Neutron Stars as Type-I Superconductors

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(Received 18 August 2003; published 16 April 2004)

In a recent paper by Link, it was pointed out that the standard picture of the neutron star core composed of a mixture of a neutron superfluid and a proton type-II superconductor is inconsistent with observations of a long period precession in isolated pulsars. In the following we will show that an appropriate treatment of the interacting two-component superfluid (made of neutron and proton Cooper pairs), when the structure of proton vortices is strongly modified, may dramatically change the standard picture, resulting in a type-I superconductor. In this case the magnetic field is expelled from the superconducting regions of the neutron star, leading to the formation of the intermediate state when alternating domains of superconducting matter and normal matter coexist.

DOI: 10.1103/PhysRevLett.92.151102 PACS numbers: 97.60.Jd, 26.60.+c, 74.25.Qt, 97.60.Gb

The conventional picture of a neutron star is that the extremely dense interior is mainly composed of neutrons, with a small amount of protons and electrons in beta equilibrium. The neutrons form $^3P_2$ Cooper pairs and Bose condense to a superfluid state, while the protons form $^1S_0$ Cooper pairs and Bose condense, as well, to give a superconductor (see, e.g., [1] for a review). It is generally believed that the proton superfluid is a type-II superconductor, which means that it supports a stable lattice of magnetic flux tubes in the presence of a magnetic field. In addition, the rotation of a neutron star causes a lattice of quantized vortices to form in the superfluid neutron state, similar to the observed vortices that form when superfluid He is rotated fast enough. In a recent paper by Link [2], it was pointed out that the precession of the neutron star hints that this picture may not necessarily be correct. In particular, Link states that the observed precession of a neutron star does not allow the proton magnetic flux tubes and neutron vortex lattice to exist simultaneously, due to the fact that the axis of rotation and the axis of the magnetic field are not aligned and the fact that these two different vortices interact quite strongly. Furthermore, Link suggests that the conventional picture of a neutron star as a type-II superconductor may have to be reconsidered. One should remark here that the conventional picture of type-II superconductivity follows from the standard analysis when only a single proton field is considered. As we shall demonstrate in this Letter, if one takes into account that the Cooper pairs of neutrons are also present in the system and that they interact strongly with the proton Cooper pairs, the superconductor may in fact be type I and exhibit the Meissner effect (total expulsion of an external magnetic field), contrary to the picture that is obtained when only the proton Cooper pair condensate is accounted for. This would support the suggestion made by Link [2] that neutron stars may in fact be type-I superconductors with the superconducting region not carrying any magnetic flux.

The core of a neutron star is a mixture of neutron and proton superfluids, as discussed above. In the presence of a magnetic field, it is well known that the type-II proton superfluid may form magnetic flux tubes. Inside the core of these vortices, the proton condensate vanishes, and the core is filled with normal protons resulting in the restoration of the broken $U(1)_{EM}$ symmetry. If the accepted estimates of the proton correlation length and the London penetration depth are used, then the distant proton vortices repel each other leading to formation of a stable vortex lattice. This is the standard picture realized in conventional type-II superconductors. However, there are many situations where this picture will be qualitatively modified. For example, if a second field or component is added, such that there is an approximate SU(2) symmetry between the original and the second fields, it may be energetically favorable for the second field to condense inside the vortex core [3], resulting in a different pattern of the vortex-vortex interaction. This behavior is known to occur in various systems: cosmic strings, high T_c superconductors, Bose-Einstein condensates, superfluid ^{3}He, and high baryon density quark matter (see Refs. [3–7]).

In the case considered in this Letter, we have a situation where there are two condensates, proton and neutron Cooper pairs, both of which are nonzero in the bulk of the matter. In what follows we shall argue that if the interactions between the proton and neutron Cooper pairs at small momentum are approximately equal (a precise condition of "approximately" will be derived below), the vortex-vortex interaction will be modified and the system will be a type-I superconductor with the magnetic field completely expelled from the superconducting regions. We believe that the approximate symmetry of proton/neutron Cooper pair interactions at large distances is somewhat justified by the original isospin symmetry of bare protons and neutrons; however this symmetry is not exactly equivalent to the conventional isotopical SU(2) symmetry. If we consider a proton vortex (magnetic flux tube) in this case, the vortex structure is nontrivial, as we will see below. The core of the proton vortex, where the proton superfluid density goes to zero, has a neutron superfluid density that is larger than at spatial infinity, far from the core. Moreover, the size of the vortex core and the asymptotic behavior of the proton condensate far from the core are also modified due to the additional neutron condensate that is present. The most important result of these effects is that the interaction between distant proton vortices may be attractive in a physical region of parameter space leading to type-I behavior: destruction of the proton vortex lattice and expulsion of the magnetic flux from the superconducting region of the neutron star. We will now elaborate on the ideas outlined above.

We start by considering the following effective Landau-Ginsburg free energy that describes a two-component Bose condensed system. In our system, we have a proton condensate described by the field ψ_1 and a neutron condensate described by the field ψ_2. The ψ_1 field with electric charge q (which is twice the fundamental proton charge, $q = 2|e|$) interacts with the gauge field $\mathbf{A}$, with $\mathbf{B} = \nabla \times \mathbf{A}$. The two dimensional free energy reads (we neglect the dependence on third direction along the vortex)

$$\mathcal{F} = \int d^2x \left[\frac{\hbar^2}{2m_c} (|(\nabla - \frac{iq}{\hbar c}\mathbf{A})\psi_1|^2 + |\nabla\psi_2|^2) + \frac{\mathbf{B}^2}{8\pi} + V(|\psi_1|^2, |\psi_2|^2) \right], \tag{1}$$

where $m_c = 2m$ and m is the mass of the nucleon. Here we have moved the effective mass difference of the proton and neutron Cooper pairs onto the interaction potential V. In the free energy given above, we have ignored the term coupling the proton and neutron superfluid velocities, which gives rise to the Andreev-Bashkin effect [8], as it is not important in our discussions. Indeed, the relevant term in the free energy can be represented as $\sim \int d^3x \vec{v}_1 \cdot \vec{v}_2$, where $\vec{v}_1$ and $\vec{v}_2$ are velocities of the superfluid components. For neutron stars which do not rotate (the case which is considered in this paper), $\vec{v}_2 = 0$, and the effect obviously vanishes. We expect that due to the small density of the neutron vortices (compared to the density of the proton vortices) the effect is still negligible for most of the flux tubes in a rotating star as well. The effect could be important only for a few of the flux tubes situated close to a neutron vortex core, where $\vec{v}_2$ strongly deviates from the constant value at interflux distance scales.

We have also ignored the fact that the neutron condensate has a nontrivial $^3P_2$ order parameter as only the magnitude of the neutron condensate is relevant to the effect described below. The free energy (1) only describes large distances and it does not describe the gap structure on the Fermi surfaces, only the superfluid component of the protons and neutrons.

The free energy (1) is invariant under a $U(1)_1 \times U(1)_2$ symmetry associated with respective phase rotations of fields ψ_1 and ψ_2, which corresponds to the conservation of the number of Cooper pairs for each species of particles. Moreover, we know that the free energy (1) describes particles interacting via the strong nuclear force and, therefore, must be approximately invariant with respect to the SU(2) isospin symmetry. Therefore, we expect that while the Fermi surfaces for protons and neutrons are very different and the gap equations are very different, the interaction between different Cooper pairs at small momentum must not be very different (the asymmetry must be proportional to $(m_d - m_u)$). This asymmetry is expressed in terms of different scattering lengths of Cooper pairs for each species. Thus, we assume, the interaction potential V can be approximately written as $V(|\psi_1|^2, |\psi_2|^2) \approx U(|\psi_1|^2 + |\psi_2|^2)$. In reality this symmetry is explicitly slightly broken, and the potential V has a minimum at $|\psi_1|^2 = n_1, |\psi_2|^2 = n_2$, where n_1 and n_2 are the proton and neutron Cooper pair densities. Hence in the ground state, $|\langle\psi_i\rangle|^2 = n_i \;\; i = 1, 2$, and both U(1) symmetries are spontaneously broken. An important quantity for the analysis that follows will be the ratio of proton to neutron Cooper pair density, $\gamma \equiv n_1/n_2$. A typical value of γ in the core of a neutron star is 5%–15%; thus, in our numerical estimates below we will often use the limit $\gamma \ll 1$, though our qualitative results do not depend on this parameter. (One should remark here that the strong deviation of γ from 1 does not imply a large difference in the interaction and Bose chemical potentials between different species of particles. This is in contrast with Fermi systems where a large difference in densities does imply a large difference in Fermi chemical potentials.)

Now let us investigate the structure of proton vortices, which exist due to the spontaneous breaking of the $U(1)_1$ symmetry. Such vortices are characterized by the phase of the ψ_1 field varying by an integer multiple of 2π as one traverses a contour around the core of the vortex. By continuity, the field ψ_1 must vanish in the center of the vortex core. Up to this point, it has been assumed that the neutron order parameter ψ_2 will remain at its expectation value in the vicinity of the proton vortex. As we have already remarked, this is not the case in many similar systems; see e.g., [3–7].

So, anticipating a nontrivial behavior of the neutron field ψ_2, let us adopt the following cylindrically symmetric ansatz for the fields describing a proton vortex with a unit winding number:

$$\psi_1 = \sqrt{n_1} f(r) e^{i\theta}, \quad \psi_2 = \sqrt{n_2} g(r), \quad \mathbf{A} = \frac{\hbar c}{q} \frac{a(r)}{r} \hat{\theta}, \tag{2}$$

where (r, θ) are the standard polar coordinates. Here we assume that the proton vortex is sufficiently far from any rotational neutron vortices, so that any variation of ψ_2 is solely due to the proton vortex. The functions f, g, and a obey the following boundary conditions: $f(0) = 0$, $f(\infty) = 1$, $g'(0) = 0$, $g(\infty) = 1$, $a(0) = 0$, and $a(\infty) = 1$. We see that the fields ψ_1 and ψ_2 approach their expectation values at $r = \infty$.

We wish to find the asymptotic behavior of fields ψ_1, ψ_2, and $\mathbf{A}$ far from the proton vortex core, as this will determine whether distant vortices repel or attract each other. The asymptotic behavior can be found analytically by expanding the fields defined in (2):

$$f(r) = 1 + F(r), \qquad g(r) = 1 + G(r), \\ a(r) = 1 - rS(r), \tag{3}$$

so that far away from the vortex core, F, G, $rS \ll 1$ and F, G, $S \to 0$ as $r \to \infty$. This allows us to linearize far from the vortex core the equations of motion corresponding to the free energy (1) to obtain

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\begin{pmatrix} F \\ G \end{pmatrix} = \mathbf{M}\begin{pmatrix} F \\ G \end{pmatrix}, \tag{4}$$

$$S'' + \frac{1}{r}S' - \frac{1}{r^2}S = \frac{1}{\lambda^2}S, \tag{5}$$

where the London penetration depth $\lambda = \sqrt{m_c c^2/4\pi q^2 n_1}$. Here all derivatives are with respect to r, and the matrix $\mathbf{M}$ mixing the fields F and G is

$$\mathbf{M} = \frac{4m_c}{\hbar^2}\begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}\begin{pmatrix} n_1 & 0 \\ 0 & n_2 \end{pmatrix}, \tag{6}$$

where the derivatives $V_{ij} \equiv \partial^2 V/(\partial|\psi_i|^2 \partial|\psi_j|^2)$ are evaluated at $|\psi_i|^2 = n_i$. Here we assume that $S^2 \ll F, G$, i.e., the superconductor is not in the strong type-II regime (this is justified since we are only attempting to find the boundary between type-I and type-II superconductivity). The solution to Eq. (5) is known to be $S = (C_A/\lambda)K_1(r/\lambda)$ where K_1 is the modified Bessel function and C_A is an arbitrary constant. The remaining Eq. (4) can be solved by diagonalizing the mixing matrix $\mathbf{M}$. In previous works the influence of the neutron condensate on the proton vortex was neglected, which formally amounts to setting the off-diagonal term V_{12} in $\mathbf{M}$ to 0. In that case, one can assume that the neutron field remains at its expectation value, i.e., $G = 0$, to obtain $F = C_F K_0(\sqrt{2}r/\xi)$, where $\xi = \sqrt{\hbar^2/2m_c n_1 V_{11}}$ is the correlation length of the proton superconductor and K_0 is the modified Bessel function. It is estimated that $\lambda \sim 80$ fm and $\xi \sim 30$ fm [2], which leads to $\kappa = \lambda/\xi \sim 3$ for the Landau-Ginzburg parameter. As is known from conventional superconductors, if $\kappa > 1/\sqrt{2}$, distant vortices repel each other leading to type-II behavior. This is the standard picture of the proton superconductor in neutron stars that is widely accepted in the astrophysics community.

However, the standard procedure described above is inherently flawed since the system exhibits an approximate U(2) symmetry, which forces approximate equality of second partial derivatives, $V_{11} \approx V_{22} \approx V_{12}$. This makes the mixing matrix $\mathbf{M}$ nearly degenerate. The general solution to Eq. (4) is

$$\begin{pmatrix} F \\ G \end{pmatrix} = \sum_{i=1,2} C_i K_0(\sqrt{\nu_i}r)\mathbf{v}_i, \tag{7}$$

where ν_i and $\mathbf{v}_i$ are the eigenvalues and eigenvectors of matrix $\mathbf{M}$, and C_i are constants to be calculated by matching to the solution of the original nonlinear equations of motion. In the limit $\gamma = n_1/n_2 \ll 1$ and $\epsilon = (V_{11}V_{22} - V_{12}^2)/V_{ij}^2 \ll 1$ one can estimate

$$\nu_1 \simeq \frac{2\epsilon}{\xi^2}, \quad \nu_2 \simeq \frac{2}{\gamma\xi^2}, \quad \mathbf{v}_1 \simeq (-1, \gamma), \quad \mathbf{v}_2 \simeq (1, 1). \tag{8}$$

The physical meaning of solution (7) is simple: there are two modes in our two-component system. The first mode describes fluctuations of relative density (concentration) of two components and the second mode describes fluctuations of overall density of two components. Notice that $\nu_1 \ll \nu_2$, and hence the overall density mode has a much smaller correlation length than the concentration mode. Therefore, far from the vortex core, the contribution of the overall density mode can be neglected, and one can write

$$\begin{pmatrix} F \\ G \end{pmatrix}(r \to \infty) \simeq C_1 K_0(\sqrt{2\epsilon}r/\xi) \cdot \begin{pmatrix} -1 \\ \gamma \end{pmatrix}. \tag{9}$$

The most important result of the above discussion is that the distance scale over which the proton and neutron condensates tend to their expectation values near a proton vortex is of order $\xi/\sqrt{\epsilon}$—the correlation length of the concentration mode. Since $\epsilon \ll 1$, this distance scale can be much larger than the proton correlation length ξ, which is typically assumed to be the radius of the proton vortex core.

We have also verified [9] the above results numerically by solving the equations of motions corresponding to (1) with a particular choice of the approximately U(2) symmetric interaction potential V. Our numerical results support the analytical calculations given above. Namely, we find that the magnitude of the neutron condensate is slightly increased in the vortex core, the radius of the magnetic flux tube is of order λ, and the radius of the proton vortex core is of order $\xi/\sqrt{\epsilon}$.

Now that we know the approximate solution for the proton vortex, we will proceed to look at the interaction between two proton vortices that are widely separated. If the interaction between two vortices is repulsive, it is energetically favorable for the superconductor to organize an Abrikosov vortex lattice with each vortex carrying a single magnetic flux quantum. As the magnetic field is increased, more vortices will appear in the material. This is classic type-II behavior. If the interaction between two vortices is attractive, it is energetically favorable for n vortices to coalesce and form a vortex of winding number n, which is expelled from the sample. This is type-I behavior. Typically, the Landau-Ginzburg parameter $\kappa = \lambda/\xi$ is introduced. In a conventional superconductor, if $\kappa < 1/\sqrt{2}$ then the superconductor is type I and vortices attract. If $\kappa > 1/\sqrt{2}$ then vortices repel each other and the superconductor is type II. As mentioned above, the typical value for a neutron star is $\kappa \sim 3$, so we would naively expect that the proton superfluid is a type-II superconductor.

Now we will present three different calculations supporting our claim that for the typical parameters of a neutron star the proton superconductor may be type I rather than type II. First of all, we follow the method suggested originally in [10] to calculate the force between two widely separated vortices. The idea of this method is to model distant vortices as point sources in a free theory, which accurately describes the behavior of fields far from the vortex cores. The methods of [10] were subsequently applied in [11] to the case similar to ours, the interaction of two widely separated vortices that have nontrivial core structure. Using the asymptotic field solutions found above, we follow the procedure of [11] to obtain the following expression for the interaction energy per unit vortex length of two distant parallel vortices:

$$U(d) \simeq \frac{2\pi\hbar^2 n_1}{m_c}[C_A^2 K_0(d/\lambda) - C_1^2 K_0(\sqrt{2\epsilon}d/\xi)], \quad (10)$$

where $d \to \infty$ is the separation between the two vortices. We see that if the first term in U dominates as $d \to \infty$, then the potential is repulsive; otherwise, if the second term dominates the potential is attractive. We introduce the new dimensionless parameter $\kappa_{\rm np} = \sqrt{\epsilon}\lambda/\xi$ into our description. In terms of this parameter, if $\kappa_{\rm np} < 1/\sqrt{2}$, then vortices attract each other and the superconductor is type I; otherwise, vortices repel each other and the superconductor is type II. Therefore, our parameter $\kappa_{\rm np} = \lambda/\delta = \sqrt{\epsilon}\lambda/\xi$ should be considered as an effective Landau-Ginzburg parameter, which determines the boundary between the type-I and type-II proton superconductivity. Because of the importance and far reaching consequences of this result, we have also calculated the vortex-vortex interaction energy in a more direct way following [12]; this calculation [9] produced the same result (10) as the above procedure, therefore confirming our picture. Our third check of the main result that for relatively small ϵ the superconductor in the neutron stars may be, in fact, type I is based on the macroscopical calculation of the critical magnetic fields. Usually one calculates the critical magnetic fields H_c and H_{c2}. We have calculated [9] the critical magnetic fields H_c and H_{c2} corresponding to the free energy (1) with a particular choice of V as a quadratic polynomial in $|\psi_i|^2$. The boundary between type-I and type-II superconductivity obtained using this procedure matches the results of our intervortex force calculation presented above.

The most important consequence of this Letter is that whether proton superconductor is type I or type II depends strongly on the magnitude of the SU(2) asymmetry parameter ϵ. Specifically, we find that the superconductor is type I when $\kappa_{np} = \sqrt{\epsilon}\lambda/\xi < 1/\sqrt{2}$, and type II otherwise. This result is quite generic and not very sensitive to the specific details of the interaction potential V. In particular, when $\epsilon \to 0$ the superconductor is type I. The parameter ϵ is not known precisely; the corresponding microscopical calculation would require the analysis of the scattering lengths (amplitudes at small momentum) of Cooper pairs for different species. We can roughly estimate this parameter as being related to the original SU(2) isospin symmetry breaking $\epsilon \sim (m_d - m_u)/\Lambda_{\rm QCD} \sim 10^{-2}$. If this is assumed to be the value of ϵ, we estimate $\kappa_{\rm np} = \sqrt{\epsilon}\lambda/\xi \sim 0.3 < 1/\sqrt{2}$, which corresponds to a type-I superconductor. From these crude estimates, we see that it is very likely that neutron stars are type-I superconductors with the superconducting region devoid of any magnetic flux, as was originally suggested in [2] to resolve the inconsistency with observations of long period precession in isolated pulsars. If this is the case, some of the explanations of glitches [13] (sudden changes of the neutron star's rotational frequency, see [13] for the original explanation of glitches), which assume a type-II proton superconductor, have to be reconsidered. Type-I superconductivity does not imply total expulsion of the magnetic field: the core structure could be composed of alternating domains of superconducting matter and normal matter [9].

We are grateful to M. Prakash, M. Alford, and J. Lattimer for bringing Ref. [2] to our attention. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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