# Radiative Intermittent Events during Fermi's Stochastic Acceleration

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(Received 28 October 2003; published 8 April 2004)

We investigate the dynamics of a realization of Fermi's relativistic acceleration mechanism that is a charged test particle oscillating between two reflecting plates that move stochastically. By allowing the charge to radiate energy during each collision, we find that the main features of the system are (1) due to the radiation drag the energy gained by the particle is bounded, and (2) the radiated energy represents a typical realization of an on-off intermittent process, due to numerous continuous encounters with a very small emission, interrupted by short and intense bursts of radiation. This intermittent radiative process exhibits non-Gaussian statistical features.

DOI: 10.1103/PhysRevLett.92.143901 PACS numbers: 41.60.-m, 98.70.Sa

Aimed at understanding both the very high energy of cosmic rays and the statistical behavior of their occurrence [1], more than half a century ago Fermi [2] proposed a simple model that describes the acceleration of charged particles to very high energies. The model is based on the estimation of the probability of the occurrence of stochastic encounters of a charged particle with magnetized interstellar clouds. Fermi's (stochastic) acceleration and some successive variations, including acceleration at shocks (see, e.g., Ref. [3]), are now considered as the classical mechanism capable of explaining how particles can be accelerated to very high energies [1].

The acceleration of particles is a ubiquitous phenomenon in the astrophysical realm, where we observe radiation released by moving charges. The emitted radiation represents therefore the basic diagnostic tool to investigate the physical properties of the environments where radiation itself has been emitted [4]. As an example among many others, consider the case of solar flares [5] where magnetic energy is released within current sheets, and isolated impulsive events due to particles that are accelerated within the flaring region are observed [6]. The statistics of isolated events exhibit power laws for the total emitted energy and the duration of bursts, as well as for the waiting times between bursts [7,8].

While standard radiative mechanisms have been extensively studied, radiation from accelerated or decelerated charges within a stochastic, or turbulent, environment has been less investigated. In this Letter we analyze a mechanism that is quite relevant in an astrophysical context, namely, the realization of the stochastic Fermi's acceleration, which we present in the framework of a relativistic *bouncing ball* model. Unlike the original model of Fermi, where a charged particle can be accelerated without limits, we consider for the charge the possibility of a drag of energy due to radiation. We show that, in this case, the total energy is bounded and the radiation is emitted as isolated bursts of energy, being the "complex" realization of an on-off intermittent process [9]. This radiative process exhibits non-Gaussian statistical features.

We consider the bouncing ball model that dates back from Fermi [10], that is, a charged particle of mass $m$ that moves between two reflecting plates of mass $M \gg m$. The plates are separated by a distance $\ell$ that cannot be zero and moves independently. If $V$ and $u$ are the velocity of the plate and the velocity of the charge before a collision, respectively, the momentum $p_a$ after a collision is related to the momentum $p_b$ before the collision by the relativistic relationship

$$p_a = -p_b \gamma^2(V)\left(1 + \frac{V^2}{c^2} - 2\frac{V}{u}\right), \qquad (1)$$

where $\gamma(V) = (1 - V^2/c^2)^{-1/2}$. The difference in energy before and after a collision $\Delta E = (2\gamma^2 E/c^2) \times (V^2 - \sigma|u||V|)$ ($E$ being the energy of the charge before the collision) depends on the relative sign $\sigma = uV/|u||V|$ between the charge speed and the plates during the collision. In this Letter we suppose that the sign $\sigma = \pm 1$ occurs stochastically for each event, due to a stochastic motion $V(t)$ of the plates. The probability of a collision with a given $\sigma$ value is proportional to $P(\sigma) \sim \nu_c \ell$, where $\nu_c \sim |u - V|/\ell$ is the frequency of collision (the distance $\ell$ is the mean-free path of the particle). Head-on collisions ($\sigma = -1$) increase the energy of the charge by a fraction $\Delta E$, while tail-on collisions ($\sigma = 1$) decrease the energy of the charge. It is easily realized that collisions do not have the same probability [2], rather $P(\sigma = -1) > P(\sigma = 1)$.

The average energy exchanged after a sequence of a large number of collisions, for a given realization of stochastic signs, is

$$\langle \Delta E \rangle = \frac{2\gamma^2(V)E}{c^2}\left(V^2 - |u||V|\sum_\sigma \sigma P(\sigma)\right).$$

In the limit of high speed [$|u| \gg |V|$ and $\gamma(V) \simeq 1$] the rate of gain of energy $dE/dt \sim \langle \Delta E \rangle/\tau$ ($\tau \sim \ell/c$ is the

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time interval between two collisions) results to be proportional to the energy itself $dE/dt \sim (4V^2/c\ell)E$. The charge energy increases, therefore, exponentially with time at a rate $4V^2/\ell c$. On the other hand, it is a simple matter to show that, in the nonrelativistic case, $\langle \Delta E \rangle = 4mV^2$, so that the total energy of the charge increases only linearly in time. It is worth noting that, in the work of Fermi, the distinction between nonrelativistic and relativistic cases is substantial. In fact, a power law $N(E_{ex}) \sim E_{ex}^{-\chi}$, for the number of cosmic rays that escape from the acceleration region with an energy $E_{ex}$ (as actually observed [1]) can be found only in the relativistic case. In the bouncing ball model, the scaling exponent $\chi \sim 1 + \ell c/4V^2\tau_e$ turns out to be proportional to the inverse of an "exit time" $\tau_e$ of particles from the acceleration region.

By introducing a discrete variable $n$ that counts successive collisions, the bouncing ball model can be written as a nonlinear map for the speed of the charge. Indeed, the momentum $p_n^*$ of the charge after the $n$th collision is calculated from (1) as a function of both the speed $u_{n-1}$ and the momentum $p_{n-1}$ of the charge after the $(n-1)$th collision. The energy of the charge is given by $E_n^* = (1 + p_n^{*2})^{1/2}$ and its speed by $u_n^* = p_n^*/E_n^*$.

Let us consider now the possibility that the charge radiates energy, and let us see what kind of radiation we can get during each collision with plates. The rate of change of radiated energy $dQ/dt$ is proportional to the square of the charge acceleration (cf., e.g., Ref. [4]). In our model the charge can radiate energy only during each quasi-instantaneous collision. In this case [4], the variation of radiated energy is proportional to the square of the variation of the charge speed in the collision, that is $\Delta Q \sim -\mu(\Delta u)^2$, where $\mu$ is a free parameter. However, a comparison with Larmor's formula [4] suggests that $\mu$ should be related to the time scale associated with the collision. This observation provides a basis for estimating the value of $\mu$ in specific applications of the model. The energy radiated during each collision can be introduced in the model by recalculating the speed of the charge after the $n$th collision as $u_n = s_n|u_n^* - \mu(u_n^* - u_{n-1})^2|$ ($s_n$ is the sign of $u_n^*$). Afterwards, we calculate the energy $E_n = (1 - u_n^2)^{-1/2}$ and the momentum $p_n = u_n E_n$, and we collect the radiated energy $Q_n = E_n^* - E_n$, as a function of the sequence of collisions.

We have performed numerical simulations of the acceleration process, each plate moving according to an uncorrelated random walk $V(t)$. As a reference, we first report results of simulations obtained without taking into account any process of energy radiation, that is, $\mu = 0$. In Fig. 1 we show the momentum and the energy $E_n^*$ as a function of $n$, obtained for a given realization of $V(t)$. We observe some periods where the momentum has very sharp variations and the charge both gains and loses energy (due to sharp acceleration and deceleration periods), thus generating a sequence of bursts. As the number of collisions increases, the frequency of these bursts increases as well. Then, a net acceleration takes place and the charge can be accelerated to an energy that, on average, increases exponentially.

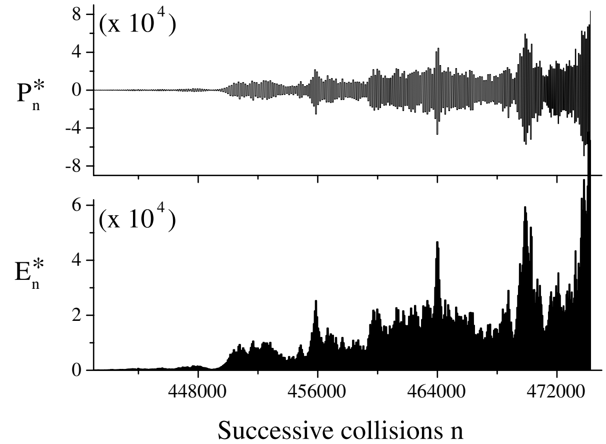


FIG. 1. The momentum (upper panel) and the energy (lower panel), gained by the charge, are reported as a function of successive collisions $n$, in the case $\mu = 0$, for a given realization of $V(t)$.

This behavior changes when we take into account the presence of the radiated energy, that is, $\mu \neq 0$. In this case the charge does not reach high energies as in the previous case, but the acceleration process is made by a sequence of isolated bursts due to rapid changes of the momentum (see Fig. 2). In fact, we observe periods where collisions do not yield to a considerable increase of the charge energy, so that the radiation emitted during these collisions is undetected. In between these periods, we observe very sharp variations of the momentum, with an increase of $|p_n|$. Each single acceleration burst represents a kind of impulsive phenomenon, and the charge

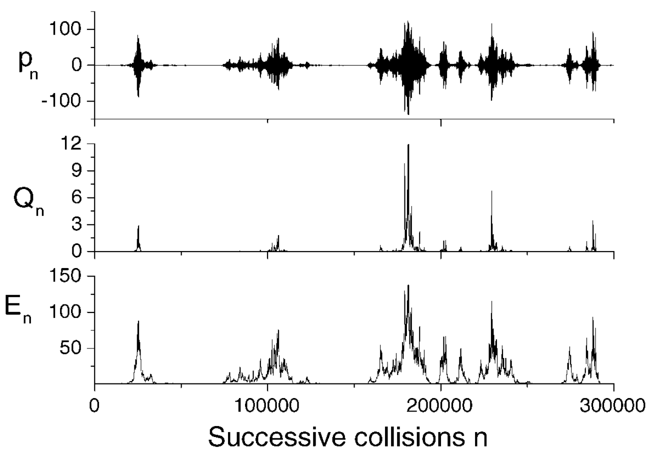


FIG. 2. The momentum (upper panel), radiated energy (middle panel), and energy gained by the charge (lower panel) are reported as a function of successive collisions $n$, in the case $\mu = 10^{-6}$, for a given realization of $V(t)$.

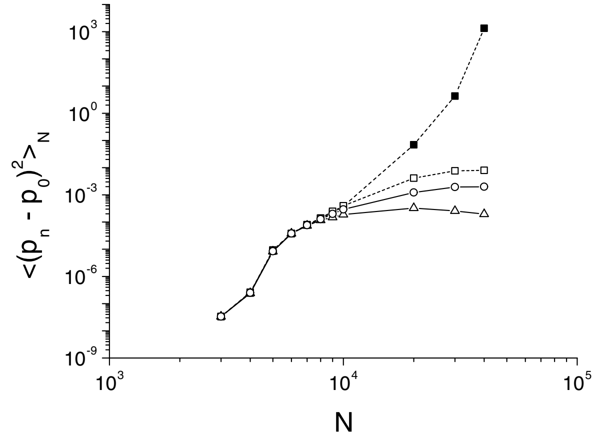


FIG. 3. The function $\langle (p_n - p_0)^2 \rangle_N$ vs $N$ for different values of the parameter $\mu$ namely, $\mu = 0$ (solid squares), $\mu = 10^{-6}$ (open squares), $\mu = 10^{-5}$ (open circles) and $\mu = 10^{-4}$ (open triangles). The average has been taken over $10^5$ realizations of $V(t)$.

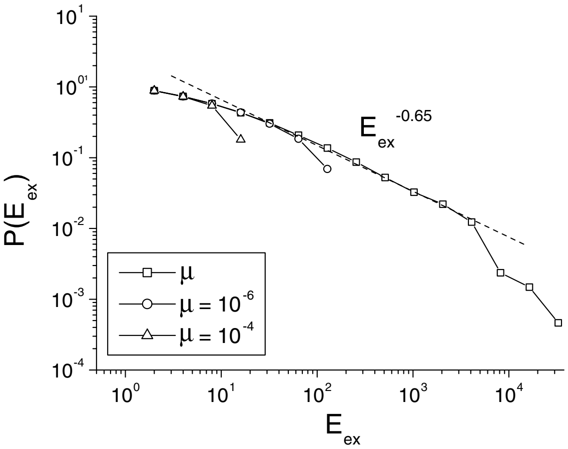


FIG. 4. The probability $P(E_{\rm ex})$ of the exit energy $E_{\rm ex}$, calculated for $10^6$ particles and for three different values of the parameter $\mu$ (reported on the figure). The dashed line refers to a power law with exponent $-0.65$.

energy $E_n$ increases, on average, exponentially. However, when the energy gained by the particle becomes high enough, the radiated energy becomes significantly different from zero, and a burst of radiation starts, thus annihilating the energy gained by the particle. By looking at the sequence $Q_n$ of radiated energy in Fig. 2, we can see it is made by bursts of very short duration, with a sharp rising phase and a slow decay. The observed burst behavior in the bouncing ball model is due to the fact that this model represents a realization of an on-off intermittent process [9]; in this process, a nonlinear map as $y(n) = z_n f(y_{n-1})$ [where $z_n$ comes from a chaotic or random process, and $f(y)$ is a nonlinear function] gives a sequence of isolated bursts due to the presence of an attracting stable subset within the phase space.

The acceleration process can be interpreted as a diffusion in momentum, due to stochastic variations of the velocity of a great number of particles [11]. Statistical information is contained in the average, over an increasing number of collisions $N$, of the squared differences between the actual momentum and the initial momentum of particles, say, $S_N = N^{-1} \sum_{n=1}^{N} (p_n - p_0)^2$. In Fig. 3 we report results obtained for $\mu = 0$ compared to results obtained with three different nonzero values of $\mu$, for $10^5$ different realizations of $V(t)$. At lower $N$ we observe an increase of $S_N$, which is independent of the value of $\mu$. This indicates on average an acceleration of the particles, even when $\mu \neq 0$. At higher $N$ values the function $S_N$ tends to a saturation when $\mu \neq 0$, thus indicating a stop of the diffusive process in the momentum space; this stop yields an upper bound for the energy gained by the charge. When $\mu = 0$ the diffusion of momentum, for higher $N$, produces a more than algebraic increase of $S_N$.

The upper bound for the energy gained by the charge has some consequence with respect to the classical Fermi's problem. We have followed the evolution of $10^6$ particles with each having a different exit time $\tau_e$. This means that each particle is free to collide up to a certain time $\tau_e$, obtained from a Poissonian stochastic process, after which we collect the exit energy $E_{\rm ex}$. The probability distribution of exit energies, shown in Fig. 4, is a power law, as expected. This is true when $\mu = 0$. On the contrary, when $\mu \neq 0$, particles cannot escape from the acceleration region with an energy greater than a given bounded value $E_b(\mu)$ (which depends on $\mu$), so that the probability of obtaining $E_{\rm ex} > E_b$ is zero, as shown in Fig. 4.

It is interesting to infer statistics on the bursts of the radiated energy $Q_n$. In general, the statistical behavior of isolated bursts, collected as a temporal point process [12],

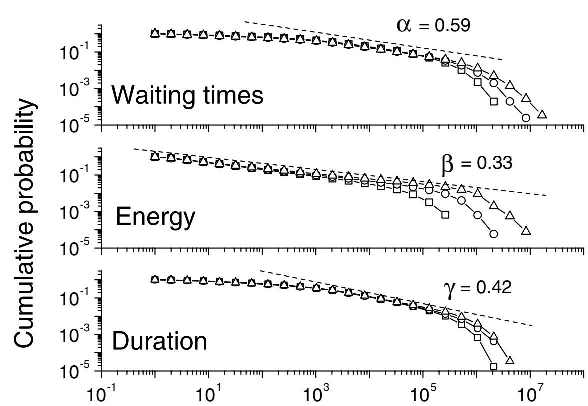


FIG. 5. The cumulative distributions for the waiting times (upper panel), the energy (middle panel), and the duration (lower panel) are reported for a threshold value $Q_{\rm th} = 10^{-3}$. Symbols refer to different values of $\mu$, namely, $\mu = 10^{-4}$ (squares), $\mu = 10^{-5}$ (circles), and $\mu = 10^{-6}$ (triangles). Distributions have been obtained using $10^6$ realizations of $V(t)$. Dashed lines refer to power laws with scaling exponents reported on the figure.

TABLE I. The values of the scaling exponents for duration ($\alpha$), energy ($\beta$), and waiting times ($\gamma$), obtained for $Q_{\rm th} = 10^{-3}$, and different values of the parameter $\mu$.

| | $\mu = 10^{-4}$ | $\mu = 10^{-5}$ | $\mu = 10^{-6}$ |
|---|---|---|---|
| $\alpha$ | $0.62 \pm 0.04$ | $0.62 \pm 0.03$ | $0.59 \pm 0.03$ |
| $\beta$ | $0.38 \pm 0.01$ | $0.35 \pm 0.01$ | $0.33 \pm 0.01$ |
| $\gamma$ | $0.36 \pm 0.02$ | $0.42 \pm 0.01$ | $0.42 \pm 0.01$ |

gives interesting information on the physical mechanism underlying the existence of bursts itself [8,12]. In order to select a burst, we use a standard threshold technique [8]; that is, we state that a burst is happening only when $Q_n$ exceed a given threshold $Q_{\rm th}$. In this way we select a sequence of bursts, and we calculate their total energy $E$ (as the integral of $E_n$), the duration $T$, and the time between two consecutive bursts $\Delta t$ (waiting time). Then, for different realizations of $V(t)$, we calculate the cumulative distribution of the above three quantities, namely $P(E)$, $P(T)$, and $P(\Delta t)$. Results for different values of $\mu$ are shown in Figs. 5. There exists an intermediate range where power laws are recovered for all quantities, that is, $P(E) \sim E^{-\beta}$, $P(T) \sim T^{-\alpha}$, and $P(\Delta t) \sim \Delta t^{-\gamma}$, all of them with scaling exponents lower than 1. As in the on-off mechanism, power laws are due to correlations between successive bursts of the radiated energy. Reference values for the scaling exponents are reported in Table I. They depend only slightly on the value of the parameter $\mu$, and do not significantly change by changing the threshold value $Q_{\rm th}$.

In conclusion, we have investigated the rich dynamics of a relativistic model reproducing Fermi's stochastic acceleration. This represents the first, simple version of a model that can be enriched by considering, for example, the dynamics in two- or in three-dimensional situations. In these cases, the dynamics could be a little bit more complicated, due to the presence of angles of collision, even if the main results remain unchanged (see, e.g., Ref. [13] for a model of the nonrelativistic Fermi's mechanism in two dimensions). In our one-dimensional case we have found that, when the charge is allowed to radiate during collisions with plates, its energy cannot increase without limits (as in the original Fermi's mechanism). Rather, the energy acquired by the particle is bounded and, interestingly enough, the radiated energy results in the realization of an on-off intermittent process; that is, there are long periods of calm between sporadic bursts of radiation. The statistical behavior of these bursts seems to be non-Gaussian, exhibiting power laws for the burst energy, their duration, and the waiting times between successive bursts.

We acknowledge discussions with A. Vulpiani, F. Lepreti, and L. Vlahos. Special thanks to P. Veltri for continuous and active interest in this work, and to C. Umeton for a critical reading of the manuscript. V. C. thanks the Observatoire de la Côte d'Azur for the kind hospitality offered during the final part of this work.

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