eN - \partial E_0/\partial V$. In the limit of strong Coulomb interaction the result of [8] is reproduced. In the opposite limit the Coulomb staircase is smeared up to an exponentially weak modulation. Employing the tight binding approximation for the Hamiltonian (14) we get $\mathcal{Q}/e = N - C_g/(C + C_g)8\sqrt{2\pi}q^{3/4} \times e^{-4\sqrt{q}}\sin(\pi N)$. The small steps of the height $C_g/(C + C_g)16\sqrt{2\pi}q^{3/4}e^{-4\sqrt{q}}$ at half-integer charge are still present as predicted in Ref. [8]. This feature is the consequence of the double-electron charge transport through the junction. The Coulomb staircase is shown in the right panel of Fig. 3.

In conclusion, we have shown that interplay between proximity and charging effects in a superconductor-normal grain tunnel junction with large normal conductance G is governed by the ratio of the charging energy E_C to the fictitious Josephson energy $E_J = G^2\delta\log(\Delta/G\delta)$. DOS and TDOS are calculated as functions of E_C/E_J and the gate voltage V. An unusual two-step shape of the TDOS is found at $E_C \sim E_J$.

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*Electronic address: ostrov@itp.ac.ru

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