Traces of Thermalization from p_t Fluctuations in Nuclear Collisions

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(Received 26 August 2003; published 21 April 2004)

Scattering of particles produced in high energy nuclear collisions can wrestle the system into a state near local thermal equilibrium. I illustrate how measurements of the centrality dependence of the mean transverse momentum and its fluctuations can exhibit this thermalization.

DOI: 10.1103/PhysRevLett.92.162301 PACS numbers: 25.75.Ld, 24.60.Ky

Fluctuations of the net transverse momentum have recently been measured, with the STAR, PHENIX, NA49, and CERES experiments reporting substantial dynamic contributions [1–4]. Such fluctuations can provide information on collision dynamics and, perhaps, the QCD phase transition [5,6]. Preliminary PHENIX and STAR data in Au + Au collisions show that p_t fluctuations increase as centrality increases [1,2]. Importantly, data from these same experiments exhibit a strikingly similar increase in the mean transverse momentum $\langle p_t \rangle$, a quantity unaffected by fluctuations [7,8].

I ask whether the approach to local thermal equilibrium can explain the similar centrality dependence of $\langle p_t \rangle$ and p_t fluctuations. My focus is on fluctuations, to develop the appropriate theoretical tools and experimental observables. Dynamic fluctuations are characterized by the observable $\langle \delta p_{t1} \delta p_{t2} \rangle$ analyzed by STAR [1], where it is termed $\sigma^2_{\langle p_t \rangle,\,\mathrm{dynam}}$, and CERES [3]. For particles of momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, one defines

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \int d\mathbf{p}_1\, d\mathbf{p}_2 \frac{\rho_2(\mathbf{p}_1, \mathbf{p}_2)}{\langle N(N-1) \rangle} \delta p_{t1}\, \delta p_{t2}, \qquad (1)$$

where $\delta p_{ti} = p_{ti} - \langle p_t \rangle$, $\langle \cdots \rangle$ is the average over events, and $d\mathbf{p} \equiv dy d^2 p_t$. This definition exploits the relation of event-by-event fluctuations to inclusive correlation functions discussed in [9]. The pair distribution is

$$\rho_2(\mathbf{p}_1, \mathbf{p}_2) = dN/d\mathbf{p}_1 d\mathbf{p}_2, \qquad (2)$$

where $\int \rho_2 d\mathbf{p}_1 d\mathbf{p}_2 = \langle N(N-1) \rangle$ for multiplicity N [9]. Observe that (1) depends only on the two-body correlation function

$$r(\mathbf{p}_1, \mathbf{p}_2) = \rho_2(\mathbf{p}_1, \mathbf{p}_2) - \rho_1(\mathbf{p}_1)\rho_1(\mathbf{p}_2) \qquad (3)$$

with $\rho_1(\mathbf{p}) = dN/d\mathbf{p}$, since the integral over $\rho_1 \rho_1$ vanishes due to the definition of δp_t. Alternative fluctuation observables Φ_{p_t}, F_{p_t}, and $\Delta\sigma_{p_t}$ proposed in [1,2,5] measure many-body correlations of all orders. These quantities are roughly equivalent:

$$F_{p_t} \approx \Phi_{p_t}/\sigma \approx \Delta\sigma_{p_t}/\sigma \approx N \langle \delta p_{t1} \delta p_{t2} \rangle / 2\sigma^2, \qquad (4)$$

when dynamic fluctuations are small compared to statistical fluctuations $\sigma^2 = \langle p_t^2 \rangle - \langle p_t \rangle^2$ [2,10].

STAR measurements of p_t fluctuations in Fig. 1(a) show an increase for low multiplicities corresponding to peripheral collisions at $s^{1/2} = 130$ GeV [1]. PHENIX measurements at 200 GeV in Fig. 1(b) also show such an increase for F_{p_t} [2]. The increase appears to peak and possibly saturate for multiplicities corresponding to mid-peripheral impact parameters. In addition, the data may show a decrease for $\Delta\sigma_{p_t}$ and F_{p_t} for the most central collisions. While these measurements are preliminary and bear large uncertainties, this centrality dependence has already been attributed to phenomena associated with the QCD transition [11,12].

I attribute the trend in Fig. 1 to the onset of thermalization in increasingly central collisions, motivated by a similar behavior of the measured $\langle p_t \rangle$ in Fig. 2 [7,8]. Thermalization occurs as scattering between particles produced in the collision drives the system toward local thermal equilibrium. The system is characterized by a phase-space density $f(\mathbf{x}, \mathbf{p}, t)$ that varies from

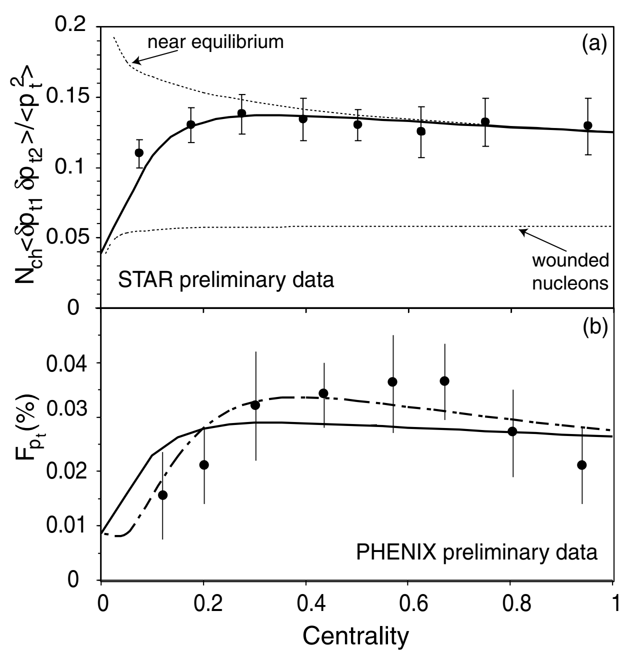

FIG. 1. (a) Dynamic p_t fluctuations computed using (6) compared to STAR data [1]. (b) Same for PHENIX data [2].

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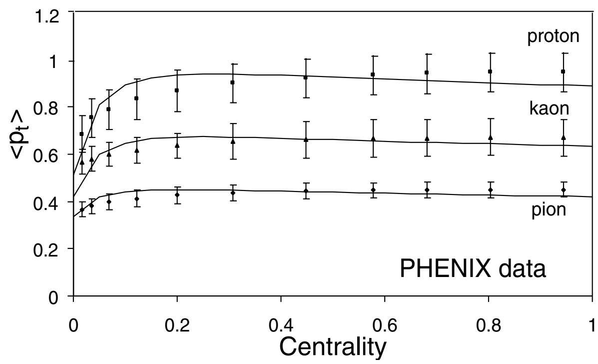

FIG. 2. Average p_t from (5) compared to data [8].

collision event to event. As the system approaches local equilibrium, the event-averaged $\langle f \rangle$ tends toward the Boltzmann-like distribution $\langle f^e \rangle$ that varies in spacetime through the temperature $T(\mathbf{x}, t)$. I show here that thermalization alters the average transverse momentum following

$$\langle p_t \rangle = \langle p_t \rangle_o S + \langle p_t \rangle_e (1 - S), \quad (5)$$

where S is the probability that a particle escapes the collision volume without scattering. Dynamic fluctuations depend on two-body correlations and, correspondingly, are described by

$$\langle \delta p_{t1} \delta p_{t2} \rangle = \langle \delta p_{t1} \delta p_{t2} \rangle_o S^2 + \langle \delta p_{t1} \delta p_{t2} \rangle_e (1 - S)^2. \quad (6)$$

The initial quantities $\langle p_t \rangle_o$ and $\langle \delta p_{t1} \delta p_{t2} \rangle_o$ are determined by the particle production mechanism, while $\langle p_t \rangle_e$ and $\langle \delta p_{t1} \delta p_{t2} \rangle_e$ depend on the state of the system near local equilibrium.

To understand how thermalization can cause the common trends in Figs. 1 and 2, observe that as centrality is increased the system lifetime increases, eventually to a point where local equilibrium is reached. Consequently, the survival probability S in (5) and (6) decreases from unity as the impact parameter decreases. Both (5) and (6) peak for impact parameters near the point where equilibrium is established. The behavior in collisions at centralities beyond that point depends on how subsequent hydrodynamic evolution changes $\langle p_t \rangle_e$ and $\langle \delta p_{t1} \delta p_{t2} \rangle_e$ as the system size and lifetime increase. Systems formed in the most central collisions can experience cooling that reduces (5) and (6).

For both the average p_t and its fluctuations to *increase* during thermalization as in Figs. 1 and 2, both $\langle p_t \rangle_e$ and $\langle \delta p_{t1} \delta p_{t2} \rangle_e$ must exceed the initial values. For the average transverse momentum, this implies that the temperature T at thermalization must be quite high, since $\langle p_t \rangle_e \propto T$. A value $\langle p_t \rangle_o \approx 350$ MeV near that measured in pp collisions implies $T \sim 400$ MeV, suggesting that partons contribute to thermalization.

In the following paragraphs, I estimate $\langle \delta p_{t1} \delta p_{t2} \rangle_o$ and $\langle \delta p_{t1} \delta p_{t2} \rangle_e$. Next, I formulate a nonequilibrium approach capable of treating fluctuations based on the Boltzmann-Langevin equation in the relaxation-time approximation. Here, I sketch the derivation of (5) and (6), leaving the details for a longer paper.

Transverse momentum and particle density fluctuations arise partly due to the particle production mechanism, e.g., string fragmentation. These fluctuations were measured in proton-proton (pp) collisions. To use these pp results to estimate $\langle \delta p_{t1} \delta p_{t2} \rangle_o$ for nuclear collisions, I apply the wounded nucleon model to describe the soft production that dominates $\langle p_t \rangle$ and $\langle \delta p_{t1} \delta p_{t2} \rangle$. The charged particle multiplicity N and other extensive quantities are assumed to scale linearly with the number of participant nucleons M, while the intensive one-body observable $\langle p_t \rangle$ is independent of M. Centrality is determined by $N/N_{\max} \approx M(b)/M(0)$ for impact parameter b, averaged over collision geometry.

To estimate the initial $\langle \delta p_{t1} \delta p_{t2} \rangle$ using the wounded nucleon model, I follow the appendix in Ref. [9] to obtain

$$\langle \delta p_{t1} \delta p_{t2} \rangle_o = \frac{2 \langle \delta p_{t1} \delta p_{t2} \rangle_{pp}}{M} \left(\frac{1 + R_{pp}}{1 + R_{AA}} \right). \quad (7)$$

The term outside the parentheses is expected because (1) measures relative fluctuations and, therefore, should scale as M^{-1}; note that pp collisions have two participants. The term in parentheses accounts for the normalization of (1) to $\langle N(N-1) \rangle \equiv \langle N \rangle^2 (1 + R_{AA})$ rather than $\langle N \rangle^2$. From [9], the robust variance R_{AA} satisfies

$$R_{AA} = \int d\mathbf{p}_1 \, d\mathbf{p}_2 \frac{r(\mathbf{p}_1, \mathbf{p}_2)}{\langle N \rangle^2} = \frac{\langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle}{\langle N \rangle^2}, \quad (8)$$

and scales as $R_{AA} \propto M^{-1}$. ISR measurements imply $\langle \delta p_{t1} \delta p_{t2} \rangle_{pp} / \langle p_t \rangle_{pp}^2 \approx 0.015$ [13]. HIJING gives $R_{pp} \sim 0.45$ and $R_{AA} \sim 0.0037$ for central Au + Au for the rapidity interval $\Delta\eta = 1.5$ studied in [1]. To compare (7) to $N \langle \delta p_{t1} \delta p_{t2} \rangle / \langle p_t \rangle^2$ in Fig. 1(a), I assume central collisions produce $N \approx 825$ charged particles in $\Delta\eta = 1.5$; i.e., $dN/d\eta \approx 550$.

Near local thermal equilibrium, dynamic fluctuations occur because initial state fluctuations result in transient spatial inhomogeneity that can survive thermalization. The inhomogeneity would eventually disappear due to diffusion and viscosity, but can be observed if freeze-out is sufficiently rapid. Inhomogeneity is essential for dynamic fluctuations, since $\langle \delta p_{t1} \delta p_{t2} \rangle$ and Φ_{p_t} would otherwise vanish for $\rho_2 = \rho_1 \rho_1$.

To see how inhomogeneity can survive thermalization, observe that local equilibrium is achieved when the average phase-space distribution of particles within a small fluid cell $\langle f \rangle$ relaxes to the local equilibrium form $\langle f^e \rangle$. The time scale for this process is the relaxation time ν^{-1} discussed later. In contrast, density differences *between* cells must be dispersed by transport from cell to cell. The time needed for diffusion to disperse a dense fluid mass of size $L \sim (|\nabla n|/n)^{-1}$ is $t_d \sim \nu L^2 / v_{\text{th}}^2$, where $v_{\text{th}} \sim 1$ is the thermal speed of particles. This time can be much larger than ν^{-1} for a sufficiently large fluid mass. The rapid expansion of the collision system further prevents inhomogeneity from being dispersed prior to freeze-out.

Inhomogeneity produces spatial correlations: It is more likely to find particles together near a dense fluid mass. These spatial correlations entirely determine the phase-space correlations when the momentum distribution at each point is thermal. I write

$$r(\mathbf{p}_1, \mathbf{p}_2) = \int d\mathbf{x}_1\, d\mathbf{x}_2\, \mathcal{P}(\mathbf{x}_1, \mathbf{p}_1, \mathbf{x}_2, \mathbf{p}_2, t) \tag{9}$$

evaluated at the freeze-out proper time τ_F, where the phase-space correlation function is

$$\mathcal{P}_{12} \equiv \langle f_1 f_2\rangle - \langle f_1\rangle\langle f_2\rangle - \delta_{12}\langle f_1\rangle, \tag{10}$$

for $\delta_{12} = \delta(\mathbf{x}_1 - \mathbf{x}_2)\delta(\mathbf{p}_1 - \mathbf{p}_2)$. A small change in density δn will initially drive the system from equilibrium by an amount $\delta f^e = f^e \delta n/n$. The corresponding phase-space correlations are described near equilibrium by

$$\mathcal{P}^e_{12} = \frac{\langle f^e_1\rangle\langle f^e_2\rangle}{\langle n_1\rangle\langle n_2\rangle} r(\mathbf{x}_1, \mathbf{x}_2), \tag{11}$$

where the spatial correlation function is

$$r(\mathbf{x}_1, \mathbf{x}_2) \equiv \langle n_1 n_2\rangle - \langle n_1\rangle\langle n_2\rangle - \delta_{12}\langle n_1\rangle. \tag{12}$$

The form (11) ensures that both $\mathcal{P}^e_{12}$ and $r(\mathbf{x}_1, \mathbf{x}_2)$ vanish in global equilibrium, where particle number fluctuations obey Poisson statistics. I use (1) and (9)–(12) to find

$$\langle \delta p_{t1}\, \delta p_{t2}\rangle_e = \int d\mathbf{x}_1\, d\mathbf{x}_2\, r(\mathbf{x}_1, \mathbf{x}_2) \frac{\overline{\delta p_t}(\mathbf{x}_1)\, \overline{\delta p_t}(\mathbf{x}_2)}{\langle N(N-1)\rangle}, \tag{13}$$

where the local transverse momentum excess, $\overline{\delta p_t}(\mathbf{x}) = \int d\mathbf{p}(p_t - \langle p_t\rangle) f(\mathbf{x}, \mathbf{p})/n(\mathbf{x})$, vanishes if the collision volume is uniform.

To estimate $\langle \delta p_{t1} \delta p_{t2}\rangle_e$ using (13), I assume that Bjorken scaling holds and that longitudinal and transverse degrees of freedom are independent. I then write the transverse coordinate dependence of (12) as

$$r(\mathbf{x}_1, \mathbf{x}_2) \propto g(r_{t1}) g(r_{t2}) c(|\mathbf{r}_{t1} - \mathbf{r}_{t2}|), \tag{14}$$

where the density is $n(x_1) \propto g(r_t)$. I parametrize g and c to be Gaussian with root-mean-square widths R_t and ξ, respectively, the transverse radius and correlation length. The momentum excess $\overline{\delta p_t}(\mathbf{x})$ in (13) depends on the temperature profile of the system, since $\int p_t f(\mathbf{x}, \mathbf{p})\, d\mathbf{p}/n(\mathbf{x}) \propto T(r_t)$. Similarly, $\langle p_t\rangle \propto \|T\|$, for the density-weighted average $\|T\| \equiv \int g(\mathbf{r}_t) T(\mathbf{r}_t)\, d\mathbf{r}_t$, so that

$$\overline{\delta p_t}(\mathbf{r}_t) = \langle p_t\rangle[\hat{T}(r_t) - 1]. \tag{15}$$

I parametrize $\hat{T}(r_t) = T(r_t)/\|T\|$ as Gaussian of width R_p and use $n \propto T^3$ to fix $R_p = \sqrt{3} R_t$.

The dynamic p_t fluctuations near local equilibrium then satisfy

$$\langle \delta p_{t1} \delta p_{t2}\rangle_e = F \frac{\langle p_t\rangle^2 R_{AA}}{1 + R_{AA}}, \tag{16}$$

where R_{AA} is given by (8). The quantity F is dimensionless and depends on the ratio of the correlation length ξ_t to the transverse size R_t. I use (14) and (15) to compute

$$F = \|c(|\mathbf{r}_{t1} - \mathbf{r}_{t2}|)[\hat{T}(\mathbf{r}_{t1}) - 1][\hat{T}(\mathbf{r}_{t2}) - 1]\|, \tag{17}$$

a double density-weighted average over $\mathbf{r}_{t1}$ and $\mathbf{r}_{t2}$. I find $F = 0.046$ for $\xi_t/R_t = 1/6$. To determine (16) for Fig. 1(a), I take $R_{AA} = 0.0037$ and $N \approx 825$ as before. I emphasize that the HIJING R_{AA} value builds in fluctuations from resonance decay and, moreover, is roughly consistent with measured net charge fluctuations [14].

Let us now describe the fast local relaxation of the phase-space density f to f^e. I start with a Boltzmann-like kinetic equation,

$$\partial f/\partial t + \mathbf{v_p} \cdot \nabla f = I[f] \approx -\nu(f - f^e), \tag{18}$$

approximating the collision term $I[f]$ using a single relaxation time ν^{-1}. Following [15,16], I use longitudinal boost invariance to write the left side of (18) as $df/d\tau$ at fixed $p_z\tau$. Longitudinal expansion further implies that the density satisfies $n(\tau) \propto \tau^{-1}$, while $\langle p_t\rangle_e \propto T \propto \tau^{-\gamma}$ for $0 < \gamma < 1/3$; see [16]. I then multiply both sides of (18) by $|\mathbf{p}_t|$ and integrate over momentum to obtain

$$\langle p_t\rangle = \langle p_t\rangle_o S + \frac{\alpha\langle p_t\rangle^0_e}{\alpha - \gamma}[S^{\gamma/\alpha} - S]. \tag{19}$$

The survival probability is

$$S = e^{-\int_{\tau_0}^{\tau_F} \nu(\tau) d\tau} \approx (\tau_0/\tau_F)^{\alpha}, \tag{20}$$

where $\nu = \langle \sigma v_{\rm rel}\rangle n(\tau)$, $\alpha = \nu_0 \tau_0$ for the formation time τ_0, the scattering cross section is σ, and $v_{\rm rel}$ is the relative velocity. For relevant values $\alpha \gg \gamma$, I approximate (19) by (5) with $\langle p_t\rangle_e \approx \langle p_t\rangle^0_e (\tau_0/\tau)^{\gamma}$.

To compute the evolution of $\langle \delta p_{t1} \delta p_{t2}\rangle$, I obtain relaxation equations for $\mathcal{P}$. Fluctuations due to scattering and drift are described by adding a Langevin force to the right side of (18) [17]. On a discrete phase-space lattice $\mathbf{p}_i, \mathbf{x}_i$, the Boltzmann-Langevin equation is

$$df_i/d\tau = -\nu(f_i - f^e_i) + \zeta_i, \tag{21}$$

where $\zeta_i(\tau)$ is a Langevin force. To incorporate the effect of fluctuations near local equilibrium, I further treat f^e_i as a stochastic variable subject to an additional Langevin force, so that $df^e_i/d\tau = \chi_i$, plus a diffusive relaxation term that I need not specify for a diffusion time scale $t_{\rm d} \gg \nu^{-1}$. The Langevin terms satisfy $\langle \zeta_i(\tau)\zeta_j(\tau')\rangle = \nu(f + f^e)\delta_{ij}\delta(\tau - \tau')$ and $\langle \zeta_i(\tau)\chi_j(\tau')\rangle = -\nu f^e \delta_{ij}\delta(\tau - \tau')$, as required by detailed balance for the relaxation-time collision term [17]. The Boltzmann equation used to compute $\langle p_t\rangle$ is the mean value of (21).

I use standard methods [17] to obtain the following two-body relaxation equations:

$$d\mathcal{P}_{ij}/d\tau = -2\nu\mathcal{P}_{ij} + \nu(C_{ij} + C_{ji}), \quad (22)$$

$$dC_{ij}/d\tau = -\nu C_{ij} + \nu\mathcal{P}^{e}_{ij}, \quad (23)$$

where I introduce the auxiliary function $C_{ij} \equiv \langle f_i f^e_j \rangle - \langle f_i \rangle \langle f^e_j \rangle$. Observe that $\mathcal{P}_{ij} = C_{ij} = 0$ in global equilibrium where the time derivatives vanish. I solve (22) and (23) assuming that C_{ij} initially vanishes and obtain $\langle \delta p_{t1} \delta p_{t2} \rangle$ from (1), (3), and (9). Equation (6) follows, but is exact only if one neglects the time dependence of (16) implied by $\langle p_t \rangle_e \propto \tau^{-\gamma}$. For $\alpha \gg \gamma$, I approximate this dependence in (6) by taking $\langle \delta p_{t1} \delta p_{t2} \rangle_e \propto \tau^{-2\gamma}$.

I now fit this transport framework together with my earlier assertion that near-equilibrium p_t correlations are induced by spatial inhomogeneity, i.e., Eq. (11). In the relaxation-time approximation $\mathcal{P}^e$ is arbitrary, as is f^e. To deduce either from transport theory, one must use (21) with the full collision term $I[f]$. Following [17] yields $\mathcal{P}^e_{12} = \langle f^e_1 \rangle \langle f^e_2 \rangle \theta$, where $\theta = a_{12} + \sum_{\mu\nu} b^{\mu\nu}_{12} p^{\mu}_1 p^{\nu}_2$, for a and $b^{\mu\nu}$ functions of x_1 and x_2. In a uniform system these coefficients are constant, so that (1), (3), and (9) imply $\langle \delta p_{t1} \delta p_{t2} \rangle \equiv 0$, confirming our intuition. Our physically motivated (11) takes $\theta \approx r(x_1, x_2)/n(x_1)n(x_2)$, which is adequate for our estimate (16).

Calculations in Figs. 1 and 2 illustrate the common effect of thermalization on one-body and two-body p_t observables. Equation (6), together with the computed (7) and (13), is in good accord with data. The solid curves in all figures are fit to STAR fluctuation data and $\langle p_t \rangle$ data (except for N, I ignore any energy dependence). I assume $\alpha = 4$ and $\gamma = 0.15$ in central collisions, and parametrize $S(M)$ by taking $\alpha \propto M^{1/3}$ and $\tau_F - \tau_0 \propto M^{1/2}$. In this Letter, it is not necessary to specify whether the equilibrating system is partonic or hadronic. That said, in Fig. 2 I take the same α for all species, as appropriate for parton scattering. Measurements of p_t fluctuations for identified particles can further test whether thermalization is species independent.

In comparing to PHENIX data in Fig. 1(b), note that the magnitude difference with Fig. 1(a) follows from the different acceptance of STAR and PHENIX. The solid curve in Fig. 1(b) agrees with the data within the uncertainty, but the dashed curve shows better agreement for $\gamma = 0.2$ and $\tau_F - \tau_0 \propto M$. While agreement with $\langle p_t \rangle$ data for the new parameters is less compelling than Fig. 2, results still fall within the uncertainty.

Preliminary data from Refs. [1,2,7,8] show tantalizing similarity to the calculations. However, experimental uncertainty must be reduced to firmly establish the low multiplicity rise as well as the behavior at high multiplicity. Contributions to $\langle p_t \rangle$ and $\langle \delta p_{t1} \delta p_{t2} \rangle$ not included in this exploratory work are diffusion, collective radial flow, Bose-Einstein (HBT) correlations, and collective hadronization. Collective effects can be important in central collisions, where the matter evolves after equilibration. Flow can enhance the fluctuations, while diffusion can reduce them. HBT effects can be experimentally estimated by cutting on each pair's relative momentum. This contribution is of order 10% at RHIC energy [18] but may be larger at lower energies [3]. While resonance and hard-scattering contributions to fluctuations are estimated by taking R_{AA} in (7) and (16) from HIJING, chemical equilibration may modify the centrality dependence for resonance production, altering $\langle \delta p_{t1} \delta p_{t2} \rangle$.

Experimental indications that nuclear collisions produce matter near local equilibrium are scant and circumstantial. Any experimental evidence of the *onset* of equilibrium — particularly at the parton level — will validate those indications. Rapidity dependence measurements can distinguish the thermalization effects proposed here from alternative explanations [11,12]. Here, the rapidity dependence arises from the dependence of (7) and (16) on R_{AA}, which is itself measurable [9].

I thank C. Pruneau, S. Voloshin, and G. Westfall for discussions. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-92ER40713.

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