Experimental Observation of Discrete Modulational Instability

J. Meier,[1] G. I. Stegeman,[1] D. N. Christodoulides,[1] Y. Silberberg,[2] R. Morandotti,[3] H. Yang,[4] G. Salamo,[4] M. Sorel,[5] and J. S. Aitchison[6]

[1]*School of Optics/CREOL, University of Central Florida, Orlando, Florida 32816, USA*
[2]*Department of Physics of Complex Systems, The Weizmann Institute of Science, 76100 Rehovot, Israel*
[3]*Institut national de la recherché scientifique, Université du Québec, Varennes, Québec J3X 1S2, Canada*
[4]*Physics Department, University of Arkansas, Fayetteville, Arkansas 72701, USA*
[5]*Department of Electrical and Electronic Engineering, University of Glasgow, Glasgow G12 8QQ, Scotland*
[6]*Department of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario M5S 3G4, Canada*
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We report the first experimental observation of modulation instability in a discrete optical nonlinear array.

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Modulational instability (MI) by which a plane wave breaks up into filaments at high intensities is a ubiquitous process that occurs in many branches of physics. Over the years, MI has been observed in various physical settings, including hydrodynamics[1,2], plasma physics [3], nonlinear optics [4,5], and quite recently in Bose-Einstein condensates [6]. MI is the outcome of the interplay between nonlinearity and dispersive/diffraction effects. It is a symmetry-breaking instability so that a small perturbation on top of a constant amplitude background experiences exponential growth, and this leads to beam breakup in either space or time. Since this disintegration typically occurs in the same parameter region where bright solitons are observed, MI is considered, to some extent, a precursor to soliton formation [7]. In nonlinear optics, MI has been experimentally demonstrated in both the temporal and spatial domain. In particular, temporal MI has been observed in optical fibers [5] as well as its spatial counterpart in nonlinear Kerr [8,9], quadratic [10], and biased photorefractive [11] media with both coherent and partially coherent beams.

In recent years, the behavior of nonlinear discrete systems has received considerable attention in areas such as biology [12], optics [13], solid state physics [14], and Bose-Einstein condensates [15]. The linear properties of this class of systems are strongly modified and as a result their nonlinear response is known to exhibit features that are otherwise impossible in the bulk/continuous regime [16]. In optics [13], arrays of nearest-neighbor coupled nonlinear waveguides have provided a fertile ground where such discrete nonlinear interactions can be experimentally observed and investigated [17–19]. Thus far, discrete spatial solitons (nonlinear eigenstates) have been successfully demonstrated in such arrays in both one-dimensional systems [17,20] as well as in two-dimensional geometries [21].

A fundamental process that is possible in such an array system is that of *discrete MI*. Modulational instability in discrete nonlinear Schrödinger-like lattices was first predicted at the base of the Brillouin zone [13], and, subsequently, this result was generalized to describe this process within the entire first band [22]. The interplay between spatial and temporal effects on the development of MI has also been investigated in detail [23]. In view of the fact that discrete MI can occur in many other physical systems in nature, its experimental observation is of importance.

The existence of MI depends on the relative signs associated with diffraction and nonlinearity. In spatially homogeneous media where the diffraction $\delta = -\partial^2 k_z/\partial k_x^2$ is always positive (normal), MI occurs only for self-focusing nonlinearities. However, in defocusing homogeneous media, beams are stable against symmetry-breaking perturbations. Because diffraction in a discrete 1D array of waveguides can be either positive or negative depending on the excitation angle, both MI and stable propagation are possible in the same array with a nonlinearity of either sign. This is a unique feature of discrete systems.

In this Letter we report the first experimental observation of discrete MI in any physical system. Using an AlGaAs waveguide array with a self-focusing Kerr nonlinearity, we found that discrete MI occurred as long as the spatial Bloch momentum vector of the initial excitation was within the normal diffraction region of the Brillouin zone. The growth rate of this instability was also experimentally determined by providing an additional weak seed wave to produce a modulation on the high intensity beam. On the other hand, in the anomalous diffraction regime, modulational instability was found to be totally absent even at very high power levels. Our observations were found to be in good agreement with theoretical predictions.

In discrete optical nonlinear waveguide arrays, the wave dynamics of the electric field amplitude at waveguide site n is known to obey the following discrete

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nonlinear Schrödinger equation [13,17]:

$$i\frac{da_n}{dz} + C(a_{n+1} + a_{n-1}) + \gamma|a_n|^2 a_n = 0, \quad (1)$$

where a_n represents the mode-field amplitude at the nth-waveguide site, C is the coupling coefficient between adjacent waveguides, and $\gamma = (\pi/\lambda_0)n_0(\varepsilon_0/\mu_0)^{1/2}n_2$ is a measure of the strength of the nonlinearity in the system. In the latter expression, n_0 is the linear refractive index of the waveguides involved, n_2 is the Kerr nonlinear coefficient of the material (in units of m^2/W), and λ_0 is the vacuum wavelength of the light wave used. We address here the experimentally relevant case of self-focusing nonlinearities, i.e., $\gamma > 0$. To theoretically analyze the MI properties of this discrete system, we assume that all waveguides in the array are equally excited with an amplitude q_0. In addition, in our analysis we allow for a phase difference Q between successive waveguides. This phase difference is associated with the spatial momentum Bloch vector of this wave $G_x = Q/D$, where D is the periodicity of the array. Under these conditions, the stationary constant amplitude solution is given by $a_n = q_0 \exp(inQ)\exp[i(2C + \mu)z]$, where without any loss of generality q_0 was taken to be real, and from Eq. (1) the nonlinear eigenvalue is found to be $\mu = \gamma q_0^2 - 4C\sin^2(Q/2)$. The stability properties of this constant amplitude wave (discrete MI) can then be determined by perturbing it by Δ_n [13,22], i.e.,

$$a_n = (q_0 + \Delta_n)\exp(inQ)\exp[i(2C + \mu)z], \quad (2)$$

where we assumed that $|\Delta_n| \ll q_0$, i.e., small perturbations. To further analyze this problem, we write

$$\Delta_n = \varepsilon_1 \exp[i(k_z z - n\theta)] + \varepsilon_2 \exp[-i(k_z z - n\theta)], \quad (3)$$

where $\varepsilon_{1,2}$, k_z are the amplitudes and spatial wave number of this perturbation. In Eq. (3) we assumed that the phase shift (among successive waveguide sites) in this "noise wave" varies by $\theta = k_x D$, where k_x is the associated spatial Bloch momentum of the two sidebands. In this case, a straightforward calculation gives the spatial perturbation wave number [13,22]

$$k_z = \pm\sqrt{8C\cos(Q)\sin^2\left(\frac{\theta}{2}\right)\left[2C\cos(Q)\sin^2\left(\frac{\theta}{2}\right) - \gamma q_0^2\right]} + 2C\sin(Q)\sin(\theta). \quad (4)$$

Equations (3) and (4) clearly indicate that this wave will be MI unstable provided that k_z is complex. Given the form of Eq. (4), then for $\gamma > 0$, the quantity under the square root is negative (k_z is complex) only if $\cos(Q) > 0$. This last result directly implies that discrete MI occurs only when the Bloch momentum lies between $-\pi/2 < Q < \pi/2$, where the array exhibits normal diffraction. On the other hand, no MI is possible in this self-focusing array when $\cos(Q) \leq 0$, i.e., where the array diffraction is anomalous. Unlike in the bulk, where MI always occurs under self-focusing conditions, in discrete systems this same instability is *totally eliminated*, provided that the Bloch momentum is within an anomalous diffraction region. Another aspect where discrete MI differs from its continuous counterpart is the fact that in the region of normal diffraction the spatial frequency θ that experiences maximum gain eventually becomes fixed at $\theta = \pi$ after a certain power threshold—a direct consequence of the limited extent of the allowed band in k_z.

The complete experimental layout is shown in Fig. 1. The light source was a Spectra Physics OPA-800CP, which produced 1.1 ps FWHM pulses at 1550 nm (below half the semiconductor's band gap) with a 1 kHz repetition rate. The signal beam from the OPA was split into two beams. One beam was used as a strong pump wave, and the second was used as a seed wave. Both beams were shaped to generate a highly elliptical beam with 250 μm width and 1.5 μm height at the front surface of the sample. A parallel plate mounted on a rotation stage before the microscope objective was used for tilting both input beams, while a rotating mirror M_1 allowed independent tilting of the weak seed beam in order to alter the modulation period. Note that the tilt angle of the pump is related to the phase-shift coefficient Q (spatial Bloch momentum), whereas the tilt angle of the perturbation field with respect to the pump is related to θ. Both beams were TE polarized. The output of the array was imaged on two cameras, a 2D Hamamatsu vidicon camera and a highly sensitive Roper-Scientific OMA-V InGaAs line camera. In addition, we separately monitored the input and output power on two Germanium detectors that were calibrated against a HP 8163A power meter.

The AlGaAs arrays were 8 and 4 mm long and had coupling constants C of 0.8 and 1.1 mm^{-1}, respectively [17]. In our samples the effective nonlinear coefficient γ for these arrays was estimated to be $\gamma = 5$ m^{-1} W^{-1}. The channel spacing D was 9 μm. For these parameters, the expected MI gain coefficients are shown in Figs. 2(a) and 2(b) for $Q = 0$ and $Q = 0.3\pi$, respectively. The power threshold (thick, solid lines) for a perturbation with spatial frequency θ decreases with increasing phase shift Q

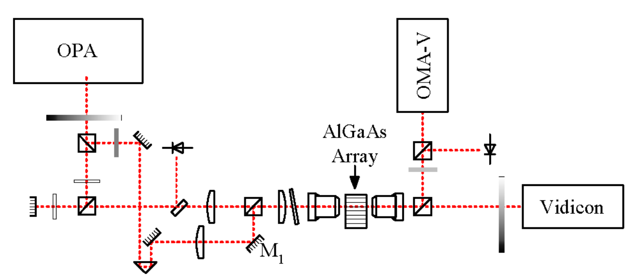

FIG. 1 (color online). Experimental setup.

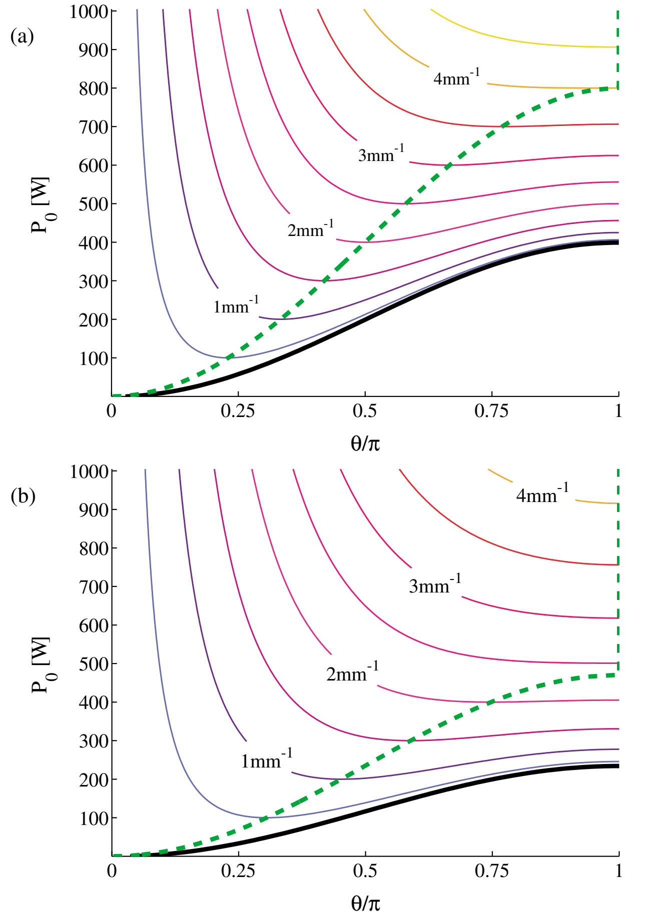

FIG. 2 (color online). MI gain coefficient Im(k_z) versus power P_0 and spatial frequency θ for (a) $Q = 0$ and (b) $Q = 0.3\pi$ and $C = 1\ \mathrm{mm}^{-1}$, $\gamma = 5\ \mathrm{m}^{-1}\,\mathrm{W}^{-1}$. The thick, solid line indicates the threshold power below which the gain is zero for a given phase shift θ; the dashed line shows the location of the maximum MI gain for a given power P_0.

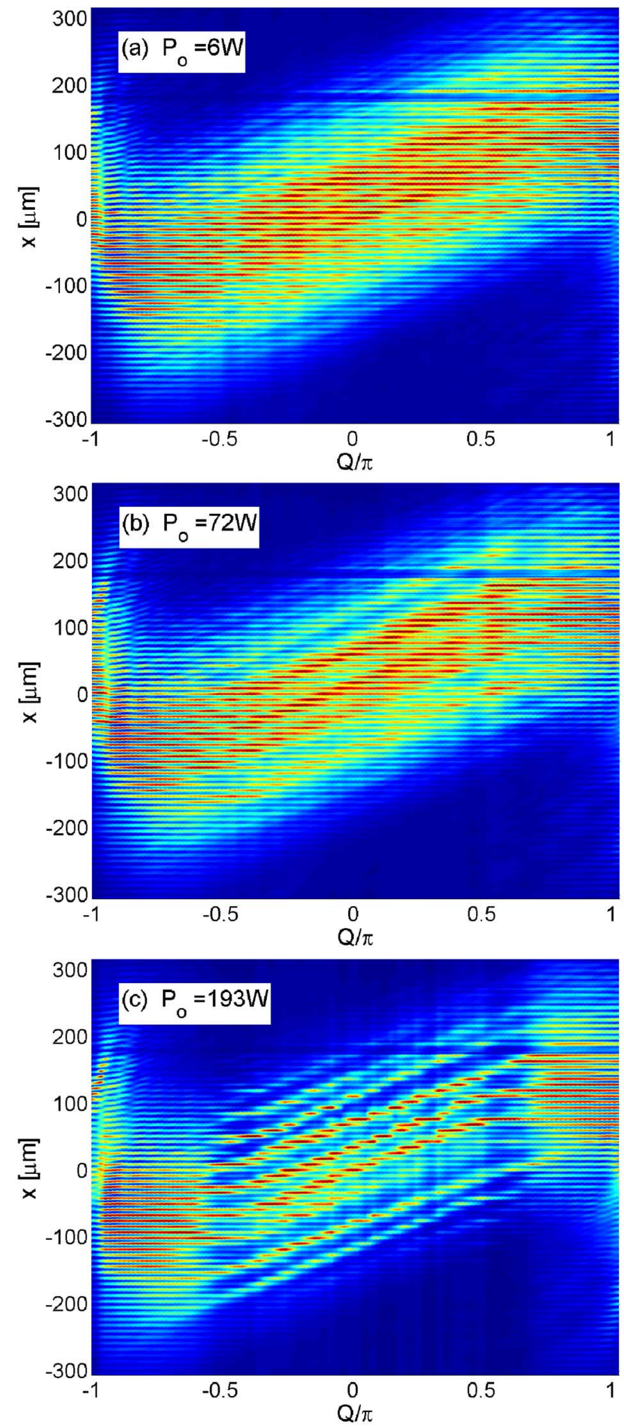

FIG. 3 (color online). Observed output intensity distribution versus phase difference Q for 6, 72, and 193 W.

between adjacent channels. For a fixed power $P_0 = \frac{1}{2} n_0 \sqrt{\varepsilon_0/\mu_0} A_{\mathrm{eff}} q_0^2$, tilting the strong wave leads to an increased gain bandwidth; however, the maximum gain possible decreases. The maximum gain (dashed lines) increases with power per channel P_0 and spatial frequency θ. At some critical value $q_0^2 = 4C/\gamma \cos Q$, maximum gain only occurs at $\theta = \pi$.

We first demonstrate the dependence of the MI gain on the propagation angle of the pump wave in the 8 mm long sample. In this experiment we used a combination of the inherent noise on the pump beam and imperfections in the sample to seed the MI. Figures 3(a)–3(c) show the observed output patterns for angles between $\pm\pi$ for three different power levels. At the lowest power of 6 W (estimated peak power in the central waveguide), we observe no MI, but only small beam distortions due to "noise." At intermediate powers of 72 W, these distortions start to grow for angles where the array exhibits normal diffraction, whereas in the negative diffraction region, the beam shows no sign of any instabilities. Note that the actual band structure of this AlGaAs array does not exactly follow the coupled mode cosine behavior as recently discussed in [24]. At the highest shown input power level of 193 W, we observe the breakup of the beam into filaments, highly localized in a few channels.

In order to evaluate the MI gain, we seeded the MI with a second, weak beam and measured its growth with

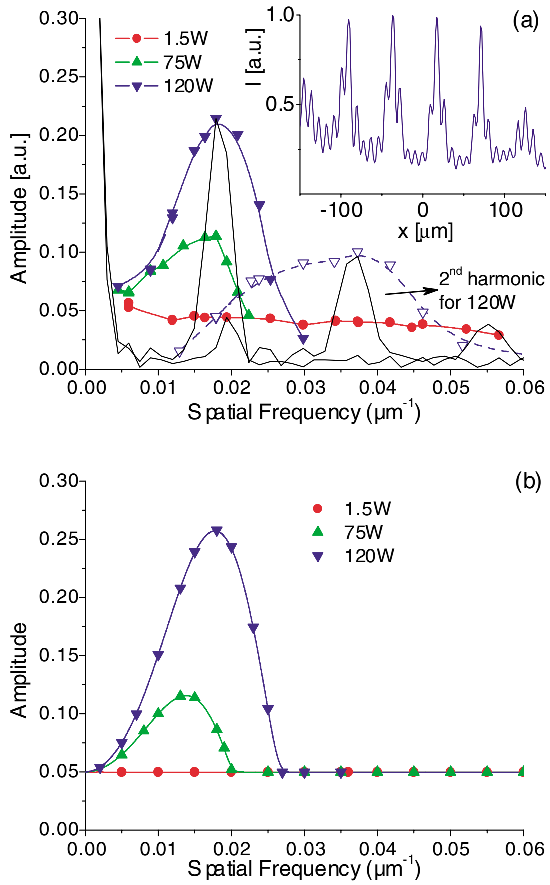

FIG. 4 (color online). Dependence of the amplified seed wave at the output on spatial frequency θ and power P_0. (a) shows the experimental data and (b) the theoretical prediction. The inset of (a) shows the output spatial intensity distribution at 120 W input pump power at the peak of the gain curve.

increasing input intensity. The amplitude of the weak wave was about 5% of the strong wave. The 4 mm long sample with the higher interchannel coupling constant and smaller propagation length (and hence smaller net gain relative to the 8 mm sample) allowed the transition from MI to beam breakup to be accurately quantified. The power distribution at the end of the sample was measured and Fourier transformed. The thin lines in Fig. 4(a) show examples of the Fourier spectra obtained at different powers at $Q = 0.3\pi$. For a power of 120 W, the beam breaks up into a sequence of highly localized filaments, shown in the inset of Fig. 4(a). The thick lines show the magnitude of the amplified seed wave and its second spatial harmonic for powers ranging from 1.5 to 120 W and seed frequencies between 0 and 0.06 $\mu\mathrm{m}^{-1}$ $(0\ldots1.08\pi)$. Above 90 W, the second spatial harmonic associated with the saturation of the MI becomes clearly visible. Figure 4(b) shows the magnitude of the amplified seed wave based on Eqs. (2)–(4) for $Q = 0.3\pi$, $C = 1.1\ \mathrm{mm}^{-1}$, and $\gamma = 5\ \mathrm{m}^{-1}\,\mathrm{W}^{-1}$. There is good agreement between the calculated and measured output magnitudes up to power levels of 75 W, beyond which MI saturation occurs, and the small signal MI theory is no longer valid

In summary, we have reported the first experimental observation of modulational instability in a discrete system. We demonstrated MI and self-localization in positive (normal) and stable beams in the negative (anomalous) diffraction regions.

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