Frequency Scales for Current Statistics of Mesoscopic Conductors

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We calculate the third cumulant of current in a chaotic cavity with contacts of arbitrary transparency as a function of frequency. Its frequency dependence drastically differs from that of the conventional noise. In addition to a dispersion at the inverse RC time characteristic of charge relaxation, it has a low-frequency dispersion at the inverse dwell time of electrons in the cavity. This effect is suppressed if both contacts have either large or small transparencies.

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What is the characteristic time scale of the dynamics of electrical transport in normal-metal mesoscopic systems if their size is larger than the screening length? The most reasonable answer is that this dynamics is governed by the RC time of the system, which describes the relaxation of piled up charge. It is this time that characterizes the admittance of a mesoscopic capacitor [1], the impedance of a diffusive contact [2], current noise in diffusive contacts [3], and charge noise in quantum point contacts and chaotic cavities [4,5]. The free motion of electrons is superseded by the effects of charge screening and therefore the dwell time of a free electron is not seen in these quantities. The only known exception is the frequency dispersion of the weak-localization correction to the conductance [6,7]. Because typical charge-relaxation times are very short for good conductors, the experimental observation of the dispersion of current noises is difficult [8].

Very recently, Reulet *et al.* measured the third cumulant of current for tunnel junctions [9]. In this case, the frequency dispersion is due only to the measurement circuit [10]. In contrast to tunnel junctions, chaotic cavities have internal dynamics. It is the purpose of this Letter to show that mesoscopic systems with such dynamics may exhibit an additional low-frequency dispersion in the third cumulant of current, which is absent for the first and second cumulants. Physically, this dispersion is due to slow, charge-neutral fluctuations of the distribution function in the interior of the system. Such charge-neutral fluctuations are similar to fluctuations of the effective electron temperature at a constant chemical potential. They do not contribute directly to the measurable current, but they modulate the intensity of noise and hence contribute to higher cumulants of current [11].

The zero-frequency electric noise in open chaotic cavities has been calculated by Jalabert *et al.* [12] using random matrix theory. More recently, these expressions were derived semiclassically by Blanter and Sukhorukov [13]. Shot-noise measurements on chaotic cavities were performed by Oberholzer *et al.* [14] The third and fourth cumulants of current were obtained by Blanter *et al.* [15] for open cavities and in Ref. [16] for cavities with contacts of arbitrary transparency. Recently, the statistics of charge in a cavity was analyzed by two of us [17].

Model.—Consider the system shown in Fig. 1. A chaotic cavity consists of a metallic mesoscopic conductor of irregular shape connected to leads L, R via quantum point contacts with conductances $G_{L,R}$ and transparencies $\Gamma_{L,R}$. We assume that $G_{L,R} \gg e^2/h$ and the bias is much larger than the temperature and frequencies. The dwell time of electrons in the cavity is given by $\tau_D = e^2 R_Q N_F$, where $R_Q = (G_L + G_R)^{-1}$ is the charge-relaxation resistance [7] and N_F is the density of states in the cavity, which is assumed to be continuous and constant. We assume that the dwell time is short as compared to inelastic scattering times. Then the mean occupation function may be written as a weighted average $f_0(\epsilon) = R_Q(G_L f_L(\epsilon) + G_R f_R(\epsilon))$ of the Fermi occupation factors $f_{L,R}$ of the leads which are step functions at zero temperature. In the absence of electrostatic interaction, fluctuations around f_0 would relax on the time scale τ_D. However, due to screening there is a second shorter time scale τ_Q, which describes fluctuations of the charge in the cavity. It is given by $\tau_Q = R_Q C_\mu$, where $C_\mu^{-1} = C^{-1} + (e^2 N_F)^{-1}$ is the electrochemical capacitance [1] and C is the geometric capacitance.

Calculation.—Our semiclassical method is based on a large separation between the time scales describing the

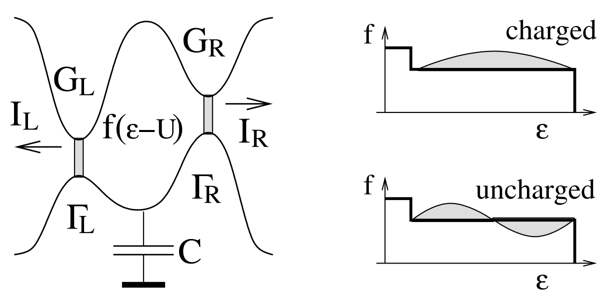

FIG. 1. Left panel: A chaotic cavity is biased from two leads and capacitively coupled to a gate. Right panel: Charged fluctuations of the distribution function f decay at the short RC time and contribute to the current noise. Uncharged fluctuations decay at a much longer time τ_D and do not contribute directly to the current noise but result in a low-frequency dispersion of the third cumulant of current.

fast fluctuations of current in isolated contacts and slow fluctuations of the electron distribution in the cavity [16]. This allows us to consider the contacts as independent generators of white noise, whose intensity is determined by the instantaneous distribution function of electrons in the cavity. Based on this time scale separation, a recursive expansion of higher cumulants of semiclassical quantities in terms of their lower-order cumulants was developed [11,16]. Very recently, such recursive relations were obtained as a saddle point expansion of a stochastic path integral [18] which will be used in the following.

Our aim is to obtain the statistics of time-dependent fluctuations of the current $I_L(t)$ flowing through the left contact of the cavity. In general, the probability $\mathcal{P}[A(t)]\mathcal{D}A$ of a time-dependent stochastic variable $A(t)$ ($t \in [0, T]$) is described by the characteristic functional $S[\chi_A]$ evaluated with an imaginary field

$$\mathcal{P}[A(t)] = \int \mathcal{D}\chi_A e^{-i\int_0^T dt A\chi_A + S[\chi_A \mapsto i\chi_A]}. \quad (1)$$

Functional derivatives $\delta^n/(\delta\chi_A(t_1)\ldots\delta\chi_A(t_n))$ of the characteristic functional $S[\chi_A]$ yield irreducible correlation functions $\langle\langle A(t_1)\ldots A(t_n)\rangle\rangle$.

To obtain the characteristic functional $S[\chi_L]$ describing the fluctuations of $I_L(t)$ we proceed in two steps. First, we consider the point contacts ($i = L, R$) as sources of white noise that depend on two common time-dependent parameters, the electron occupation function of the cavity $f(\epsilon - eU)$ and the electrostatic potential of the cavity U. Their characteristic functionals are given by $S_i[\tilde{\chi}_{i,\epsilon}] = \int dt \int d\epsilon H_i(\tilde{\chi}_{i,\epsilon})$ with

$$H_i = \langle\langle \tilde{I}_i \rangle\rangle_\epsilon \tilde{\chi}_{i,\epsilon} + \frac{1}{2}\langle\langle \tilde{I}_i^2 \rangle\rangle_\epsilon \tilde{\chi}_{i,\epsilon}^2 + \frac{1}{6}\langle\langle \tilde{I}_i^3 \rangle\rangle_\epsilon \tilde{\chi}_{i,\epsilon}^3 + \ldots. \quad (2)$$

The cumulants $\langle\langle \tilde{I}_i^n \rangle\rangle_\epsilon$ must be taken from a quantum-mechanical calculation [19,20]

$$\begin{aligned}\langle\langle \tilde{I}_i^n \rangle\rangle_\epsilon = \frac{G_i}{e\Gamma_i}\frac{\partial^n}{\partial\chi^n}\ln\{1 + \Gamma_i f_i(\epsilon)[1 - f(\epsilon - eU)](e^{-e\chi} - 1) \\ + \Gamma_i f(\epsilon - eU)[1 - f_i(\epsilon)] \\ \times (e^{e\chi} - 1)\}|_{\chi=0}. \quad (3)\end{aligned}$$

In a second step, we take into account that the occupation function $f(\epsilon - eU)$ and the potential U are not free parameters but are fixed by the kinetic equation

$$\frac{df}{dt} = \frac{dU}{dt}\frac{\partial f}{\partial U} - \frac{1}{eN_F}(\tilde{I}_{L,\epsilon} + \tilde{I}_{R,\epsilon})$$

and the charge-conservation law

$$C\frac{dU}{dt} = -\int d\epsilon(\tilde{I}_{L,\epsilon} + \tilde{I}_{R,\epsilon}). \quad (4)$$

The two conservation laws are expressed by path integrals over Lagrange multipliers λ_ϵ, ξ and integrated over the fluctuations of occupation function and potential to obtain the following result for the generating functional:

$$\begin{aligned}e^{S[i\chi_L]} = \int \mathcal{D}\lambda_\epsilon \mathcal{D}\xi \mathcal{D}f \mathcal{D}U \\ \times \exp\{S_L[i\lambda_\epsilon + i\xi - i\chi_L] + S_R[i\lambda_\epsilon + i\xi] + S_C\}, \quad (5)\end{aligned}$$

where the conservation laws are expressed by the following dynamical action:

$$S_C = -i\int_0^T dt\left\{\xi C\dot{U} + eN_F\int d\epsilon\lambda_\epsilon\left(\frac{df}{dt} - \frac{dU}{dt}\frac{\partial f}{\partial U}\right)\right\}. \quad (6)$$

In the semiclassical regime, this path integral may be evaluated in the saddle point approximation [18,21] to obtain current correlation functions $\langle\langle I_L(t_1)\ldots I_L(t_n)\rangle\rangle$ of arbitrary order. We choose a large time interval $[0, T]$ and neglect transient effects. It is then most convenient to work in Fourier space where the correlation functions may be written in terms of spectral functions P_n

$$\begin{aligned}\langle\langle I_L(\omega_1)\ldots I_L(\omega_n)\rangle\rangle = 2\pi\delta(\omega_1 + \cdots + \omega_n) \\ \times P_n(\omega_1\cdots\omega_{n-1}). \quad (7)\end{aligned}$$

In the following, we analyze the dispersion of the spectral functions P_2 and P_3.

Results.—We first briefly discuss the frequency dependence of current noise to connect our theory to earlier results [4]. Solving the relevant saddle point equations, we may express the spectrum as

$$P_2 = |Z_Q(\omega)|^2\int d\epsilon[\langle\langle \tilde{I}_L^2\rangle\rangle_0|\tilde{G}_R(\omega)|^2 + \langle\langle \tilde{I}_R^2\rangle\rangle_0 G_L^2], \quad (8)$$

where we introduced $Z_Q(\omega) = (G_L + G_R + i\omega C_\mu)^{-1}$ and $\tilde{G}_R(\omega) = G_R + i\omega C_\mu$. The zero subscript indicates that the cumulants $\langle\langle \tilde{I}_i^n\rangle\rangle_\epsilon$ are evaluated at $f(\epsilon - eU) = f_0(\epsilon)$. To complete Eq. (8), it remains to insert the bare noise cumulants $\langle\langle \tilde{I}_i^2\rangle\rangle_\epsilon$ from Eq. (3). At low frequencies, the noise spectrum P_2 shows correlations between left and right point contact. At high frequencies, these correlations disappear and we observe the bare noise of the left contact. The transition frequency is given by the RC time τ_Q. We emphasize that the dwell time τ_D does not appear in the dispersion of Eq. (8).

We now turn to the much more complicated frequency dependence of the third order correlator. For the third order contribution to the action (6) we find

$$\begin{aligned}S_3[\chi_L] = \frac{1}{6}\int dt d\epsilon\{\langle\langle \tilde{I}_L^3\rangle\rangle_0(\xi_1 - \chi_L)^3 + \langle\langle \tilde{I}_R^3\rangle\rangle_0\xi_1^3 \\ + 3[\langle\langle \tilde{I}_L^2\rangle\rangle_0(\xi_1 - \chi_L) + \langle\langle \tilde{I}_R^2\rangle\rangle_0\xi_1] \\ \times(\lambda_2 + \xi_2)\}, \quad (9)\end{aligned}$$

where $\xi_1 = (1 + i\omega\tau_Q)^{-1}G_L R_Q \chi_L$ and where the fields λ_2, ξ_2 satisfy the following relations derived from the saddle point equations:

$$(1 + i\omega\tau_D)\lambda_2 = R_Q A_L(\eta_1 - \chi_L)^2 + R_Q A_R \eta_1^2 - \xi_2,$$
$$(1 + i\omega\tau_Q)\xi_2 = R_Q B_L(\eta_1 - \chi_L)^2 + R_Q B_R \eta_1^2. \tag{10}$$

We abbreviated $A_i = \partial\langle \tilde{I}_i^2\rangle_0/\partial f$, and $B_i = (C_\mu/C) \int d\epsilon(-\partial f_0/\partial\epsilon)A_i$. Functional derivatives of Eq. (9) with respect to χ_L yield the third order spectral function P_3. Alternatively, this result may be obtained from a recursive diagrammatic expansion of the third cumulant [11,16]. Since Eqs. (10) depend on both time scales τ_Q and τ_D, it is clear that the third order spectral function P_3 displays dispersion on two frequency scales in strong contrast to the noise spectrum (8).

A typical frequency dispersion of P_3 obtained from an analytical evaluation of Eqs. (9) and (10) is shown in Fig. 2. In general, $P_3(\omega_1, \omega_2)$ exhibits a complicated behavior due to the twofold dispersion at the inverse RC time and at the inverse dwell time. Additional peculiarities appear at the scale τ_D^{-1} if one of the frequencies or their sum tends to zero. The shape of $P_3(\omega_1, \omega_2)$ essentially depends on the parameters of the contacts. In particular, for a cavity with one tunnel and one ballistic contact, it exhibits a nonmonotonic behavior as one goes from $\omega_1 = \omega_2 = 0$ to high frequencies (see Fig. 3). A relatively simple analytical expression for this case may be obtained if conductances are equal $G_L = G_R = G$, if $\tau_D \gg \tau_Q$, and if one of the frequencies is zero:

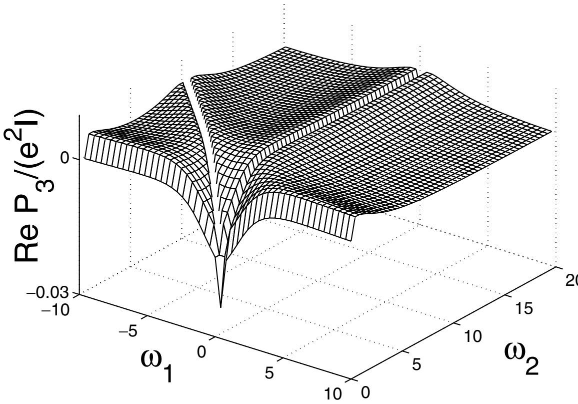

FIG. 2. A 3D plot of Re $P_3(\omega_1, \omega_2)$ for $G_L/G_R = 1/2$, $\Gamma_L = \Gamma_R = 3/4$, $\tau_Q = 1/3$, and $\tau_D = 10$ (dimensionless units).

$$P_3(\omega, 0) = -\frac{1}{32} e^2 I \frac{1 + 2\tau_D^2\omega^2 + \tau_D^2\tau_Q^2\omega^4}{(1 + \omega^2\tau_D^2)(1 + \omega^2\tau_Q^2)^2}. \tag{11}$$

The $P_3(\omega, 0)$ curve shows a clear minimum at $\omega \sim (\tau_D\tau_Q)^{-1/2}$ and the amplitude of its variation tends to $P_3(0, 0)$ as $\tau_Q/\tau_D \to 0$ (see Fig. 3).

In what follows, we will be interested in the most typical case when the dwell time τ_D is much larger than the RC time τ_Q. Even for this case the general expression for P_3 is long and we present only the low-frequency behavior, $\omega_i \ll \tau_Q^{-1}$. In this limit, P_3 is of the form [22]

$$P_3(\omega_1,\omega_2) = e^2 I\Bigg\{ 3G_L G_R \frac{[(1-\Gamma_R)G_L^2 - (1-\Gamma_L)G_R^2]^2}{(G_L+G_R)^6} - 2\frac{\Gamma_R^2 G_L^5 + \Gamma_L^2 G_R^5}{(G_L+G_R)^5} + 3\frac{\Gamma_R G_L^4 + \Gamma_L G_R^4}{(G_L+G_R)^4} - \frac{G_L^3 + G_R^3}{(G_L+G_R)^3}$$
$$- G_L G_R \frac{[(1-\Gamma_R)G_L^2 - (1-\Gamma_L)G_R^2](\Gamma_R G_L^2 - \Gamma_L G_R^2)}{(G_L+G_R)^6} \times \left[\frac{1}{1+i\omega_1\tau_D} + \frac{1}{1+i\omega_2\tau_D} + \frac{1}{1-i(\omega_1+\omega_2)\tau_D}\right]\Bigg\}. \tag{12}$$

The last term in the curly brackets shows a strong dispersion at the scale $\omega_{1,2} \sim 1/\tau_D$. This dispersion vanishes for symmetric cavities and cavities with two tunnel or two ballistic contacts, but is in general (see Fig. 2) of the same order as the frequency-independent contribution. Note that charge neutrality forces low-frequency dispersion to be the same in both contacts. Equation (12) is therefore symmetric with respect to indices L and R.

Discussion.—In the spirit of the cascade approach [11,16], the low-frequency dispersion of the third cumulant can be explained by charge-neutral fluctuations of the electron occupation function f. To illustrate this, we Fourier transform Eq. (12) and find that the dispersive part of the third order correlation function without prefactors and permutations is given by

$$\langle\langle I_L(t_1)I_L(t_2)I_L(t_3)\rangle\rangle \propto \delta(t_2 - t_3)\Theta(t_2 - t_1)e^{-(t_2-t_1)/\tau_D}. \tag{13}$$

An initial fluctuation of current at t_1 is correlated with two subsequent fluctuations at $t_{2,3}$ which happen almost

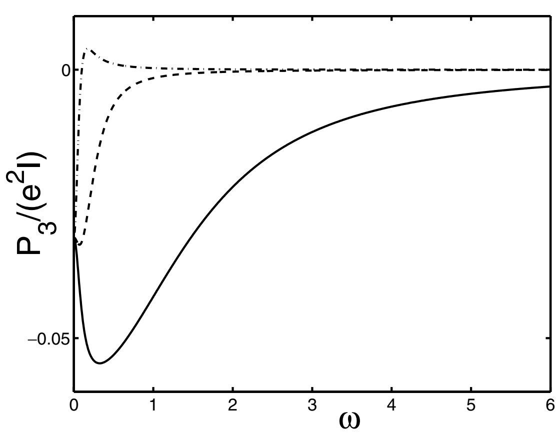

FIG. 3. $P_3(\omega, 0)$ as a function of ω for $G_L/G_R = 1$, $\Gamma_L = 1$, $\Gamma_R = 0$, and $\tau_D = 10$ (dimensionless units). The solid, dashed, and dash-dotted curves correspond to $\tau_Q = 1/2$, $\tau_Q = 3$, and to weak electrostatic coupling $\tau_Q = \tau_D$ $(C = \infty)$.

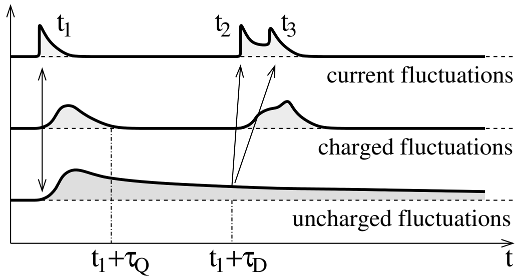

FIG. 4. Charge neutral process leading to the low-frequency dispersion of the third cumulant.

instantaneously within a short time τ_Q. The process leading to this correlation is shown in Fig. 4: A current fluctuation at t_1 leads to an accumulation of charge inside the cavity which decays quickly. But since currents are fluctuating independently at each energy, the same current fluctuation may induce as well a charge-neutral fluctuation of the electron occupation function f. During the decay of this neutral fluctuation, f differs from its mean value f_0 and changes the probability that another pair of current fluctuations will occur at later times t_2 and t_3. The low-frequency dispersion of P_3 originates from the slow decay of neutral fluctuations at the scale τ_D. Qualitative arguments can as well explain why the dispersion vanishes for certain configurations. For a cavity with two ballistic contacts, Eq. (2) depends only on f and not on f_i. For this reason, the initial current at t_1 may not create charge-neutral fluctuations of f. For a cavity with two tunnel contacts, the white noise functionals (3) depend only on charged fluctuations. Charge-neutral fluctuations thus cannot create a pair of currents at $t_{2,3}$. A similar argument applies to symmetric cavities.

The fluctuations of the current in the left-hand contact of the cavity can be measured as fluctuations of voltage across a small resistor attached to it. Based on the parameters of a chaotic cavity used in shot-noise experiments [14], our estimates give an inverse dwell time of the order of 10 GHz, which is well within the experimental range for measuring the frequency dependence of noise [8]. We also believe that similar low-frequency dispersion of the third cumulant may be observed in other semiclassical systems such as diffusive wires, where the dwell time is of the same order or larger. Hence our results are of direct experimental interest.

Conclusions.—We have shown that the third cumulant of current in mesoscopic systems may exhibit a strong dispersion at frequencies much smaller than the charge-relaxation time of the system. This effect has a purely classical origin and the variations of the cumulant may be of the order of its zero-frequency value even if the number of quantum channels in the system is large.

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[21] The saddle point equations are nonlinear differential equations for the four fields $f(\epsilon - eU), U, \lambda_\epsilon, \xi$ that contain the external statistical field χ_L in inhomogeneous source terms. To obtain second and third order correlation functions of the current I_L it is sufficient to solve the saddle point equations up to second order in the external field χ_L. The result can then be substituted into the action (5) to obtain the current correlation functions.
[22] $P_3(\omega_1, \omega_2)$ is complex if none of the frequencies ω_1, ω_2, and $\omega_1 + \omega_2$ are zero.