Incoherent Solitons in Instantaneous Response Nonlinear Media

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We show theoretically and experimentally in an optical fiber system that solitons can be spontaneously generated from incoherent light in an instantaneous response nonlinear Kerr medium. The theory reveals that the unexpected existence of these incoherent solitons relies on a phase-locking mechanism, which leads to the emergence of a mutual coherence between the incoherent waves that constitute the soliton.

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For many years solitons have been regarded as being inherently coherent localized structures, and the recent discovery of incoherent solitons in optics has created a major breakthrough in nonlinear science [1–3]. The incoherent soliton consists of a phenomenon of self-trapping of spatially and temporally incoherent light in a biased photorefractive crystal. Since their first experimental observation in 1996, incoherent solitons have become a blooming area of research [2](for a review see Ref. [3]). The self-trapping of an incoherent beam is possible because of the *noninstantaneous* photorefractive nonlinearity that averages the field fluctuations provided that its response time τ is much longer than the correlation time t_c that characterizes the incoherent beam fluctuations, i.e., $\tau \gg t_c$ [1–3]. In this way the medium responds to the time-averaged intensity and not to the instantaneous speckles that constitute the incoherent beam. Conversely, an instantaneous nonlinearity that responds to the individual speckles will generate multiple filaments that fragment the beam [3]. Then, the generally accepted opinion is that an instantaneous nonlinearity cannot support an incoherent soliton structure.

In opposition with this common belief, we show in this Letter that incoherent solitons exist in instantaneous response nonlinear media. We demonstrate theoretically and experimentally the existence of incoherent solitons in optical Kerr media. Our demonstration is made with silica optical fibers in which the optical Kerr effect has a response time τ of a few femtoseconds. The correlation time t_c of the incoherent fields that we consider is much longer than this response time, and we are thus in a situation opposite to that of the photorefractive incoherent soliton, namely, $t_c \gg \tau$. The new incoherent soliton can be supported by an instantaneous nonlinearity because its structure is quite different from that of the conventional solitons considered in Refs. [1–3]. It consists of a steady-state field envelope that results from a balance between the Kerr nonlinearity and the convection that is due to the group velocity difference between interacting waves. Several solitons of this kind have been extensively investigated in various branches of nonlinear science [4], in particular, in nonlinear optics [5–7]. The type of soliton we deal with here is called domain-wall soliton (DWS) because of its resemblance with the domain walls of ferromagnetic materials [6,7]. We show that, despite the quasi-instantaneous response of the fiber Kerr nonlinearity, these solitons can be spontaneously generated with incoherent optical waves.

The existence of this incoherent soliton relies on a mechanism of phase locking that makes the copropagating wave components of the soliton mutually coherent, regardless of their degree of incoherence. In other terms, each wave component does not contain phase information, but there is a strong phase correlation between distinct wave components. In this respect, the nature of these incoherent solitons is analogous to that of incoherent parametric solitons that have been predicted recently in quadratic nonlinear media [8]. The existence of the new incoherent soliton appears counterintuitive because it involves nonlinear wave interactions in the fully incoherent regime, where $t_c \ll \tau_0$, τ_0 being the characteristic time of nonlinear interaction. This regime is usually treated theoretically through the *random phase approximation* [9], in which the rapidly fluctuating phases are considered as being not significant to the interaction and are thus averaged out. In contrast with this usual approach, by showing the existence of incoherent solitons our theory reveals that coherent phase effects can play an essential role in nonlinear systems of incoherent waves.

Let us consider two beams that counterpropagate in an optical fiber. Each beam is described by two scalar fields that represent their orthogonal polarization components. The optical Kerr effect couples their evolution, and, following a standard procedure [10], one can derive the four coupled equations that govern the evolution of the slowly varying envelopes $E_j^{f,b}$ of their two components $j = 1, 2$ [7]:

$$D_j^{f,b} E_j^{f,b} = (-1)^j 2\gamma \bar{E}_j^{b,f} E_{3-j}^{f,b} E_{3-j}^{b,f} + i\gamma Q_j^{f,b} E_j^{f,b}, \quad (1)$$

where $D_j^f = \partial_t + v_j^f \partial_x$, $D_j^b = \partial_t - v_j^b \partial_x$, v_1^f (v_1^b), v_2^f (v_2^b) being the group velocities of the two components of the forward (backward) wave, respectively. In Eq. (1), $\bar{E}_j^{f,b}$ denotes the complex conjugate of $E_j^{f,b}$, and the parameter γ is the usual nonlinear Kerr coefficient [10]. The first term of the right-hand side of Eq. (1) describes the phase-matched four-wave interaction, while the second term describes a nonlinear phase modulation that can be decomposed into a self-action part and an interaction part, $Q_j^{f,b} = Q_{j,\mathrm{self}}^{f,b} + Q_{j,\mathrm{int}}^{f,b}$, with $Q_{j,\mathrm{self}}^{f,b} = |E_j^{f,b}|^2 + 2|E_{3-j}^{f,b}|^2$ and $Q_{j,\mathrm{int}}^{f,b} = 2(|E_j^{b,f}|^2 + |E_{3-j}^{b,f}|^2)$.

The system of Eq. (1) admits a great variety of coherent soliton solutions $E_j^{f,b} = A_j^{f,b}(x,t)$, $j = 1, 2$ [see, e.g., Fig. 1(a)], that propagate at some velocity V [6]. To obtain a physical insight into incoherent solitons, let us first consider the idealized situation where only the phase of the fields fluctuates randomly; i.e., $E_{j,0}^{f,b} = A_j^{f,b} \times \exp[i\phi_j^{f,b}(x,t)]$, where $\phi_j^{f,b}(x,t)$ are random functions and $A_j^{f,b}$ are the known coherent soliton envelopes. Substitution of these expressions into Eq. (1) readily shows that $E_{j,0}^{f,b}$ are solutions provided that the random phases $\phi_j^{f,b}$ satisfy the phase-matching condition $\phi_1^f + \phi_1^b = \phi_2^f + \phi_2^b$ and that they travel with the constant velocity $v_j^{f,b}$; i.e., they have the form $\phi_j^f(x - v_j^f t)$, $\phi_j^b(x + v_j^b t)$. Clearly, these conditions are satisfied for any realization of the random functions $\phi_j^{f,b}$, provided that $\phi_1^f = \phi_2^f$, $\phi_1^b = \phi_2^b$ and $v_1^f = v_2^f$, $v_1^b = v_2^b$.

This simple observation reveals that incoherent soliton solutions to Eq. (1) exist when the group velocities of the copropagating waves are *matched*. Conversely, when the velocities are not matched, the random phases prevent any *coherent wave interaction,* and solitons can no longer be generated [8]. The existence of incoherent solitons can be generalized to envelope amplitude fluctuations, so that

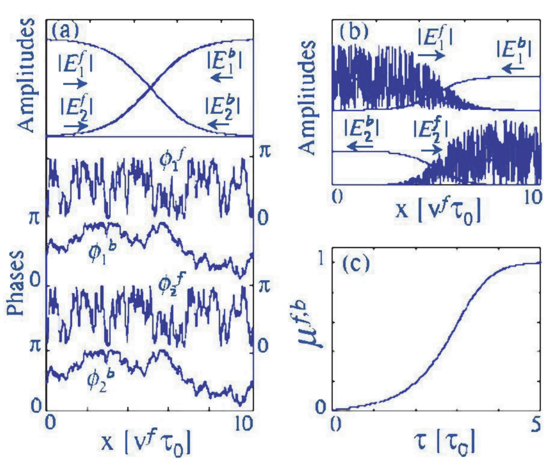

FIG. 1 (color online). (a) Amplitudes $|E_j^{f,b}|$ and phases $\phi_j^{f,b}$ of the incoherent soliton with pure random phase fluctuations. (b) Soliton composed of two incoherent ($E_{1,2}^f$), and two coherent ($E_{1,2}^b$) waves. (c) Evolution of the degree of mutual coherence $\mu^{f,b}(\tau)$ [from Eq. (4)] between the copropagating waves E_1^f and E_2^f ($\tau_0 = 1$ ns). In (a) and (b), $v_1^f = v_2^f$, $v_1^b = v_2^b$.

they can be *generated spontaneously* from incoherent waves thanks to a phase-locking mechanism that cancels out the phase difference of the copropagating waves. Let us note that, in this way, the propagation properties of the incoherent soliton (e.g., its velocity V) result to be the same as that of their coherent counterpart. We verified the existence of incoherent solitons by numerical integration of Eq. (1). Figures 1(a) and 1(b) show an incoherent DWS generated from incoherent waves entering a piece of optical fiber from both ends. As for the coherent DWS, the soliton consists of a transition between two domains of stable mutual polarization arrangements of the counterpropagating waves [6]. Figure 1(a) reveals that the phases of copropagating waves are locked and thus follow the same random fluctuations ($\phi_1^f = \phi_2^f$, $\phi_1^b = \phi_2^b$), while the intensity profiles $|E_j^{f,b}|$ are identical to those of the analytical soliton solution.

Let us now show that this mutual coherence between the copropagating waves *emerges spontaneously* in the system. This result is important for the generation of the incoherent DWS in our experiment, which is achieved in practice by means of two counterpropagating pump waves $E_1^{f,b}$ [6,7]. This configuration is parametrically unstable with respect to the growth of the daughter waves $E_2^{f,b}$ from noise fluctuations. The initial waves are thus *mutually incoherent,* and the theory we are going to present shows that the copropagating waves become mutually coherent as a result of their parametric interaction.

We developed the theory for the simpler situation where the soliton is composed of two incoherent and two coherent waves, as shown in Fig. 1(b). It is obvious that this situation constitutes a particular case of the incoherent DWS, since it can be obtained by increasing the correlation time of the incoming backward wave E_1^b to infinity. Furthermore, we consider the linear regime of parametric interaction where the pump waves $E_1^{f,b}$ are assumed to be unaffected by the daughter waves $E_2^{f,b}$. It proves convenient to analyze the evolution of the fields in the reference frame of the incoherent pump E_1^f ($z = x - v_1^f t$, $\tau = t$), where the pump intensity $|E_1^f|^2(z)$ is a stochastic function of the single variable z. In the following, we assume that the stochastic fields obey a Gaussian statistics and that the pump E_1^f is of zero mean ($\langle E_1^f(z,\tau)\rangle = 0$) and translationally invariant with the mean intensity $\langle |E_1^f|^2\rangle = |E_1^b|^2 = e_0^2$. The linearized Eq. (1) then yield the following equations for the instantaneous mutual coherence functions $\Gamma^{f,b}(z,\tau) = E_1^{f,b}(z,\tau)\bar{E}_2^{f,b}(z,\tau)$:

$$\partial\Gamma^b/\partial\tau - w\partial\Gamma^b/\partial z = \frac{2}{\tau_0}\bar{\Gamma}^f - \frac{i}{\tau_0}\Gamma^b, \tag{2}$$

$$\partial\Gamma^f/\partial\tau = \gamma(2\bar{\Gamma}^b - i\Gamma^f)|E_1^f|^2(z), \tag{3}$$

where $\tau_0 = 1/(\gamma e_0^2)$. In order to make possible the phase locking of the copropagating waves, we implicitly assumed that their velocities are matched, $v_{1,2}^f = v^f$, $v_{1,2}^b = v^b$, so that $w = v^f + v^b$.

The equations for $\Gamma^{f,b}$ can be treated by means of the spatial Fourier expansion $\Gamma^{f,b}(k,\tau)$ of the mutual coherence functions, whose ensemble average satisfies $\langle\Gamma^{f,b}(k,\tau)\rangle = \Lambda^{f,b}(\tau)\,\delta(k)$, where $\Lambda^{f,b}(\tau) = \langle E_1^{f,b}(z,\tau)\bar{E}_2^{f,b}(z,\tau)\rangle$ is the (spatial) average mutual coherence function at time τ. Now, by following the usual procedure based on the property of factorizability of stochastic Gaussian fields [11], one can achieve a closure of the hierarchy of the moments' equations for $\langle\Gamma^{f,b}(k,\tau)\rangle$. The analysis reveals that when $v^f t_c \ll w\tau_0$ the intensity fluctuations $|E_1^f|^2$ of the incoherent pump can be averaged out by the convection w, whenever the pump fluctuations are sufficiently rapid (small values of t_c). This effect of convection-induced averaging of pump fluctuations has been widely discussed in Refs. [8]. It reveals, in particular, that the incoherent soliton cannot be generated if the pump is not sufficiently incoherent. In the present case, the averaging process takes place efficiently thanks to the large convection inherent to the counterpropagating configuration of the interaction ($w \simeq 2v^f$). As a result, if $t_c \ll \tau_0$, the equations for $\Lambda^{f,b}(\tau)$ are *similar* to those obtained in the presence of a fully coherent pump: $d\Lambda^f/d\tau + \frac{2i}{\tau_0}\Lambda^f = \frac{2}{\tau_0}\bar{\Lambda}^b$, $d\Lambda^b/d\tau + \frac{i}{\tau_0}\Lambda^b = \frac{2}{\tau_0}\bar{\Lambda}^f$. Their solutions read $\Lambda^f(\tau) = U(\tau)\Lambda_0^f + V(\tau)\bar{\Lambda}_0^b$, $\Lambda^b(\tau) = \exp(i\tau/\tau_0)[U(\tau)\Lambda_0^b + V(\tau)\bar{\Lambda}_0^f]$, where $\Lambda_0^{f,b} = \Lambda^{f,b}(\tau = 0)$ and $U = \exp(-i\tau/2\tau_0)[\cosh(\sqrt{7}\tau/2\tau_0) - i(3/\sqrt{7})\sinh(\sqrt{7}\tau/2\tau_0)]$, $V = (4/\sqrt{7})\exp(-i\tau/2\tau_0)\times\sinh(\sqrt{7}\tau/2\tau_0)$. To calculate the evolution of the degree of mutual coherence between the copropagating waves, we normalize $\Lambda^{f,b}$ as $\mu^{f,b}(\tau) = |\Lambda^{f,b}(\tau)|/[\langle|E_1^{f,b}|^2\rangle\times(\tau)\langle|E_2^{f,b}|^2\rangle(\tau)]^{1/2}$ [11]. Noting that $\frac{d}{d\tau}\langle|E_2^{f,b}|^2\rangle = 2\gamma(\Lambda^f\Lambda^b + \bar{\Lambda}^f\bar{\Lambda}^b)$, one obtains, after integration,

$$\mu^{f,b}(\tau) = \frac{|\Lambda^{f,b}(\tau)|}{[|\Lambda^{f,b}(\tau)|^2 + q^{f,b} - |\Lambda^{f,b}(0)|^2]^{1/2}}, \quad (4)$$

where $q^{f,b} = e_0^2\langle|E_2^{f,b}|^2\rangle$ $(\tau = 0)$. The function $\mu^{f,b}(\tau)$ exhibits a monotonic growth and tends asymptotically to the unity value, $\mu^{f,b} \to 1$, as illustrated in Fig. 1(c). This means that the copropagating waves become mutually coherent during the interaction; i.e., E_2^b becomes coherent, whereas E_2^f remains incoherent but becomes mutually coherent to E_1^f. Note that, due to the nonlinear coupling, the assumption of Gaussian statistics is known to break down for large interaction lengths, so that the mutual coherence achieved by the copropagating waves may no longer be complete, but only partial.

We first investigated in our experiment (see Fig. 2) incoherent DWS composed of two coherent $E_{1,2}^b$ and two incoherent $E_{1,2}^f$ waves, as shown in Fig. 1(b). The counterpropagating beams were obtained from a Q-switched frequency-doubled Nd-doped yttrium aluminum garnet (Nd:YAG) laser, emitting 6.5-ns pulses at $\omega_p = 563.6$ THz. The laser output was subsequently split into two beams. The first beam was used as the *coherent* backward pump, whereas the *incoherent* forward pump was generated from the amplified spontaneous emission (ASE) of a dye amplifier pumped by the second Nd:YAG beam. A diffraction grating followed by an adjustable slit allowed us to control the spectral width of the incoherent pump, whose central frequency was around $\omega_s = 513$ THz. The backward coherent (forward incoherent) beam E_1^b (E_1^f) is injected into the fiber with a left- (right-) hand circular polarization (CP). The daughter wave E_2^b (E_2^f) thus corresponds to the right- (left-) hand circularly polarized component of the backward (forward) beam. In order to satisfy the velocity matching condition required for the phase-locking mechanism, the fiber has ultralow birefringence (spun fiber), which implies that the CP's are polarization eigenstates of identical velocities ($v_1^f = v_2^f$, $v_1^b = v_2^b$).

Figure 3 shows the experimental output spectra (in logarithmic scale) in the right (E_1^f) and left (E_2^f) CP components of the incoherent forward beam, when the backward pump E_1^b is not injected into the fiber 3(a), and when it is launched into the fiber 3(b). The spectral width of the incoherent pump was 3 THz and the peak powers of both beams were fixed to 180 W. As can be seen in Fig. 3(a), with a single beam in the fiber ($E_1^b = 0$), the input right CP component of the forward beam is maintained unchanged at the fiber output ($E_2^f = 0$), as can be expected from the fact that a circular polarization is a stable eigenstate of the system [6,7]. When both waves counterpropagate, we clearly see in Fig. 3(b) a significant polarization switching (from the right to the left CP components) induced by the backward pump E_1^b. This polarization switching is associated with the generation of a DWS [7]. During this process, E_2^f acquires the same coherence as the pump E_1^f, and, as will be discussed later, the two waves become mutually coherent. Incoherent DWS have been observed for a large range of spectral widths of the incoherent pump ($0.3 \leq \Delta\nu \leq 3$ THz) with a degree of polarization switching almost independently on $\Delta\nu$, in agreement with theory. Let us note that the incomplete polarization switching visible in Fig. 3(b) is due to the short fiber length ($L = 1.2$ m), which was

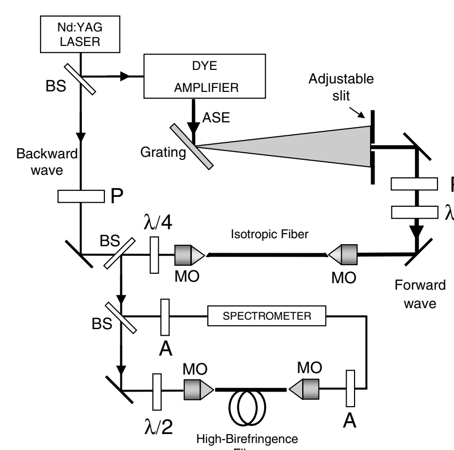

FIG. 2. Schematic of experimental setup. P, polarizer; A, analyzer; MO, microscope objective; BS, beam splitter.

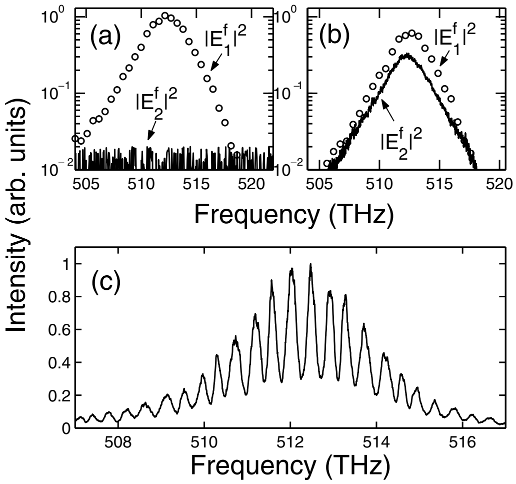

FIG. 3. Experimental generation of the incoherent DWS: output spectra in the right (empty dots) and left (solid curve) CP components of the forward incoherent beam, when the backward coherent pump is turned (a) off and (b) on. Experimental demonstration of the mutual coherence between the incoherent components (E_1^f, E_2^f) of the output forward beam: typical output spectrum from the PM fiber, modulated because of interference between E_1^f and E_2^f.

limited by the requirement of achieving a full spatial superposition of the two pulses over the whole fiber.

To study the generation of the fully incoherent DWS, in which both the forward and backward beams are incoherent, the ASE beam at the dye amplifier output was split into two equal intensity beams that were used as backward and forward waves. The spectral width of both waves was fixed to 3 THz ($t_c \simeq 300$ fs). In these conditions we obtained a degree of polarization switching roughly equal to that obtained with a coherent backward wave [Figs. 3(a) and 3(b)]. Let us emphasize that the generation of DWS takes place in the *fully incoherent regime of interaction* because the characteristic interaction time $\tau_0 = 1/\gamma e_0^2$ is in the nanosecond range ($\gamma = 3.4 \times 10^6\ \mathrm{W}^{-1}\,\mathrm{s}^{-1}$), so that $t_c/\tau_0 \simeq 2 \times 10^{-4} \ll 1$.

To demonstrate the phase-locking mechanism we measured the mutual coherence of the copropagating waves at the fiber output. To this end we used a frequency domain interferometry technique whose principle is explained in Ref. [12]. This technique exploits the propagation in a high birefringence fiber (see Fig. 2) to perform a spectral interference between E_1^f and E_2^f. Figure 3(c) shows a typical spectrum, whose strong modulation demonstrates that the two incoherent waves have acquired a high degree of mutual coherence.

In summary, we showed both theoretically and experimentally that solitons can be generated from incoherent light in an instantaneous response nonlinear Kerr medium. These incoherent solitons exist owing to a phase-locking mechanism that is responsible for the spontaneous emergence of a mutual coherence between the incoherent waves. Importantly, the mutual coherence is shown to arise in the fully incoherent regime of interaction ($t_c \ll \tau_0$), where one usually applies the random phase approximation, which implicitly assumes the waves to be mutually incoherent during the interaction [9]. In contrast with this standard approach, we showed that the wave system exhibits critical coherent features in spite of the incoherence of the fields. Besides the context of optics, the new incoherent solitons are relevant to many branches of nonlinear science. For instance, our results are of potential interest to the emerging area of multicomponent Bose-Einstein condensates (BEC). Considering that partially condensed BEC at finite temperature can be described by the Gross-Pitaevskii equation with incoherent matter waves [13], we can reasonably infer the existence of these incoherent solitons in spinor BEC's in which resonant four-wave interaction in the backward configuration has recently been studied [14].

[1] M. Mitchell *et al.*, Phys. Rev. Lett. **77**, 490 (1996); M. Mitchell and M. Segev, Nature (London) **387**, 880 (1997).
[2] See, e.g., D. N. Christodoulides *et al.*, Phys. Rev. Lett. **78**, 646 (1997); Phys. Rev. Lett. **80**, 5113 (1998); N. N. Akhmediev and A. Ankiewicz, Phys. Rev. Lett. **83**, 4736 (1999); B. Hall *et al.*, Phys. Rev. E **65**, 035602 (2002).
[3] M. Segev and D. N. Christodoulides, in *Spatial Solitons*, edited by S. Trillo and W. Torruellas (Springer, Berlin, 2001).
[4] D. J. Kaup, A. Reiman, and A. Bers, Rev. Mod. Phys. **51**, 275 (1979).
[5] See, e.g., K. Drühl *et al.*, Phys. Rev. Lett. **51**, 1171 (1983); E. Picholle *et al.*, Phys. Rev. Lett. **66**, 1454 (1991); S. Trillo, Opt. Lett. **21**, 1111 (1996); A. Picozzi and M. Haelterman, Phys. Rev. Lett. **84**, 5760 (2000).
[6] V. E. Zakharov and A. V. Mikhailov, JETP Lett. **45**, 349 (1987); S. Wabnitz and B. Daino, Phys. Lett. A **182**, 289 (1993). For a review, see D. David, D. D. Holm, and M. V. Tratnik, Phys. Rep. **187**, 281 (1990).
[7] S. Pitois, G. Millot, and S. Wabnitz, Phys. Rev. Lett. **81**, 1409 (1998); J. Opt. Soc. Am. B **18**, 432 (2001).
[8] A. Picozzi and M. Haelterman, Phys. Rev. Lett. **86**, 2010 (2001); A. Picozzi, C. Montes, and M. Haelterman, Phys. Rev. E **66**, 056605 (2002).
[9] V. N. Tystovich, *Nonlinear Effects in Plasma* (Plenum, New York, 1970); A. Hasegawa, Phys. Fluids **20**, 2155 (1977).
[10] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, New York, 2001).
[11] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).
[12] G. Millot and J. M. Dudley, Appl. Opt. **41**, 2589 (2002).
[13] For example, M. Davis *et al.*, Phys. Rev. Lett. **87**, 160402 (2001).
[14] J. Heurich *et al.*, Phys. Rev. A **63**, 033605 (2001).