

Ambiguities in the Up-Quark Mass

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It has long been known that no physical singularity is encountered as up-quark mass is adjusted from small positive to negative values as long as all other quarks remain massive. This is tied to an additive ambiguity in the definition of the quark mass. This calls into question the acceptability of attempts to solve the strong CP problem via a vanishing mass for the lightest quark.

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The standard $SU(3)$ non-Abelian gauge theory of the strong interactions is quite remarkable in that, once an arbitrary overall scale is fixed, the only parameters are the quark masses. Using a few pseudoscalar meson masses to fix these parameters, the non-Abelian gauge theory describing quark confining dynamics is unique.

When multiple quark masses vanish, the theory acquires exact chiral symmetries. These manifest themselves through Goldstone bosons arising from spontaneous symmetry breaking in the vacuum. As the physical light quarks do have masses, these symmetries are only approximate, but the pions are believed to be the remnants of this Goldstone boson structure.

The case where only one of the quarks is massless is particularly interesting in that anomalies break all chiral symmetries. A mass gap is dynamically generated, no Goldstone bosons are expected, and no singularities occur as the mass passes through zero. Because of the anomalies, negative and positive quark masses are not equivalent. Indeed, when a single quark mass is made sufficiently negative a spontaneous breakdown of parity will appear [1–3].

The smooth behavior around small quark masses raises the question of how to define the quark mass. Because of confinement, it is not obvious how experiments involving only physical particles can determine whether a nondegenerate quark mass vanishes. If this cannot be done, then the concept of a massless quark is unphysical.

Whether the lightest quark is massless might be regarded as an academic point. Phenomenologically, despite ambiguities in the chiral Lagrangian [4], it appears that a massless up-quark seems untenable [5,6]. Nevertheless, even in principle the mass is just some parameter adjusted to give the correct long distance physics. From this point of view a vanishing value is just a point in parameter space, as good as any other. But the issue gains significance when a special value of the parameter is used to solve a more complex problem. This is the case with the up-quark mass, which continues to be proposed as a possible solution to what is known as the strong CP problem [7,8].

The purpose of this Letter is to emphasize how confinement and chiral anomalies make it unclear whether the

concept of a vanishing m_u is well posed. Reference [8] raises some of these issues, pursuing $m_u = 0$ anyway as an accidental symmetry. A preliminary unpublished version of these arguments is contained in Ref. [9].

Because renormalization is required, the concept of an “underlying basic Lagrangian” does not exist. The continuum theory is specified in terms of basic symmetries and a few renormalized parameters. In practice, the definition of a field theory relies on a limiting process from a cutoff version. As the lattice is the best understood non-perturbative cutoff, it provides the most natural framework for such a definition. Thus I denote my cutoff parameter as a , representing a lattice spacing or minimum length.

This is only a notational issue. Any regulator must accommodate the known chiral anomalies, and thus chiral symmetry breaking terms of some form must appear in the cutoff theory. These effects can come in many guises. With a Pauli-Villars scheme, there is a heavy regulator field. With dimensional regularization the anomaly is hidden in the fermionic measure. For Wilson lattice gauge theory there is the famous Wilson term. With domain wall fermions there is a residual mass from a finite fifth dimension. With overlap fermions things are hidden in a combination of the measure and a certain nonuniqueness of the operator. A lattice regulator also introduces a dimensionful parameter, the lattice spacing a . This feature also is not special to the lattice. The scale anomaly, through the phenomenon known as “dimensional transmutation” [10], is responsible for masses of hadrons such as the proton and glueballs, even in the massless quark limit. For such physics, any complete regulator must introduce a scale.

For the issue being raised here, the heavier quarks play no crucial role. Thus I imagine them to be “integrated out” and consider the theory reduced to a single flavor. This allows me to consider only two bare parameters, the coupling and quark mass. Including the other quarks is straightforward, but unnecessarily complicates the equations.

The renormalization process tunes all relevant bare parameters as a function of the cutoff while fixing a set of renormalized quantities. As I need to renormalize both

the bare coupling and quark mass, I need to fix two physical observables. For this purpose I choose the lightest boson and the lightest baryon masses. As both are expected to be stable, this precludes any ambiguity from particle widths. From its roots in the multiflavor theory, I denote the lightest boson the π , and the lightest baryon as p . Because of confinement, the values of their masses are inherently nonperturbative quantities.

With the cutoff in place, the physical masses are functions of (g, m, a) , the bare charge, the bare coupling, and the cutoff. Holding the masses constant, the renormalization process determines how g and m flow as the cutoff is removed. Because of asymptotic freedom, this flow eventually enters the perturbative regime and we have the famous renormalization group equations [11]

$$a \frac{dg}{da} \equiv \beta(g) = \beta_0 g^3 + \beta_1 g^5 + \dots, \quad (1)$$

$$a \frac{dm}{da} \equiv m\gamma(g, m) = m(\gamma_0 g^2 + \gamma_1 g^4 + \dots) + \text{nonperturbative}. \quad (2)$$

The “nonperturbative” term should vanish faster than any power of the coupling. I include it explicitly in the mass flow because it will play a crucial role in the latter discussion. The values for the first few coefficients β_0 , β_1 , and γ_0 are known [12] and independent of regularization scheme.

The solution to these equations shows how the bare coupling and bare mass are driven to zero as the cutoff is removed

$$a = \frac{1}{\Lambda} e^{-1/2\beta_0 g^2} g^{-\beta_1/\beta_0^2} [1 + O(g^2)], \quad (3)$$

$$m = M g^{\gamma_0/\beta_0} [1 + O(g^2)]. \quad (4)$$

The quantities Λ and M are integration constants for the renormalization group equations. I refer to Λ as the overall strong interaction scale and M as the renormalized quark mass. Their values depend on the explicit renormalization scheme as well as the physical values being held fixed in the renormalization process, i.e., the proton and pion masses. This connection is highly nonperturbative. Indeed particle masses are long distance properties, and thus require following the renormalization group flow far out of the perturbative regime.

Turning things around, we can consider the physical particle masses to be functions of these integration constants. Simple dimensional analysis tells us that the dependence of physical masses must take the form

$$m_p = \Lambda f_p(M/\Lambda), \quad (5)$$

$$m_\pi = \Lambda f_\pi(M/\Lambda), \quad (6)$$

where the $f_i(x)$ are dimensionless functions whose detailed form is highly nonperturbative.

For the case of degenerate quarks we expect the square of the pion mass to vanish linearly as the renormalized quark mass goes to zero. This means we anticipate a square root singularity in $f_\pi(x)$ at $x = 0$. Indeed, requiring the singularity to occur at the origin removes any additive nonperturbative ambiguity in defining the renormalized mass.

With a nondegenerate quark, things are more subtle. As discussed above, we expect physics to behave smoothly as the quark mass passes through zero. That is, we do not expect $f_i(x)$ to display any singularity at $x = 0$. Nonperturbative dynamics generates an additional contribution to the mass of the pseudoscalar meson, thus the $M = 0$ flow generically corresponds to a positive value of m_π . While a $m_\pi = 0$ flow line can exist, it represents the boundary of a CP violating phase and has little to do with massless quarks.

Now I come to the question of scheme dependence. Given some different renormalization prescription, i.e., a modified lattice action, the precise flows will change. Although the behavior dictated in Eqs. (3) and (4) will be preserved, the integration constants (Λ, M) and the function $f(x)$ will in general be modified. Marking the new quantities with tildes, matching the schemes to give the same physics requires

$$m_i = \Lambda f_i(M/\Lambda) = \tilde{\Lambda} \tilde{f}_i(\tilde{M}/\tilde{\Lambda}). \quad (7)$$

Upon the removal of the cutoff, two different valid cutoff schemes should give the same result for the physical masses. If the concept of a massless quark has physical meaning, this means that $M = 0$ should match with $\tilde{M} = 0$. Otherwise the continuum limit in one scheme for the massless quark theory would correspond to the continuum limit taken in another scheme where the quark mass is not zero. The issue raised in this Letter is the absence of any known reason for the vanishing of M to require the vanishing of \tilde{M} .

On changing schemes, we introduce new definitions for the coupling and mass. To match onto the perturbative limit, it is reasonable to restrict these definitions to agree at leading order. Thus I require

$$\tilde{g} = g + O(g^3), \quad (8)$$

$$\tilde{m} = m[1 + O(g^2)] + \text{nonperturbative}. \quad (9)$$

Here the nonperturbative terms should vanish faster than any power of the coupling, but are not in general proportional to m . In particular, a nonperturbative additive shift in the up-quark mass follows qualitatively from the analysis of classical gauge configurations, i.e., “pseudo-particles” or “instantons” [13]. As shown some time ago by 't Hooft [14], these configurations generate an effective multifermion vertex where all flavors of quark flip

their spin. If we take this vertex and tie together the massive quark lines with mass terms, then the resulting process generates an effective mass term for the light quark. This process is illustrated in Fig. 1. The strength of this term is proportional to the product of the masses of the more massive quarks. If there are no additional massive quarks, the scale is set by Λ , the strong interaction scale.

The requirements for the perturbative limit apply at fixed cutoff. Indeed, the interplay of the $a \rightarrow 0$ and the $g \rightarrow 0$ limits is rather intricate. As $g \rightarrow 0$ at fixed a the quarks decouple and we have a theory of free quarks and gluons. The limit $a \rightarrow 0$ at fixed g brings on the standard divergences of relativistic field theory. The proper continuum limit follows the renormalization group trajectory with both a and g going together in the appropriate way and gives a theory with important nonperturbative effects such as confinement.

Assuming only the matching conditions in Eqs. (8) and (9) leaves the freedom to do some amusing things. As a particularly contrived example, consider

$$\tilde{g} = g, \quad (10)$$

$$\tilde{m} = m - Mg^{\gamma_0/\beta_0} \times \frac{e^{-1/2\beta_0 g^2} g^{-\beta_1/\beta_0^2}}{\Lambda a}. \quad (11)$$

The last factor vanishes than any power of g , but is crafted to go to unity along the renormalization group trajectory. Note that a power of the scale factor is necessary for nonperturbative phenomena to be relevant to the continuum limit [14]. With this form, one can immedi-

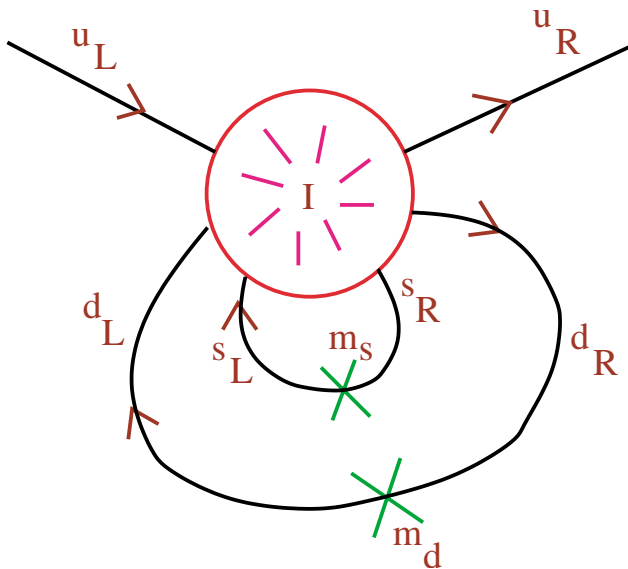


FIG. 1 (color online). Nonperturbative classical gauge configurations will generate an effective mass term for the up quark. The magnitude of this mass is proportional to the product of the heavier quark masses. The precise value, however, is scheme and scale dependent.

ately relate the old and new renormalized masses

$$\tilde{M} \equiv \lim_{a \rightarrow 0} \tilde{m} \tilde{g}^{-\gamma_0/\beta_0} = M - M = 0. \quad (12)$$

Thus for any M , another scheme always exists where the renormalized quark mass vanishes. The possibility of such a transformation is the root of the claim that masslessness is not a physical concept for a nondegenerate quark.

Now I turn to some observations on the relevance of this conclusion to lattice gauge theory. Recently there has been considerable progress with lattice fermion formulations that preserve a remnant of exact chiral symmetry [15]. With such, the multiple flavor theory will preserve the $m_\pi = 0$ contour as the $m = 0$ axis. The important point is that this is not true for the one flavor theory, where the $m_\pi = 0$ flow delves into the negative mass regime. The motivation for extending these chiral fermion actions to the one flavor case seems extremely perverse; indeed, in this situation we do not expect any exact chiral symmetry to survive. But if we are to give a massless quark any scheme independent meaning, this may be the only route. Nevertheless, even with these actions, there is still no reason to expect $M = 0$ to give scheme independent physical masses.

To begin with, the chiral fermion actions are not themselves unique. For example, the overlap operator [16] is constructed by a projection process from the conventional Wilson lattice operator. The latter has a mass parameter which is to be chosen in a particular domain. On changing this parameter, the massless Dirac operator still satisfies the Ginsparg-Wilson relation [17], but this condition does not guarantee that physical particle masses shift in a way that preserves their ratio.

As another way to see that this nonuniversality might be expected, consider that the dynamics of the one flavor case generates a mass gap in the π channel. This means that the eigenvalues of the Dirac operator important to low energy physics are not near the origin, but dynamically driven a finite distance away [18]. Indeed, the absence of massless particles in the regulated theory requires the density of these eigenvalues to vanish at the origin. Changing the projection procedure generating the overlap operator will modify the size of this gap, changing the π mass. If the baryon mass is not equally affected by exactly the same factor, this modification cannot be absorbed in an overall scale factor. The effects on the baryon mass, however, are expected to be less dependent on chiral issues since the baryon remains massive even when several quarks are massless.

Finally, the action used for the pure gauge field is not unique. It has been demonstrated that different gauge actions can strongly modify the topological structures at finite lattice spacing [19]. The effect of such structures is expected to dominate the boson mass generation but play a lesser role for the baryon.

This nonperturbative ambiguity in the quark mass carries over to the explicitly CP violating case where the mass is complex with phase θ . The above discussion shows that even the sign of the up-quark mass can be ambiguous. Thus different schemes can have an ambiguity of θ between 0 and π , a particularly severe example. For other values of θ , to fix the continuum theory uniquely we need to introduce another renormalized quantity. For example, this could be a three meson coupling or the electric dipole moment of a baryon. Beyond the lowest order in the chiral expansion, the precise dependence of the renormalized parameter on θ is scheme dependent.

Although I have phrased the discussion in terms of a mass parameter, the conclusions are unchanged if the mass is generated via a Higgs mechanism involving unifying fields. The Yukawa coupling of the Higgs field to the up-quark receives an additive shift induced by the heavier quarks interacting with nonperturbative gauge fields. This shift gives a vanishing Yukawa coupling a significance similar to a vanishing fundamental quark mass.

In summary, I have argued that the concept of a single massless quark is mathematically ill posed. Admittedly this is not a rigorous proof. But any conclusion depending fundamentally on the concept of a massless up-quark, such as the preservation of CP symmetry in unified theories, should address how the miracle of a vanishing M gives scheme independent physical particle masses.

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